Supply Chain Replenishment for Multi-period Newsvendor Products

Chun-Chin Wei †
Department of Marketing and Distribution Management
Chien Hsin University of Science & Technology, Chungli, Taiwan
Tel: (+886) 3-458-1196 ext. 7500, Email: d887801@uch.edu.tw

Liang-Tu Chen
Department of Commerce Automation and Management
National Pingtung University, Pingtung, Taiwan
Tel: (+886) 8-766-3800 ext. 32100, Email: ltchen@mail.nptu.edu.tw

Abstract. Traditionally, a newsvendor problem is a single-period model for a retailer and can be applied in the replenishment decision for a product with a short life cycle. However, many fashionable commodities are seasonal; not all of these products must be sold within a single period of a selling season and they can be replenished once during each cycle. When a selling season consists of multiple ordering cycles, these commodities can be replenished once during each cycle. For example, many shops order commodities, such as computer, communication, consumer electronic, and fashion goods once a week. In this article, a novel multi-period model is developed from the classical single-period newsvendor environment to determine multiple ordering replenishment decisions for a product over a short selling season. This study not only demonstrates profit function of a retailer, but also provides those of both manufacturer and channel-wide for a supply chain problem. Furthermore, the proposed multi-period ordering model provides explicit insights into how the ordering decisions of the retailer are affected at a specific period by considering unsold inventory or unsatisfied demand of a previous period. A numerical analysis and the simulation results illustrate the feasibility of applying the proposed model.

Keywords: Supply chain management, Inventory management, Newsvendor

1. INTRODUCTION

Supply chain management (SCM) is seen as a set of practices aimed at managing and coordinating the whole supply chain from raw material suppliers to end customers and which develop greater synergy through collaboration along the whole supply chain. If supply chain members always make a decision to obtain their own maximal profits from the market, they cannot achieve the coordination. The focus of SCM is now not only limited to increasing the internal efficiency of organizations but has been broadened to include methods of reducing waste and adding value across the entire supply chain. Then, the members can use some incentives, like quantity discount, buyback policies, return policy and sales revenue sharing, etc. to enhance their own profits and other supply chain members' profits. That is, neither the retailer nor the manufacturer would be willing to accept less profit after coordination were achieved than before it were achieved. If the coordination results can increase the profits of the retailer, manufacturer and the channel, the supply chain can be said to be coordinated.

The newsvendor model is a single-period single-product model for a retailer and can be applied on the replenishment decisions for a product with a short shelf or demand life (Khouja, 1999; Peidro et al., 2009). Practical examples are retailers selling seasonal or fashionable goods, newsstands, and food retailers selling dairy products before expiration dates. In the conventional newsvendor problem, the unit selling price, cost, salvage value and shortage penalty of an inventory item, and the density function of the item's stochastic demand are assumed known as prior knowledge (Lau and Lau, 1988). The critical property of these products is a deadline after which selling must stop. The leftover stock becomes worthless if not sold by a specific deadline. The key issue is how to determine the optimal order quantity under a given optimization objective for maximizing the retailer's profit. Numerous studies have investigated this issue, including Nahmias and Morion (1993), Zhou and Yang (2003), Chen and Liu (2008), Su and Pearn (2011), Merzifonluoglu and Feng, (2014), and Wang and Chen (2015).

Many fashionable commodities are seasonal; however, not all of these products must be sold within a single period of a selling season. When a selling season consists of multiple ordering cycles, these commodities can be replenished once during each cycle. For example, many shops order products, such as computer, communication, consumer electronic, and fashion goods once a week. In this case, determining the ordering quantity for each period to maximize the retailer's profit during a short selling season is a critical problem. According to these characteristics, retailers cannot accurately forecast market demand and fluctuating product supply, thus resulting in either excessive inventory or product shortages. Then, retailers must reduce such risk and increase profits by adopting novel contractual agreements with suppliers. Some studies further extended the single-period model to two-period (Taylor, 2001; Lee, 2007) or multi-period problems (Matsuyama, 2006; Perakis and Sood, 2006). In contrast with the conventional single-period newsvendor model, a multi-period newsvendor problem must consider product quantity unsold or unsatisfied demand during a specific period. Uncertain demand not only influences order quantity for a period, it also affects ordering decisions for the subsequent period. Retailers must determine the replenishment level for the next period based on sales during the current period. Hence, some portion of unsold commodities may be stored as inventory and the quantity of unsatisfied demand may be supplemented in the next period.

This study investigates ordering decisions faced by a retailer and a manufacturer in the channel for a commodity in a short life cycle or selling season with multiple ordering periods. A multi-period hierarchical inventory model will be developed based on the newsvendor problem and an ordering cycle with an identical length to examine replenishment decisions and maximize the retailer expected profit. In the multi-period replenishment model, some portion of unsold commodities will be stored and sold in next period or some portion of unsatisfied demand also will be backordered in next period. For understanding the profits of the manufacturer and the entire channel, their profit models simultaneously are also developed.

2. ASSUMPTIONS

This study considers a channel in a supply chain, in which a manufacturer produces a specific commodity for sale by a retailer. The commodity produced has a relatively short life cycle and we assume the retailer can sell it during a short selling season. A selling season H consists of n successive ordering periods. Furthermore, the retailer orders the commodity each period and only places one order to the manufacturer in a period. We assume the manufacturer has unlimited capacity to supply goods. The quantity is delivered to the retailer at the beginning of each period. Generally, the probability density function of demand is a function of selling price. To simplify the model, this paper assumes that selling price is fixed and demand is independent of selling price. The probability density function of demand may be known as a prior knowledge and random demand occurs in each period. No salvage value exists at the end of the selling season. Table 1 lists the circumstances and assumptions of this replenishment problem.

Table 1: Problem the circumstances and assumptions.

Items	Circumstances
Channel structure	two-tier supply chain, including a manufacturer and a retailer
Product characteristic	1. one commodity with a short life cycle and short selling season 2. fixed selling price for the commodity
Objective function	retailer's profit is maximized
Decision variables	replenishment quantity sent by the manufacturer to the retailer
Demand function	uncertain demand
Cost	manufacturing cost, setup cost, goodwill cost, and inventory holding cost
Time interval	1. short selling season (limited planning horizon) 2. multi-period in the selling season 3. same and fixed duration for each period

3. MECHANISMS

The following notations are utilized to formulate the problem.

x_i = total amount of market demand during the i^{th} period, $i = 1, 2, \cdots, n$;
$f(x_i)$ = probability density function of demand x_i;
q_i = ordering quantity by the retailer from the manufacturer at the beginning of the i^{th} period;
l_i = initial inventory level of the retailer at the beginning of the i^{th} period;
p = retailer selling price per unit;

w = manufacturer wholesale price per unit;
c = manufacturing cost per unit;
h = inventory holding cost per unit item and per unit of time paid by the retailer;
g = goodwill cost per unit item and per unit of time due to sellout by the retailer;
CS = retailer setup cost;
MS = manufacturer setup cost;
$EP_R(q_i)$ = retailer expected profit in the i^{th} period;
$EP_M(q_i)$ = manufacturer expected profit in the i^{th} period;
$EP_T(q_i)$ = entire channel expected profit in the i^{th} period.

The relationships between the values are assumed:

$$c < w < p \tag{1}$$

First, we introduce the decision rules in the multiple period situation. The objective is to determine an ordering plan such that the retailer can maximize expected profit as the conventional newsboy model. In the multi-period newsboy model, the retailer determines the ordering quantity at the start of each period based on real sales volume of the previous period based on the traditional wholesales price contract. The following ordering rules are utilized to transform the conventional single-period newsboy model into a multi-period model.

1. If unsold commodities exist in a certain period i, demand x_i is less than the initial inventory level l_i and then some portion of unsold commodities is stored by the retailer (Matsuyama, 2006). Since the retailer can determine the replenishment order for each period, the quantity ordered for the next period must be reduced to sell out the inventory. In this case, the retailer must bear inventory holding cost. That is, the ordering quantity for the next period becomes less than that when the retailer has no inventory.
2. If demand is unsatisfied in a certain period i, demand x_i is larger than the initial inventory level l_i, and the retailer then loses sale opportunities. The goodwill cost is borne by the retailer. Furthermore, the retailer can request that the manufacturer offer additional goods to complement some portion of unsatisfied demand when the customers are willing to wait. The manufacturer is also willing to satisfy the backordered quantity to enhance profits. Then, the quantity ordered for the next period exceeds that when the retailer has no inventory (Pal et al., 2006).

Based on the rules, the models can be formulated iteratively as follows.

3.1 When *i* = 1

Assume that the retailer has no initial inventory before the start of the selling season; the retailer orders q_1 from the manufacturer at the beginning of the selling season. Then, $q_1 = l_1$. The retailer expected profit during the 1$^{\text{st}}$ period, $EP_R(q_1)$, is given as follows:

$$EP_R(q_1) = -wq_1 + \int_0^{l_1}[x_1 p]f(x_1)dx_1 + \int_{l_1}^{\infty}[pq_1 - g(x_1 - q_1)]f(x_1)dx_1 - CS \tag{2}$$

That is, if the demand x_1 is smaller than or equal to l_1, the amount of the product which the retailer can sell is x_1. If the demand x_1 is larger than l_1, the maximum quantity which the retailer can sell is l_1, then the retailer needs to burden the goodwill cost due to sale opportunity losing. In the traditional single-period newsvendor model, to obtain optimal ordering quantity q_1 to maximize the retailer profit, $EP_R(q_1)$ is differentiated with respect to q_1 and the computational result is set to 0. We get

$$F(q_1^*) = \frac{p+g-w}{p+g} \tag{3}$$

And

$$q_1^* = F\left(\frac{p+g-w}{p+g}\right)^{-1} \tag{4}$$

The retailer expected profit, $EP_R(q_1)$, is maximum when the retailer orders quantity q_1 in the classical single-period newsvendor model. On the other hand, the manufacturer expected profit, $EP_M(q_1)$, can be expressed as

$$EP_M(q_1) = (w-c)q_1 - MS \tag{5}$$

To simplify the hierarchical multiple period model, this study does not consider the capacity of the manufacturer.

By combining Eqs. (2) and (5), the total profit for the entire channel is

$$EP_T(q_1) = -cq_1 + \int_0^{l_1}[x_1 p]f(x_1)dx_1 + \int_{l_1}^{\infty}[pl_1 - g(x_1 - l_1)]f(x_1)dx_1 - CS - MS \tag{6}$$

Similarly, in the conventional newsvendor model, by substituting q_1^* into Eqs. (5) and (6), the expected profits of the manufacturer and entire channel when the retailer profit is maximal can be obtained.

3.2 When *i = 2, 3, …, n-1, n*

According to the two ordering rules, we also can transfer the single newsvendor model to a multi-period model. As space is limited, we only present the framework of the methodology as Figure 1.

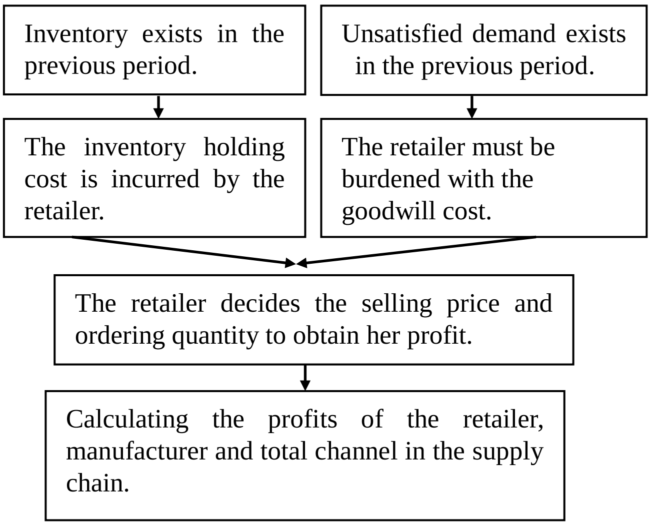

Figure 1: the ordering methodology of the channel coordination.

4. EXAMPLE

The following numerical example may serve to illustrate the concepts. The selling season of a product is 3 months, which consists of 12 periods. The retailer replenishes stock once per week during the selling season. The retail price of the product is 50 and per unit manufacturing cost is 12. The unit wholesale price paid by the retailer is 30. If the retailer has inventory, the unit holding cost per period is 6 and the ratio of amount of stock to the amount unsold stock, α , is 0.9. Conversely, if there is unsatisfied demand in a period, the ratio of amount sold at the beginning of the current period to the amount of unsatisfied demand during the previous period, β , is 0.5, and the goodwill cost per unit is 14 due to sellout incurred by the retailer. In practice, the values of α and β are not constant. They change from period to period. For simplifying the simulation process and understanding the influence of the two variables on the profits of the supply chain members, this simulation assumes their values are fixed. This study assumes the probability density functions of demand x_i of all period are independent and follow normal distribution. Additionally, the means of $f(x_i)$ decline as time goes on, $\mu_x = 500 - 20 \times (i-1)$, $1 \le i \le 12$, and the standard deviation σ_x is 70. The complex procedures for determining the ordering quantity for each period and expected profits of the retailer, manufacturer, and channel were performed using Mathematica in all numerical trials for the multi-period scenarios. Table 2 lists simulation results for the first trial by using the proposed model (variable quantity obtained from the models).

The columns of variable quantity in Table 2 shows the ordering quantity in each period is determined based on the amount of inventory or unsatisfied demand from the previous period by real customer demand. For example, when the retailer initial inventory for Period 3 exceeds customer demand, retailer inventory increases. The quantity ordered for Period 4 is reduced to balance the inventory and the retailer should bear the inventory holding cost. Conversely, when some demand cannot be satisfied in Period 2 and the ordering quantity for Period 3 is increased to fulfill some unsatisfied demand from Period 2. Additionally, the retailer can earn some profits from the backordered quantity in Period 3, but the retailer has to burden the goodwill cost in Period 2. This study also illustrated the profits of the manufacturer and the whole channel for the multi-period model.

Under the same conditions as those in trail 1, ten trials were carried out with various demands. Table 3 shows the total ordering quantity and profits for the retailer, manufacturer and channel under the variable quantity from the proposed models.

Table 2: Simulation results of trial 1.

period (i)	demand (x_i)	variable quantity obtained from models						fixed quantity									
		l_i	q_i	$EP_R(q_i)$	$EP_M(q_i)$	$EP_T(q_i)$		l_i^s	q_i^s	$EP_R(q_i^s)$	$EP_M(q_i^s)$	$EP_T(q_i^s)$	l_i^b	q_i^b	$EP_R(q_i^b)$	$EP_M(q_i^b)$	$EP_T(q_i^b)$
1	502	478	478	8824	8004	16828		300	300	2772	4800	7572	450	450	7872	7500	15372
2	493	457	470	8510	7860	16370		199	300	1484	4800	6284	424	450	7634	7500	15134
3	451	538	556	6345	9408	15753		153	300	1428	4800	6228	416	450	8135	7500	15635
4	427	519	440	7280	7320	14600		151	300	1736	4800	6536	432	450	8325	7500	15825
5	433	398	315	9069	5070	14139		162	300	1806	4800	6606	455	450	7718	7500	15218
6	398	478	496	5495	8328	13823		165	300	2331	4800	7131	470	450	5881	7500	13381
7	377	458	386	6438	6348	12786		183	300	2859	4800	7659	514	450	5339	7500	12839
8	351	439	365	5763	5970	11733		203	300	3528	4800	8328	574	450	2910	7500	10410
9	350	317	239	7412	3702	11114		226	300	3864	4800	8664	650	450	2401	7500	9901
10	322	297	314	5544	5052	10596		238	300	4174	4800	8974	720	450	580	7500	8080
11	295	278	290	5162	4620	9782		258	300	5082	4800	9882	809	450	-1299	7500	6201
12	266	285	294	4505	4692	9197		282	300	5849	4800	10649	912	450	-3370	7500	4130
total			4643	80347	76374	156721			3600	36913	57600	94513		5400	52126	90000	142126

Table 3**:** Simulation results of ten trials.

trial	variable quantity obtained from models				fixed quantity							
	q_i	$EP_R(q_i)$	$EP_M(q_i)$	$EP_T(q_i)$	$q_i^s = 300$	$EP_R(q_i^s)$	$EP_M(q_i^s)$	$EP_T(q_i^s)$	$q_i^b = 450$	$EP_R(q_i^b)$	$EP_M(q_i^b)$	$EP_T(q_i^b)$
1	4643	80847	76374	156721	3600	36913	57600	94513	5400	52126	90000	142126
2	5084	87384	84276	171660	3600	18807	57600	76407	5400	84300	90000	174300
3	4655	81818	76590	158408	3600	37132	57600	94732	5400	53917	90000	143917
4	4787	84866	78943	163809	3600	31323	57600	88923	5400	62763	90000	152763
5	4590	73927	75420	149347	3600	40583	57600	98183	5400	49899	90000	139899
6	4831	72012	79794	151806	3600	24341	57600	81941	5400	52586	90000	142586
7	4801	70205	79218	149423	3600	23112	57600	80712	5400	57799	90000	147799
8	4625	70119	76032	146151	3600	35644	57600	93244	5400	52276	90000	142276
9	5201	87576	86400	173976	3600	14710	57600	72310	5400	75849	90000	165849
10	4705	76541	77490	154031	3600	27884	57600	85484	5400	31968	90000	121968

For comparing the results of the variable quantity obtained from the proposed models with the results of the fixed quantity scenarios, this study also simulated the results of the retailer, manufacturer, and whole channel by using a smaller fixed quantity ($q_i^s = 300$) and a bigger fixed quantity ($q_i^b = 450$). The results of these simulations also are listed in Table 2. Simulation results demonstrate that the total profit of the retailer in the selling season under the proposed variable quantity model is better than that generated by the fixed quantity scenarios.

Similar to trail 1, the profits of the retailer under the variable quantity conditions are better than those under the fixed quantity conditions. The retailer can reduce risk associated with accumulating inventory or undesired shortages by adopting the proposed multi-period model.

5. CONCLUSIONS

This study demonstrated a model for developing the profit of the overall channel and those of both the retailer and manufacturer under a multi-period newsvendor environment. The traditional single-period newsvendor model was effectively extended to a multi-period model for meeting real challenges encountered when dealing with a short product life cycle. The proposed model can handle replenishment decisions for a product with a short selling season or life cycle and allow the retailer to place multiple orders and replenish stock in each period during the selling season. If there are some inventories, then a part of unsold commodities is stored and the retailer will burden the inventory holding cost. The remaining inventory can be sold during the next period. On the other hand, if some unsatisfied demands exist, then the retailer should bear the goodwill cost due to losing sale opportunities. And the retailer can request that the manufacturer offers additional goods to complement some portion of unsatisfied demand when customers are willing to wait. The inventory holding cost and backordering to meet unsatisfied demand are integrated into the proposed models. We ran a series of numerical trials to simulate the proposed model. The results clearly indicate that the proposed multi-period model is effective in the profits of the retailer and outperforms those in the fixed ordering quantity conditions. The proposed multi-period model provides an opportunity for solving the replenishment decision of a short life cycle product with multiple period and fixed ordering duration. Additionally, the proposed model contributes to a research base on the effective integration and cooperation of supply chain members.

ACKNOWLEDGMENTS

The authors wish to thank the National Science Council of the Republic of China, Taiwan, for financially supporting this research under Contracts No. NSC 98-2221-E-231-013 and No. NSC101-2221-E-231-008.

REFERENCES

Chen, S.L., and Liu, C.L. (2008) The optimal consignment policy for the manufacturer under supply chain co-ordination. *International Journal of Production Research*, **46**, 5121-5143.

Khouja, M. (1999) The single-period (news-vendor) problem: literature review and suggestions for future research. *Omega*, **27**, 537-553.

Lau, A.H.L., and Lau, H.S. (1988) The newsboy problem with price-dependent demand distribution. *IIE Transaction*, **20**, 168-175.

Lee, C.H. (2007) Coordination on stocking and progressive pricing policies for a supply chain. *International Journal of Production Economics*, **106**, 307-319.

Matsuyama, K. (2006) The multi-period newsboy problem. *European Journal of Operational Research*, **171**, 170-188.

Merzifonluoglu, Y., and Feng, Y. (2014) Newsvendor problem with multiple unreliable suppliers. *International Journal of Production Research*, **52**, 221-242.

Nahmias, S., and Morion, T.E. (1993) Near myopic heuristic for the fixed-life perishable problem. *Management Science*, **39**, 1490-1498.

Pal, A.K., Bhunia, A.K., and Mukherjee, R.N. (2006) Optimal lot size model for deteriorating items with demand rate dependent on displayed stock level (DSL) and partial backordering. *European Journal of Operational Research*, **175**, 977-991.

Peidro, D., Mula, J., and Poler, R. (2009) Quantitative models for supply chain planning under uncertainty: a review. *International Journal of Advanced Manufacturing Technology*, **43**, 400-420.

Perakis, G., and Sood, A. (2006) Competitive multi-period pricing for perishable products: a robust optimization approach. *Mathematical Programming*, **107**, 295-315.

Su, R.H., and Pearn, W.L. (2011) Product selection for newsboy-type product with normal demands and unequal costs. *International Journal of Production Economics*, **132**, 214-222.

Taylor, T.A. (2001) Channel coordination under price protection, midlife returns, and end-of-life returns in dynamic markets. *Management Science*, **47**, 1220-1234.

Wang, C., and Chen, X. (2015) Optimal ordering policy for a price-setting newsvendor with option contracts under demand uncertainty. *International Journal of Production Research*, **53**, 6279-6293.

Zhou, Y.W., and Yang, S.L. (2003) An optimal replenishment policy for items with an inventory-level-dependent demand rate and fixed lifetime under the LIFO policy. *Journal of the Operational Research Society*, **54**, 585-593.