Maintenance Contract

With Imperfect Preventive Maintenance

In Dynamic Operating Condition

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Abstract. This paper develops a mathematical model for a two-dimensional maintenance contract of a repairable item with imperfect preventive maintenance. The contract coverage is characterized by two parameters – age and usage and also provides preventive maintenance and corrective actions during the contract period. A penalty cost incurred when the time required to perform an imperfect repair exceeds a target. This strategy will reduce equipment failures and hence it decreases the penalty cost and maintenance cost during the contract. We use one dimensional approach to model the age and usage of the item. Finally we find the optimal time between preventive maintenance which maximizes the expected profit using non-cooperative game theory and give a numerical example.

Keywords: maintenance contract, imperfect repair, preventive maintenance, non-cooperative game theory

1. INTRODUCTION

Maintenance contracts have received much attention in the literature. Jackson and Pascual (2008) and Wang (2010) studied maintenance contracts for repairable items, which involve preventive maintenance policies. Those papers studied maintenance contract with consider a penalty based on down time for each failure – i.e. a penalty cost incurs the agent or Original Equipment Manufacturer (OEM) when the actual down time to fix the failed equipment is greater than the target value. Iskandar *et al*. (2014) have studied maintenance contracts with availability as a key measure, where a penalty cost incurs when the actual availability falls below the target (or total down time in a given period exceeds the target).

According to the literature review, as the OEM or an external agent normally offers a variety of maintenance contracts, then the maintenance actions preventive maintenance (PM) and/or corrective maintenance (CM) can be outsourced to the OEM (or an external agent). From the owner's viewpoint, maintenance programs are aimed at not only to reach the performance target (e.g. 90% availability) but also to achieve an optimal profit. In order to reach the optimal profit, the maintenance contract offered by the OEM should not just to ensure the performance target but also to achieve a higher performance which is beyond the target. This in turn will results in optimal profits for both the owner and the OEM. The decision problems for the owner are (i) to select the maintenance contract option that can reach the higher performance of the equipment with reasonable maintenance costs, and (ii) determine an attractive cost. And for the OEM or an external agent the decision problem is to determine the optimal price for each options offered.

Many maintenance contract models studied so far, it is implicitly assumed that the item is in continuous use without considering both external environment and operational modes of the item (Ashgarizadeh and Murthy, 2000; Iskandar et.al., 2013; Iskandar et.al., 2014; Jackson and Pascual, 2008; Mirzahosseinian and Piplani, 2011). The failure rate of an item when in normal condition can be different from that when severe condition. In order to evaluate the maintenance contract costs from a realistic viewpoint, we should study the failure models under various usage patterns and dynamic environment.

In this paper, we focus on maintenance contract cost during the product life cycle for items used in dynamic environment. The item can be either use in normal condition or severe condition and the failure depends on the usage intensity the unit has been used. The usage intensity varies across the population of users and is modeled as a continuous random variable. The product degradation and failure depends on the usage intensity and this in turn has an impact on the expected maintenance contract cost. This needs to be taken into account in determining the price of maintenance contract and the optimal option decisions.

This paper is composed as follows. Section 1 and 2 deal with background and model formulation for the maintenance contracts studied. Sections 3 gives model analysis to obtain the optimal number of preventive maintenance and the optimal maintenance contract profit. In Section 4, we give numerical example to illustrate the model and finally in section 5 we conclude with topics for further research .

2. MODEL FORMULATION

2.1 Maintenance Contract

We consider two maintenance contract options. A high availability (or a low downtime) of the truck is critical factor for achieving a monthly production target of a company. Performance based maintenance contracts offers a penalty cost (i.e. if the actual downtime is above the target) to motivate the original equipment manufacturer (OEM) to increase the performance. Two maintenance contract options are considered as follows.

Option 1(O_1): The owner performs a PM action in-house but a CM action is outsourced to the OEM. Under this option, if the truck fails the owner calls the OEM to fix the truck. The OEM will charge the owner a fixed cost C_S for each repair (CM). No penalty cost incurs the OEM if the downtime caused by a failure falls above the target as the OEM only performs CM.

Option 2(O_2): For a fixed price of maintenance contract P_G, the OEM agrees to perform full maintenance including PM and CM actions during the contract. The OEM promises that the down time for each failure is less then a target value stated in the contract (note that downtime is repair time plus waiting time). As the maintenance is full coverage (PM and CM), then a penalty cost incurs the OEM if the actual down time caused by each failure falls above the target. If the down time for each failure over the contract be X_{ji} is more than the down time target ξ, then the OEM should pay a penalty cost. The penalty cost, C_P is viewed as a compensation received by the owner.

We consider that a company operates an item –i.e a dump truck. The dump truck is offered with a two-dimensional maintenance contract with the contract characterised by a rectangle region $\Omega_T = [0, \Gamma_0) \times [0, U_0)$ where Γ_0 and U_0 are the time and the usage limits (e.g. the maximum coverage for Γ_0 (e.g. 1 year) or U_0 (e.g. 50.000 km) (see Fig.1). For a given usage rate y of the dump truck, the lease contract ceases at $\Gamma_y = \Gamma_0$ for $y \le U_0/\Gamma_0$, or $\Gamma_y = U/y$ for $y > U_0/\Gamma_0$, whichever occurs first. We consider that the maintenance contract given by the OEM also covers PM action, and hence, during the maintenance period CM and PM actions are done by the OEM without any charge to the owner.

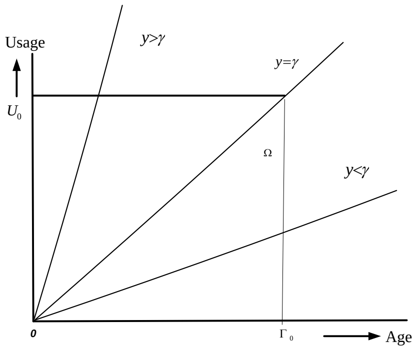

Figure 1. The two-dimensional maintenance contract

As the maintenance contract is full coverage (PM and CM), then a penalty cost incurs the OEM if the actual down time falls above the target ($\mathfrak{T}$) . If $\mathcal{D}$ is down time (consisting repair time and waiting time) for each failure occuring during the contract, then the OEM should pay a penalty cost when $\mathcal{D} > \mathfrak{T}$. The amount of the penalty cost is assumed to be proportional to $\Delta = \mathcal{D} - \mathfrak{T}$. The penalty cost ($\mathcal{C}_{\mathcal{P}}$) is viewed as a penalty given by the OEM. The decision problem for the OEM is to determine the optimal number of PM such that to minimize the expected cost and will lead to maximize the expected profit.

2.2 Failure modelling

2.2.1 Approaches to modelling usage

In this paper, we refer to the model presented by Husniah et.al. (2015). The item is assumed to be used in normal stress and high stress condition. As a result, at time t, $0 \le t \le W$ the item can be either in normal stress(N) or severe or high stress (H). The transitions from N to H and form H to N occur in a random manner . So we model the transitions by a two state continuous time Markov chain formulation $X(t)$. Here $X(t)$=1 if the item is in high stress at time t and $X(t)$=0 if the item is normal stress. Conditional on the usage rate $U=u$, the probabilities $\{X(t+\delta t)=j|X(t)=i\}, 0\le i,j\le 1$ are given by the following matrix:

$$X(t)\begin{cases}\overbrace{\begin{matrix}1 & 1-\lambda_1\delta t & \lambda_1\delta t\\ & & \\ 0 & \lambda_0\delta t & 1-\lambda_0\delta t\end{matrix}}^{X(t+\delta t)}\end{cases}$$

2.2.2 Approaches to modelling failures

For the cost analysis of two-dimensional maintenance contract policies, we model item failure using one dimensional approach. This approach assumes that the usage rate Y varies from customer to customer but is constant for a given customer. For $Y=y$, the conditional hazard function for the time to first failure is given by $r_y(t)$ which is a non-decreasing function of t (the age of the item) and y.

As age and usage are considered as major factors to influence failure, hence we use the accelerated failure time (AFT) model which allows to incorporate the effect of usage rate on degradation of the item. Let y_0 denotes the nominal usage rate value associated with component reliability. Using the AFT formulation, if $T_0[T]$ denotes the time to first failure under usage rate $y_0[y]$ then we have $T_y=(y_0/y)^\eta T_0$ where η indicates the environment of mining, such as hilly, incline or decline land contour. If the distribution function for T_0 is given by $F_0(T, \alpha_0)$, where α_0 is the scale parameter, then the distribution function for T_y is the same as that for T_0 but with a scale parameter given by $\alpha_y=(y_0/y)^\eta\alpha_0$ with $\eta\ge 1$. Hence, we have $F(t,\alpha_y)=F_0((y_0/y)^\eta t,\alpha_y)$.

For a given usage rate y, $r(t|y)\ge 0$ represents the conditional hazard (failure rate) function for the time to first failure and it is a non-decreasing function of the item age t and y. We model failures over time by a counting process. As failed items are repaired are 'minimal' and repair times are negligible, then a conditional intensity function $\lambda(t|y)$ same as a conditional hazard function $r(t|y)$, $\lambda(t|y)=r(t|y)$.

We assume the failure rate function when the unit is in normal stress and in high stress follow the weibull form

$$r_y^i(t)=\frac{\beta}{\alpha_i\left(u/u_0\right)^{\eta_i}}t^{\beta-1}, i=0,1 \tag{1}$$

where i=0 for normal and 1 for high stress.

The failure rate when normal is always less than the failure rate when in high. Using Eqs. (1), we have

$$r_y(t)=r_y^1(t|X(t)=1)p_1(t)+r_y^0(t|X(t)=0)\,p_0(t) \tag{2}$$

where $p_i(t,u)$, $0\le i\le 1$ is the probability the Markov chain $X(t)$ is in state i at time t. From the theory of Markov chains, we have

$$\mathrm{p}_1(t)=\frac{\lambda_0}{\lambda_0+\lambda_1}+\frac{\lambda_1}{\lambda_0+\lambda_1}e^{-(\lambda_0+\lambda_1)t} \tag{3}$$

and

$$\mathrm{p}_0(t)=\frac{\lambda_0}{\lambda_0+\lambda_1}-\frac{\lambda_1}{\lambda_0+\lambda_1}e^{-(\lambda_0+\lambda_1)t} \tag{4}$$

Using Eqs. (3) and (4) in (2), we have

$$r_y(t)=\begin{cases} r_y^1(t|X(t)=1)\left[\frac{\lambda_0}{\lambda_0+\lambda_1}+\frac{\lambda_1}{\lambda_0+\lambda_1}e^{-(\lambda_0+\lambda_1)t}\right] \\ +r_y^0(t|X(t)=0)\left[\frac{\lambda_0}{\lambda_0+\lambda_1}-\frac{\lambda_1}{\lambda_0+\lambda_1}e^{-(\lambda_0+\lambda_1)t}\right]\end{cases} \tag{5}$$

Finally we can obtain $F_y(t)$ and $f_y(t)$,

$F_y(t)=1-\exp\left\{-\int_0^t r_y(t)dt\right\}$ and $f_y(t)=r_y(t)\exp\left\{-\int_0^t r_y(t)dt\right\}$.

To control the degradation of the equipment, PM is conducted regularly during the life cycle. PM can be done in-house or by the OEM (or an agent). We consider that PM done in-house is less effective than that of the OEM, and model the effect of PM through the failure rate function as follows. If $r_{1y}(t)$ represents the failure rate function for a given usage rate y with PM done in-house (Option O_1), and it is given by

$$r_{1y}(t)=\rho r_y(t) \quad 0\le t<\Gamma_y \tag{6}$$

where $\rho>1$. For PM done by OEM, the failure rate function is given by $r_{2y}(t)=r_y(t) \quad 0\le t<\Gamma_y$.

Note that $\rho>1$ meaning that the failure rate function increases with a higher rate or the PM done in-house is less effective than PM by OEM ($\rho=1$).

Preventive Maintenance Policy:

We define periodic PM policy for a given $Y=y$. PM policy for a given y, is characterised by single parameter τ_y. The equipment is periodically maintained at $k.\tau_y$. Any failure occurring between PM is minimally repaired (see Fig. 2). Note $(k+1)\tau_y=\Gamma_0$ where k is an integer value.

Modeling of PM effect:

For a given usage rate y, the effect of PM actions on the intensity function is given by $r(t_j)=r(t_{j-1})-\Delta_j$ with $0\le\Delta_j\le r(t_{j-1})-\sum_{i=0}^{j}\Delta_i$. Δ_j denotes the reduction of the intensity function after j^{th}, $j\ge 1$, PM action. If the PM action is done at $j^{th}, j \geq 1$ the intensity function is reduced by Δ_j, then for $t_j \leq t < t_{j+1}$ the intensity function is given by $r_j(t) = r(t) - \sum_{i=0}^{j} \Delta_i$ with $\Delta_0 = 0$. For simplicity we assume that for each PM action $\Delta_j = \Delta_{j+1} = \Delta$ then $r_j(t) = r(t) - j\Delta$ (See Fig. 3).

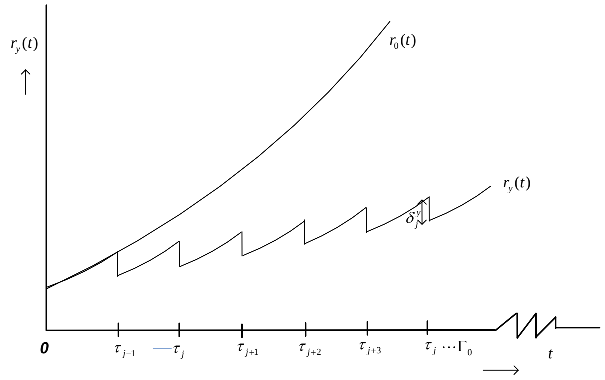

Figure 2. The effect of PM action on failure rate function for $Y = y$

If any failure occurring between pm is minimally repaired, then expected total number of minimal repairs in $([t_{j-1}, t_j), 1 \leq j \leq k_y + 1)$ is given by

$$N = \sum_{j=1}^{k_y+1} \int_{t_{j-1}}^{t_j} r_{j-1}(t')dt' = R(\Gamma_0) - \sum_{j=1}^{k_y} (\Gamma_0 - j\tau)\Delta_j \tag{7}$$

For $t_j - t_{j-1} = \tau_y$ then

$$N(k_y, \tau_y) = R(\Gamma_0) - \sum_{j=1}^{k_y+1} \left[(\Gamma_0 - j\tau_y)\right]\left[r(j\tau_y) - r((j-1)\tau_y)\right]. \tag{8}$$

As the contract is full coverage (PM and CM), then a penalty cost incurs the OEM if the actual down time falls above the target $(\mathfrak{S})$. If $\mathcal{D}$ is down time (consisting repair time and waiting time) for each failure occuring during the contract, then the OEM should pay a penalty cost when $\mathcal{D} > \mathfrak{S}$. The amount of the penalty cost is assumed to be proportional to $\Delta = \mathcal{D} - \mathfrak{S}$. The penalty cost ($\mathcal{C}_{p}$) is viewed as a penalty given by the OEM. The decision problem for the OEM is to determine the optimal price structure and maintenance level such that to minimize the expected cost.

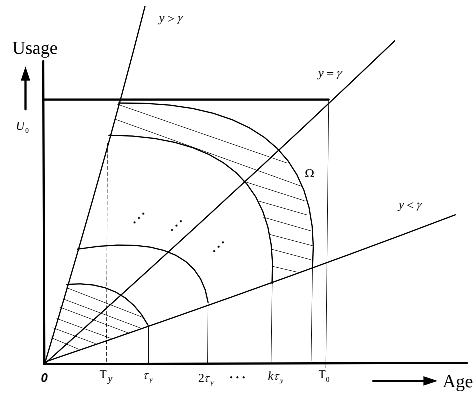

Figure 3. Preventive Maintenance region

Notations:

X_i	:Downtime caused by the *i*-th failure
$\Omega_T = [0, \Gamma_0) \times [0, U_0)$	:Maintenance contract coverage, time and usage limits
Δ_y	:Preventive maintenance level
$\mathfrak{S}$	:Down time target
pi(t,u)	:Probability that the Markov chain *X(t)* is in state *i* at time *t* conditional on the usage
$\mathcal{D}$	:Total downtime in (*0,t*]
F(t)	:Distribution function of downtime
K, τ.	:Revenue, maintenance contract time
Y	:Usage rate
C_m	:Repair cost done by OEM
C_s	:Repair cost charged to the owner for each failure
C_{pm}	:Preventive maintenance cost per unit of time
$\mathcal{C}_{p}$	:Penalty cost per unit of time
C_0	:Preventive maintenance cost
C_v	:Degree of preventive maintenance cost
$\mathcal{C}_{p}$	:Penalty cost per unit of time
C_b	:The annual product cost over the

	contract period
P_0	:PM cost done in-house over the contract period
$F(t,\alpha_y)$	:Conditional failure distribution for a given usage rate y
$r_y(t)$, $R_y(t)$	:Hazard, and Cumulative hazard functions associated with $F(t, \alpha_y)$
k	:Number of PM during maintenance contract
Δ_y	:Preventive maintenance level

3. MODEL ANALYSIS

Owner decision problem

Option-1: For case $y \le \gamma$, if K is the revenue ($/hour) received by the owner as a result of transporting mining materials from a mining site to a processing unit, C_b and P_0 are the annual cost of the item and PM cost over the contract period, respectively, then the expected profit is given by

$$E\left[\phi_y(O_1;C_s)\right] = \\ = K\left\{\Gamma_0 - R_{1y}(\Gamma_0)E[X_{ji}]\right\} - C_s R_{1y}(\Gamma_0) - P_0 - C_b \quad (9)$$

For case $y > \gamma$, as the contract ceases at Γ_0 then the expected profit of the owner is given by (9) replacing Γ_0 with Γ_y.

Option-2:For $y \le \gamma$, the expected profit of the owner is

$$E\left[\phi_y(O_2;P_G)\right] = \\ = K\left\{\Gamma_0 - R_{2y}(\Gamma_0)E[X_{ji}]\right\} + R_{2y}(\Gamma_0)EP(\Gamma_0) - P_G - C_b, \quad (10)$$

$$EP(\Gamma_0) = C_p\bar{G}(\text{S})N(k_y,\tau_y)$$

$EP(\Gamma_0)$ is the expected penalty viewed as a compensation received by the owner. For case $y > \gamma$, as in Option 1, the expected profit of the owner is given by (10) replacing Γ_0 with Γ_y.

OEM decision problem

Option-1: For $y \le \gamma$, the expected profit of OEM is given by

$$E\left[\pi_y(O_1;C_s)\right] = (C_s - C_m)R_{1y}(\Gamma_0) \quad (11)$$

For $y > \gamma$, the expected profit of OEM is given by (11) replacing Γ_0 with Γ_y.

Option-2: for $y \le \gamma$, when the down time of the item is above the target, the OEM incurs a penalty cost, and hence costs incurred by the OEM are penalty cost, repair cost and PM cost. Whilst the revenues of the OEM will be the price of the contract. We obtain expected penalty cost, the expected repair cost, and expected PM cost in Γ_0 per item as follows

<u>Expected cost with PM and minimal repair,</u>

$$C(k_y,\tau_y) = \\ C_r R(\Gamma_0) + k_y C_0 - \sum_{j=1}^{k_y+1}\left[C_r(L - j\tau_y) - C_v\right]\left[r(j\tau_y) - r((j-1)\tau_y)\right] \quad (12)$$

<u>Expected penalty cost:</u>

The OEM incurs penalty cost when the down time caused by a failure exceeds the predetermined target. Let $\mathscr{D}$ denote down time (consisting repair time and waiting time) for each failure occuring during the contract and down time allowed. The expected penalty cost is given by $C_p\bar{G}(\mathfrak{S})N(k_y,\tau_y)$ where $\mathscr{C}_p$ is the penalty cost and $N(k_y,\tau_y)$ denotes the expected number of failure in interval $(0,\Gamma_0]$.

As a result, the total expected profit of the OEM is

$$E\left[\pi(O_2)\right] = P_G - C(k_y,\tau_y) - \mathscr{C}_p\bar{G}(\mathfrak{S})N(k_y,\tau_y) \quad (13)$$

For $y > \gamma$,the expected profit of the OEM choosing O_2 is given by (13) replacing Γ_0 with Γ_y.

We consider a situation where both the owner and the OEM want to negotiate and determine jointly the terms and condition of the maintenancecontract to achieve a win-win solution. As a result, we can use a Nash solution of the bargaining game to obtain the optimal solution.

Proposition 1.

For case $y \le \gamma$, there exist C_S^*, and P_G^* (which is unique and finite) such that $E[\phi_y(O_i)] = E[\pi_y(O_i)]; i = 1,2$ given by

$$C_S^* = \frac{\left[K\left\{\Gamma_0 - R_{1y}(\Gamma_0)E\left[X_{ji}\right]\right\} + C_m R_{1y}(\Gamma_0) - P_0 - C_b\right]}{2R_{1y}(\Gamma_0)} \quad (14)$$

$$P_G^* = \frac{1}{2}\left[\begin{array}{l} K\left[\Gamma_0 - R_{2y}(\Gamma_0)E\left[X_{ji}\right]\right] + 2EP_y(\Gamma_0) \\ +C(k_y,\tau_y) - C_b \end{array}\right] \quad (15)$$

Using the results from Proposition 1, we obtain the optimal expected profit of each party (the owner or the OEM) for O_1 and O_2 given by

$$E[\pi_y(O_1;C_S^*) = \frac{1}{2}\left[K\left\{\tau - R_{1y}(\Gamma_0)E\left[X_{ji}\right]\right\} - P_0 - C_b - C_m R_{1y}(\Gamma_0)\right] \quad (16)$$

$$E[\pi_y(O_2;P_G^*) = \frac{1}{2}\left[K\left\{\tau - R_{2y}(\Gamma_0)E\left[X_{ji}\right]\right\} - C(k_y,\tau_y) - C_b\right] \quad (17)$$

For $y > \gamma$ the optimal repair cost, C_s^* and P_G^* and the optimal expected profit are given in Proposition 1 and equations (14)-(17) by replacing Γ_0 with Γ_y .

4. NUMERICAL EXAMPLE

We consider that the failure distribution is given by the Weibull distribution, $F(t;\alpha_y) = 1 - \exp(-t/\alpha_y)^{\beta}$, and its hazard function is $r_y(t) = \beta(t^{\beta-1}/(\alpha_y)^{\beta})$. Let the parameter values be as follows. α_0 = 236.28 (days), β = 2.06, U=2 ($3\text{x}10^4$Km) (γ=U/ Γ_0=1), ρ=2.5, Γ_0=3(years), $C_0 = 0.7 \times 10^6$, $C_v = 4.3 \times 10^6$, $C_m = 6.8 \times 10^6$, $C_b = 269 \times 10^6$, P_o=1/2 $C_m \Gamma_0$, and the transition matrix is given as

$$P = \begin{bmatrix} 0.474 & 0.526 \\ 0.567 & 0.433 \end{bmatrix}$$

Table I and Table II show the optimal number of PM and the optimal expected profit in option 1 and option 2 for the owner and the OEM. The expected profit for the owner and the OEM increase with the increasing of revenue *K*. The increase of revenue *K*, may lead to change the optimal option from option 1 to option 2 (see Fig. 4). This is due to the increase of profit as the number of failure decreases.

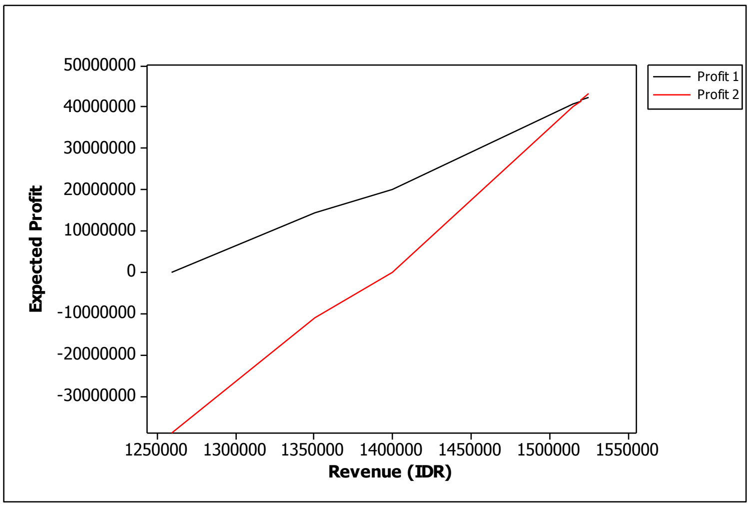

Figure. 4 Expected profit option 1 and option 2 for various revenue

Table I: Expected profit and expected maintenance cost for option 1 and option 2

	Option 1		Option 2	
k	$E[C]$	$E[\pi]$	$E[C]$	$E[\pi]$
1	$3{,}0573 \times 10^8$	$2{,}7937 \times 10^7$	$3{,}3319 \times 10^8$	$1{,}3840 \times 10^7$
2	$3{,}0235 \times 10^8$	$3{,}0097 \times 10^7$	$3{,}2894 \times 10^8$	$1{,}9219 \times 10^7$
3^*	$3{,}0208 \times 10^{8^*}$	$3{,}0343 \times 10^{7^*}$	$3{,}2847 \times 10^{8^*}$	$1{,}9965 \times 10^{7^*}$
4	$3{,}0257 \times 10^8$	$3{,}0121 \times 10^7$	$3{,}2892 \times 10^8$	$1{,}9576 \times 10^7$
5	$3{,}0331 \times 10^8$	$2{,}9752 \times 10^7$	$3{,}2966 \times 10^8$	$1{,}8834 \times 10^7$

Table II: Expected profit and repair cost for option 1 and option 2 with various revenue

Revenue (IDR)	$\boldsymbol{E[C]}$		$E[\pi]$		$\boldsymbol{C_{sc}}$	$\boldsymbol{C_m}$
	Option 1 ($\times 10^8$)	Option 2 ($\times 10^8$)	Option 1 ($\times 10^7$)	Option 2 ($\times 10^7$)	Option 1 ($\times 10^8$)	Option 2 ($\times 10^6$)
1.259.206,71	3,0208	3,2847	0,0000*	−3,8835	3,0208	5,9997
1.350.076,71	3,0208	3,2847	1,4446*	−1,0842	3,1652	6,5777
1.400.076,71	3,0208	3,2847	2,0090*	0,0094686	3,2217	6,8026
1.515.076,71	3,0208	3,2847	4,0677*	3,9989	3,4275	7,6253
1.519.685,71	3,0208	3,2847	4,1409*	4,1409*	3,4349	7,6546
1.520.076,71	3,0208	3,2847	4,1471	4,1529*	3,4355	7,6570
≥ 1.525.076,71	3,0208	3,2847	≥ 4,2266	≥ 4,3070*	≥ 3,4434	≥ 7,6888

5. CONCLUSIONS

We have studied a maintenance contract for a repairable item with dynamic operating condition. The optimal price structure for the OEM and the optimal maintenance option for the owner of the item are obtained. In this paper, we only consider item without warranty with two players -i.e. OEM and owner. In many cases, a repairable item is sold with warranty and the maintenance contract provider has more than one agent included the OEM and offers more options -partial, moderate, and full coverage of maintenance contract. These further research topics are under investigation.

ACKNOWLEDGMENTS

This work is funded by the Ministry of Research, Technology, and Higher Education of the Republic of Indonesia through the scheme of "PUPT 2016".

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