

# Forecasting of Purchase Dependent Power Demands using Vector Autoregressive Model as Basis for Inventory Policy in a Retailer

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**Abstract.** Retailer is the stage in the supply chain that has responsibility to deliver the product to end customer. Nowadays, due to the advanced of the information technology, e-retailer (electronic retailer) is increasing in number. However, sometimes a retailer uses both e-retailer and traditional retailer to sell the same product to the end customer. This situation increases the complexity of managing inventory in that type of retailer. In addition, the type of the product sold by retailer, i.e. the product that have purchase dependent power, make the problem of managing inventory in the retailer become more complex. In this paper, the preliminary research was conducted in an apparel store in order to observe that two different products have purchase dependent power, i.e. demand of product A is affecting demand of product B, and vice versa. Then, a vector autoregressive VAR(1) model is being used as the reference forecasting model for the demand pattern. It is expected that the forecasting model can be used as basis for determining inventory policy for two products with purchase dependent power.

**Keywords:** purchase dependent power, forecasting, vector autoregressive, VAR(1) model

## 1. INTRODUCTION

In the entire supply chain, retail plays an important role to deliver the product to end customers. As an important stage in the supply chain, a retailer has to manage themselves in order to provide the goods at the right quantity, in the right time and in the right quality. If a retailer is not able to provide product at the right quantity, a stock out condition might be occur.

The complexity of managing a retail might arise especially when a retail use both traditional retail and electronic retail (e-retail) to sell the product. To avoid stock out, when they plan their inventory, this type of retail has to consider demand from both traditional retail and e-retail.

In an e-retail, a customer gets interaction with the retailer by using website that contains several catalogs. A catalog contains much information about the product such

as price, size, and its availability. If a customer has found the product that they want to buy then they can easily select the product and the product automatically will be added to his/her order. However, some times in order to save the delivery order, it is believed that some customer have a tendency to buy more than one product at one order.

When a customer buy product A he/she also tend to buy product B, then product A and product B have tendency to be appeared in a single order, and when one of the product is not available then the customer cancel all the order is called as purchase dependence (Bala 2008; Bala 2010; Bala et al, 2010; Bala 2012; Park and Seo, 2013; Park, 2015).

If certain items are purchase dependence, then, it affects the way retail determine its inventory policy (Park and Seo, 2013). Therefore, in order to have better inventory policy retail is suggested to know what kinds of items that

are purchase dependences. To the best of author knowledge, research on inventory policy under purchase dependences had been addressed conducted by Bala (2008; 2010), Bala et al. (2010), Park and Seo (2013), and Park (2015). However, in those researches the process to conclude that certain items are purchase dependences had not been discussed. As according to Park and Seo (2013), purchase dependence related to the customer behavior, therefore the research in this paper is trying to study about the customer behavior of apparel product to see which apparel products that have purchase dependences. The details of this study will be explained in Section 2.

We have a hypothesis that the purchase dependent power affected the demand of products. In other word, if product A and B have purchase dependency, then the demand of product A and B are also have demand dependency, i.e. demand of product A is affected by demand of product B and vice versa demand of product B is affected by demand of product A. For modeling the relationship, a vector autoregressive model VAR(1) (Reinsel, 2003) is applied. The model is explained in Section 3. The model is then will be confirmed with the data of a retail apparel located at Yogyakarta, Indonesia.

In addition, numerical experiments on the vector autoregressive model are presented in Section 4. These experiments are conducted in order to study the behavior of the models. Finally, a concluding remark is presented in Section 5

## 2. CASE STUDY OF PURCHASE DEPENDENCE

We conducted research on a female retail apparel located at Yogyakarta, Indonesia which are selling common fashion apparel such as shirts, short dress, long dress, sweaters, tank tops, blouses, and accessories. This retail apparel sells the same products via traditional and online channel.

In order to study the purchase dependency among the products, we collected questionnaire from 90 respondents. The respondents profile are as follow: 47% are male and 53% are female, 61% are equal or less than 21 years in age and 39% are more than 21 years in age, 33% are high school students, 33% are university students, and 33% are professional worker.

Some interesting results related to online shop from the questionnaire are presented below:

1. the majority (49 respondents) choose to shop online due to its practicality and convenience
2. major factors for selecting an online shop are reliability of the shop in term of payment and delivery (21 respondents) and product variety and availability (19 respondents)

3. the common products being purchased online are apparel (54 respondents) and shoes (20 respondents)
4. the common number of items being purchased in single order in apparel online shop are 1-2 products (70 respondents)
5. **when a goods are not in stock, majority (47 respondents) decide not to purchase the item, while the other (30 respondents) decide to purchase the item in another shop, and only limited person (10 respondents) are waiting for the goods**
6. some of pair of products that tend to be purchase together, or has purchase dependency, selected by respondents are presented in Table 1. It is empirically shown that shirt and trouser has tendency to be purchase dependence.

Table 1: Product Dependency

Product 1	Product 2	Respondents
Shirt	Trouser	43
Shirt	Short	19
Shirt	Jacket	16
Shirt	Skirt	13
T-Shirt	Skirt	8

## 3. VECTOR AUTOREGRESSIVE MODEL

Based on introduction above, let us consider two products, which are product 1 and 2. These products have purchase dependent power, i.e. demand of product 1 is affecting demand of product 2, and vice versa. Therefore, we can have a hypothesis that the demand of product 1 at time period  $t$  is influenced by the demand of product 1 and the demand of product 2 at time period  $(t-1)$ . And vice versa, the demand of product 2 at time period  $t$  is also influenced by the demand of product 2 and the demand of product 1 at time period  $(t-1)$ . If the influence is represented by linear relationship, a vector autoregressive model VAR(1) can be utilized for representing these relationship among the demand of product 1 and 2.

The vector autoregressive VAR(1) model can be mathematically formulated as follow

$$Y_{1,t} = \delta_1 + \phi_{1,1}Y_{1,t-1} + \phi_{1,2}Y_{2,t-1} + \varepsilon_{1,t} \quad (1)$$

$$Y_{2,t} = \delta_2 + \phi_{2,1}Y_{1,t-1} + \phi_{2,2}Y_{2,t-1} + \varepsilon_{2,t} \quad (2)$$

where

$Y_{i,t}$  : demand of product  $i$  at period  $t$

$\phi_{i,j}$  : an autoregressive coefficient, which represents the effect of demand product  $i$  at period  $(t-1)$  on

demand product  $j$  at period  $t$

$\delta_i$  : an autoregressive coefficient, which represents the part of demand product  $i$  at period  $t$  that is not affected by past demand

$\varepsilon_{i,t}$  : error term of autoregressive equation  $i$ , which represents the part of demand product  $i$  at period  $t$  that cannot explained by the autoregressive equation

Multiple linear regression methodology (i.e. Hair, 2010) can be used for determining the autoregressive coefficient. Since there are three coefficients in each equation, therefore, at least four periods of demand data are required for conducting the regression.

#### 4. NUMERICAL EXPERIMENTS ON VAR(1)

In order to study the behavior of the VAR(1) model, numerical experiments are conducted involving some demand patterns that usually exist, which are horizontal, linear increasing, linear decreasing, and seasonal demand patterns. For each demand pattern, two sets of data consists

of ten periods of time are being used for the experiments. The demand datasets for the experiments are visualized in Figures 1 – 4.

Table 2: Demand patterns of each experiment

Experiment	Product 1	Product 2
I	horizontal	horizontal
II	horizontal	linear increasing
III	horizontal	linear decreasing
IV	horizontal	seasonal
V	linear increasing	linear increasing
VI	linear increasing	linear decreasing
VII	linear increasing	seasonal
VIII	linear decreasing	linear decreasing
IX	linear decreasing	seasonal
X	seasonal	seasonal

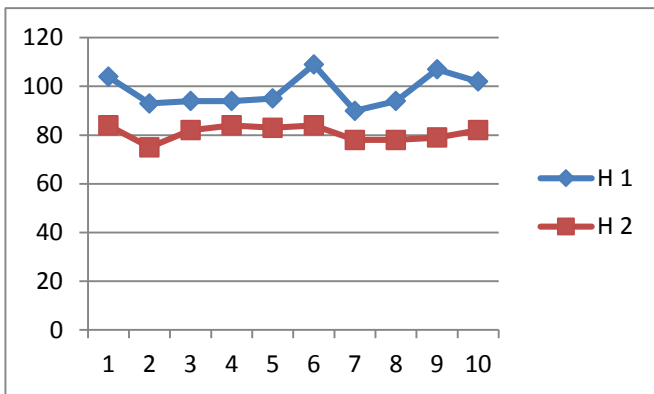


Figure 1. Horizontal data pattern

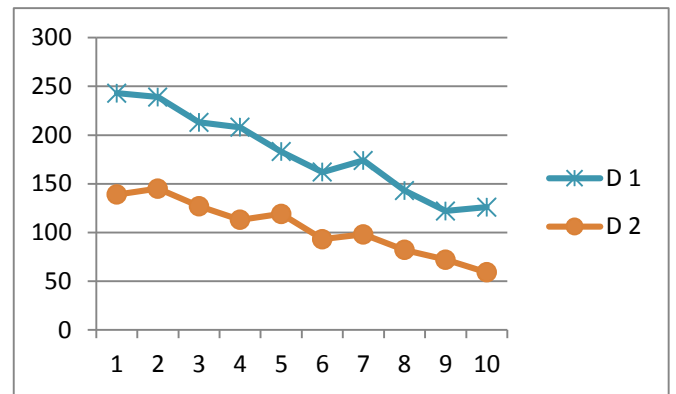


Figure 3. Linear decreasing demand pattern

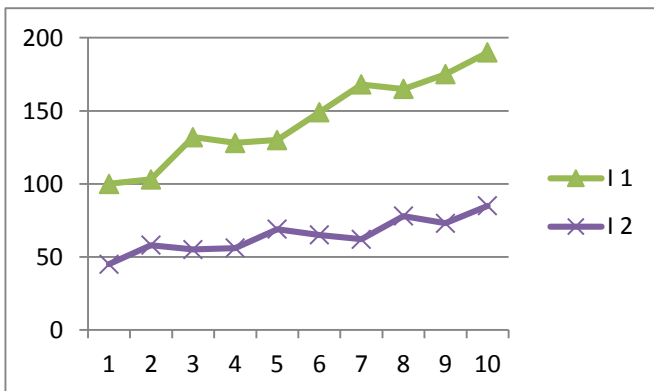


Figure 2. Linear increasing demand pattern

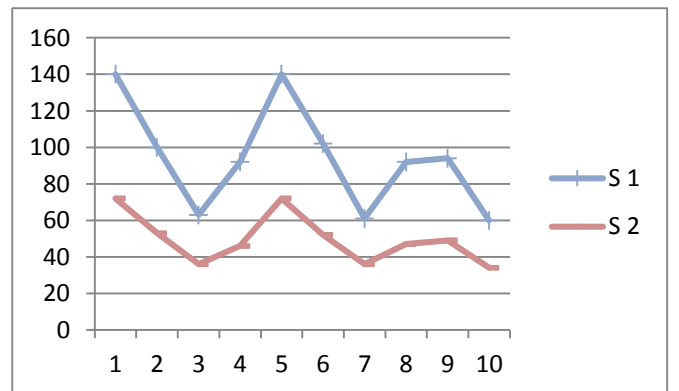


Figure 4. Seasonal demand pattern

The experiments consist of ten individual experiment, in which each experiment is trying to assume that a pair of demand products is following VAR(1) model. The demand pattern of each experiment are presented in Table 2. Using multiple linear regression, the autoregressive coefficients of each equation can be determined. The regression results are presented in Table 3, along with several important metrics such as the correlation coefficient ( $r$ ), the coefficient of determination (adjusted  $r^2$ ), standard error, F and sig-F of ANOVA test.

It is known from statistical knowledge that (1) the correlation coefficient shows how strong the linear relationship, i.e. a value close to 1 means close to perfect positive relationship, (2) the coefficient of determination shows how close the data fit to the model, i.e. a value close to 1 means almost of the data can be completely explained by the model, and (3) the sig-F indicates the result of hypothesis testing on whether all the regression coefficients are zero, that is a small value of sig-F (i.e. sig-F less than

0.05) indicates that at least one regression coefficient is not zero.

Based on three indicators above, it is concluded from the result in Table 2 that these following product demand can be fitted into VAR(1) model: experiment II on product 2 (linear increasing demand pattern), experiment II on product 2 (linear decreasing demand pattern), experiment V on product 1 and 2 (both are linear increasing demand pattern), experiment VI on product 1 and 2, experiment VII on product 1, experiment VIII on product 1 and 2, experiment IX on product 1 (linear decreasing demand pattern). Therefore, it can be concluded that the VAR(1) model is matching for linear demand pattern, either increasing or decreasing.

## 5. CONCLUDING REMARK

In this research, we are partially proven that purchase dependency has relationship with demand dependency.

Table 3. Regression results of the experiments

Experiment	I		II		III		IV		V	
Product $i$	1	2	1	2	1	2	1	2	1	2
$r$	0.2149	0.3688	0.3975	0.9234	0.6039	0.9882	0.4510	0.5506	0.9673	0.8513
adjusted $r^2$	-0.2718	-0.1520	-0.1227	<b>0.8036</b>	0.1529	<b>0.9688</b>	-0.0622	0.0709	<b>0.9141</b>	<b>0.6328</b>
standard error	7.5883	3.3988	7.1296	12.3186	6.1932	7.1430	6.9349	24.6550	8.1448	6.2808
F	0.1452	0.4722	0.5630	17.3658	1.7219	125.3174	0.7659	1.3052	43.5867	7.8945
sig-F	0.8678	0.6450	0.5969	<b>0.0032</b>	0.2565	<b>0.0000</b>	0.5055	0.3384	<b>0.0003</b>	<b>0.0209</b>
$\delta_i$	128.6993	91.0916	107.0757	-25.59920	150.69515	-208.1577	121.9647	276.3819	-10.7149	23.1284
$\phi_{i,1}$	-0.1457	-0.1819	-0.2194	0.4784	-0.3584	1.9925	-0.3571	-2.1939	0.5533	0.3367
$\phi_{i,2}$	-0.2092	0.0897	0.0859	0.9195	-0.0966	1.0018	0.1069	0.2797	1.3277	-0.0500
Experiment	VI		VII		VIII		IX		X	
Product $i$	1	2	1	2	1	2	1	2	1	2
$r$	0.9603	0.9425	0.9326	0.2384	0.9330	0.9682	0.9369	0.3146	0.2541	0.3410
adjusted $r^2$	<b>0.8962</b>	<b>0.8512</b>	<b>0.8263</b>	-0.2575	<b>0.8272</b>	<b>0.9166</b>	<b>0.8371</b>	-0.2014	-0.2472	-0.1783
standard error	8.9546	15.6068	11.5830	28.6835	16.8153	7.9873	16.3274	28.0359	28.5655	12.7373
F	35.5417	23.8797	20.0344	0.1808	20.1548	44.9555	21.5594	0.3295	0.2072	0.3949
sig-F	<b>0.0005</b>	<b>0.0014</b>	<b>0.0022</b>	0.8390	<b>0.0022</b>	<b>0.0002</b>	<b>0.0018</b>	0.7315	0.8185	0.6901
$\delta_i$	230.8824	148.2766	-15.2261	129.3905	4.8445	-18.7655	-4.0413	53.2842	105.8610	55.9078
$\phi_{i,1}$	0.1487	-0.5648	1.0372	-0.2473	0.9368	0.9718	0.8847	0.1931	2.0121	1.2169
$\phi_{i,2}$	-0.5476	0.5581	0.2043	-0.0581	-0.0546	-0.5694	0.1288	-0.0014	-4.1629	-2.4922

However, some efforts still required in order to explain clearly these relationship.

Another conclusion that can be made from the experiments conducted in this research is the demand dependency can be mathematically represented by VAR(1) model, especially whenever the demand of product is following either linear increasing or decreasing demand pattern. Other efforts still also required in order to representing another demand pattern, i.e. horizontal and seasonal demand pattern.

Finally, how to utilize these knowledge in order to improve inventory policy is open to be explored as another research direction.

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