Solving the Capacitated Facility Location Problem Using a Cuckoo Search

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Abstract. Facility location problem (FLP) is a critical issue in strategic planning for modern enterprises. It is a classical optimization problem to determine the number and locations of a set of facilities (factories, warehouses, etc.) and assign customers to these facilities in such a way that the total cost is minimized. The FLP with a limited capacity is called the capacitated FLP (CFLP) and it belongs to the class of NP-hard combinatorial optimization problem. Most of the previous studies for solving the CFLP developed a heuristic framework such as genetic algorithm, particle swarm optimization (PSO) algorithm, colony algorithm, Tabu search, etc. The cuckoo search (CS) algorithm idealized cuckoos breeding behavior and can be applied for various optimization problems. It has less parameters, better search route, and strong search ability compared to other metaheuristic algorithms. This research therefore uses cuckoo search via Lévy flights to solve the CFLP. Several datasets in OR-library are tested. Experimental results show the cuckoo search performs well with effectiveness for CFLP.

Keywords: Cuckoo Search, Capacitated Facility Location Problem, Lévy Flights

1. INTRODUCTION

In modern enterprises, decisions about the distribution system are usually a strategic issue. The core component of distribution system design concerns the problem of locating facilities (warehouses, plants, machines, etc) and allocating customers to these in such a way the total cost is minimized. The facility location problem (FLP) is a classical, combinatorial optimization problem. There are two kinds of FLP, depending on the restrictions on the amount of service provided by every facility. When capacity of each facility is infinite, the FLP is called the uncapacitated FLP (UFLP). On the other hand, the FLP with a limited capacity is called the capacitated FLP (CFLP). In both the UFLP and the CFLP, the number of feasible solutions grows exponentially with the instance size. As most practical instances are large-sized, finding optimal solutions becomes extremely difficult.

The UFLP and CFLP both belong to the class of NP-hard combinatorial optimization problem (Mirchandani & Francis, 1990; Charikar & Guha, 2005). In the past decades, lots of algorithms, exact and heuristic, have been developed to solve the UFLP and CFLP. Exact approaches based on mathematical programming and enumeration techniques have been proposed by Erlenkotter (1978), Barcel´o, et al. (1991); Akinc U, Khumawala B. 1997, Holmberg, K., Rnnqvist, M., Yuan, D. (1999); Diaz, J.A., Fern´andez, E. (2002). However, there exists hardly any algorithm to find optimal solutions efficiently for an arbitrary instance (Tohyama et al., 2011). In many cases, optimal solutions are not necessarily required for UFLP and CFLP. Satisfactorily accurate approximate solutions usually prove acceptable if they can be found within a reasonable time. Many metaheuristic approaches have been presented for this purpose, such as tabu search (Filho & Galvo, 1998; Delmaire, et al. 1999; Al-Sultan and Al-Fawzan, 1999; Michel, & Hentenryck, 2004), genetic algorithms (GA) (Kratica, 1999; Kratica et al. 2001; Mari´c , 2010; Tohyama et al. 2011), ant colony optimization (Chen & Ting, 2008; Lina et al., 2012) or particle swarm optimization (PSO) (Sevkli & Guner, 2006;Wang & Watada,2012).

Modern Metaheuristic algorithms such as PSO, ant colony optimization, or cuckoo search (CS), are mainly bio inspired. Biological systems run in such a mechanism that presents remarkable adaptation, robustness, and reliability in the dynamic environments in spite of individual simplicity, especially in social birds, fishes, and insects. Therefore they are often used to solve NP-hard problems. Among them, cuckoo search which was proposed by Yang and Deb in 2009, is the latest addition to the family of metaheuristic algorithms. The inspiration of this algorithm came from the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds. It has attracted great attention due to its promising efficiency in solving many optimization problems of sciences, engineering and industry (Yang and Deb 2010, 2013; Kanagaraj et al., 2012; Tiwari, 2012; Wang et al., 2012; Yildiz, 2013).

In this paper a cuckoo search based algorithm is proposed to solve the capacitated facility location problem. The rest of the paper is organized as follows. Section 2 presents the CFLP model. The proposed method using cuckoo search via Lévy flights is described in section 3. Section 4 describes experiments, analysis and comparison results and in section 5 conclusions are drawn.

2. CAPACITATED FACILITY LOCATION PROBLEM

Consider a set of facilities (servers) indexed by $i \in I$ and a set of customers (clients) indexed by $j \in J$. The problem is to choose some sites from I as facility locations to provide services to customers in J. The facility setup cost f_i ($\in$ R) occurs when facility i is opened. Let u_i be the capacity of the facility i and let d_j be the demand of customer j. Let x_{ij} be the flow of the demand from facility i to the customer j. In addition, the service cost c_{ij} ($\in$ R) is determined for every customer j using the facility i. Let y_i = 1 if facility i is opened, and y_i = 0 otherwise. In this case, the CFLP is the problem of determining facility placement and allocating customers so as to minimize the total costs of use (for customers) and facility installation (for providers). The capacitated facility location problem can therefore be formulated as follows:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} f_i y_i \tag{1}$$

$$\text{subject to } \sum_{i=1}^{m} x_{ij} \geq d_j\,, \qquad j \in J \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} \leq y_i\,, \qquad i \in I \tag{3}$$

$$x_{ij} \geq 0 \text{ integer} \quad i \in I, j \in J \tag{4}$$

$$y_i \in \{0,1\} \quad i \in I \tag{5}$$

The objective function (1) minimizes the total costs: the setup costs of opened facilities and the costs of serving customers by the opened facilities. Constraints (2) ensure that the demand of each customer is satisfied; constraints (3) express that the service provided by a facility cannot exceed its capacity; constraints (4) and (5) are the integrality constraints.

3. CUCKOO SEARCH ALGORITHM

3.1 Introduction to Cuckoo Search Algorithm

Cuckoo Search is a novel metaheuristic algorithm recently developed by Yang and Deb (2009). The CS is inspired by the special lifestyle and aggressive reproduction strategy of some species of bird family called cuckoo. Cuckoos lay their eggs in the nests of other host birds (almost other species) with remarkable abilities such as selecting the recently spawned nests and removing existing eggs that increase hatching probability of their eggs. On the other hand, some of host birds are able to combat this parasite behavior of cuckoos and throw out the discovered foreign eggs or build their new nests in new locations.

CS algorithm contains a population of nests or eggs. It is a population based optimization technique and as many other metaheuristic algorithms it starts with random initial population. For simplicity, the following representations are used, where each egg in a nest represents a solution and a cuckoo egg represents a new one. If the cuckoo egg is very similar to the host's, then this cuckoo egg is less likely to be discovered. The aim is to employ the new and potentially better solutions (cuckoo eggs) to replace a not-so-good solution in the nests. It is usually combined with the Lévy flight behavior of some birds and fruit flies (Yang and Deb, 2009; Kaveh & Bakhshpoori, 2011).

For simplicity in describing the CS algorithm, three idealized rules can be used (Yang and Deb, 2009):

(1) Each cuckoo lays one egg at a time and dumps it randomly in a randomly chosen host nest.
(2) The best nests with high quality of eggs are carried over to the next generations (a sort of eclecticism).
(3) The number of available host nests is fixed. The egg, which is laid by a cuckoo, is discovered by the host bird with a probability of $p_a \in [0,1]$.

The first rule can be considered as a randomization process so that a new solution can be randomly generated either by random walk or by Lévy flights. When generating new solutions $x^{(t+1)}$ for a cuckoo i, a Lévy flight random walk is performed as mentioned below:

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \oplus Levy(\lambda) \tag{6}$$

where α >0 is known as the step size and should be chosen considering the scale of the problem. The product

$\oplus$ stands for entrywise multiplication. In this algorithm the major advantage is the use of Lévy flight, which is more efficient in exploring the search space as its step length is much longer in long run for both local and global searching. The random step length is drawn from a Lévy distribution which has an infinite variance and an infinite mean.

$$\text{Lévy} \sim u = t^{-\lambda}, (1 < \lambda \leq 3) \quad (7)$$

Another advantage of the CS is that only one parameter p_a, the fraction of nests to be threw away, needs to be adjusted. Yang and Deb (2009) suggested the value of $p_a = 0.25$, but it can be any value between 0 and 1.

3.2 Proposed method

This paper applied the cuckoo search to solve the capacitated facility location problem. Feasible solutions to the CFLP are evaluated by the total of facility opening costs and service costs (Eq. 1). Instead of randomized generating initial solution, this research got the initial solution by assign the facility with the minimal service cost to any arbitrary customer demand. If the customer demand couldn't be satisfied, the facility with the second lowest service cost will be chose to provide the demand. Figure 1 shows the pseudo code of the proposed method.

Begin
  Objective function $F(x), x = (x_{11}, x_{12}, \dots, x_{mn})$
  Generate initial population of *d* host nests;
    $x_i (i = 1, \dots, d)$
  while (*t* < *MaxGeneration*) or (*stop criterion*)
    Get a cuckoo randomly by Levy flights;
    Evaluate its fitness F_i;
    Choose a nest among *d* (say *j*) randomly;
    If $F_i \leq F_j$
      replace *j* by the new solution;
    end
    Abandon a fraction (p_a) and build new ones;
    Keep the best solutions;
    Rank the solutions and find the current best;
  end while
end

Figure 1: Pseudo code of the proposed CS

4. NEMERICAL EXPERIMENTS

The CS method proposed to solve the CFLP in this paper was verified by numerical experiments. Extensive computational experiments were taken to test the proposed CS method. The algorithm was implemented in Matlab. Benchmark problems from the OR-Library (Beasley, 1988) were used in the experiments, as shown in Table 1.

The parameters of CS were set according to the results of preliminary experiments. The values of the parameters thus determined are given in Table 2.

This paper used the ratio of the obtained solution to the optimal solution for comparison. Table 3 shows the results among different instances.

Table1: Test Instance from the OR-Library

Dataset	Number of potential facility locations	Number of customers	Facility capacity	Fixed cost
cap41	16	50	5000	7500
cap43	16	50	5000	17500
cap51	16	50	10000	17500
cap61	16	50	15000	7500
cap81	25	50	5000	7500
cap91	25	50	15000	7500
cap111	50	50	5000	7500

Table 2: Parameters

Parameter	Value
Nest size	5, 15, 30, 50
Terminal condition (*MaxGeneration*)	1000, 3000, 5000, 10000
Step size (α)	0.01

From Table 3, it can be observed that the CS algorithm can obtain the good solution in almost all instances except cap111 which is a large sized problem.

5. Conclusions

In this paper, the cuckoo search algorithm is presented and successfully implemented to the optimization of capacitated facility location problems. The new solution is found via Lévy flights. Benchmark problems from the OR-Library were used in the experiments, and the result indicated the effectiveness of the CS. In this paper, the initial solution of the CS algorithm is obtained by assign the facility with minimal service cost. In the future, different initial solution generating methods will be studied, and the computational results by CS algorithm will be compared with other metaheuristics such as GA, Tabu search, PSO, and so on. Other possible future work is the application of CS to other types of FLP.

Table 3: Experimental results

Instance	Max. generation	Solution(total cost)	Ratio to optimal solution
cap41	1000	1065813.3250	1.0244
	3000	1060704.4750	1.0195
	5000	1183804.6875	1.0267
	10000	1052511.8500	1.0116
cap43	1000	1145037.4750	1.1005
	3000	1178956.9750	1.0225
	5000	1164083.4250	1.0096
	10000	1169149.4625	1.0140
cap51	1000	1032841.2125	1.0074
	3000	1031252.9375	1.0059
	5000	1031252.9375	1.0059
	10000	1031731.9375	1.0064
cap61	1000	950470.1875	1.0191
	3000	950470.1875	1.0191
	5000	950470.1875	1.0191
	10000	950470.1875	1.0191
cap81	1000	846145.9375	1.0091
	3000	846125.2000	1.0091
	5000	846111.2875	1.0091
	10000	846111.2875	1.0091
cap91	1000	832291.1500	1.0447
	3000	832291.1500	1.0447
	5000	832291.1500	1.0447
	10000	832291.1500	1.0447
cap111	1000	991679.5625	1.2004
	3000	991576.4000	1.2003
	5000	991484.9375	1.2002
	10000	991491.7125	1.2002

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