The Pickup and Delivery Multi-depot Vehicle Routing Problem

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Abstract. We address in this paper the pickup and delivery multi-depot vehicle routing problem (PDMDVRP). In this problem, a single commodity type is collected from a set of pickup customers to be delivered to a set of delivery customers by vehicles dispatched from multiple depots. The proposed problem has many applications in practice such as distribution of money between the branches of a bank, and moving a certain type of products from warehouses with extra supply to others that are short of the same product in a distribution network. The objective of PDMDVRP is to minimize the travel cost of capacitated vehicles in order to transport the commodity from pickup customers to delivery customers. We present a mathematical formulation for PDMDVRP and use CPLEX to solve small-scale instances. We propose a simulated annealing (SA) algorithm to solve the larger scale instances. Results show that the proposed algorithm is capable of producing high-quality PDMDVRP solutions.

Keywords: vehicle routing, multi-depot, pickup and delivery, simulated annealing.

1. INTRODUCTION

This paper presents the pickup and delivery multi-depot vehicle routing problem (PDMDVRP). In this problem, multiple depots are considered with a set of customers that are divided into two groups according to the service required (delivery or pickup). Each delivery customer requires a given amount of the product and each pickup customer provides a given amount of the product. The product collected from a pickup customer can be supplied to a delivery customer, on the assumption that there is no deterioration in the product.

For example, the customers can be branches of a bank in an area providing or requiring a known amount of money (the product), and the depots are the main branches of the bank. Clearly, this is a very simple variant of a more realistic problem where several commodities (e.g., bills and coins) could be considered, but it is still an interesting problem in its own right. It is assumed that the vehicle has a fixed upper-limit capacity and must start and end the route at the same depot. Also, we allow the initial load of the vehicle leaving from the depot to be any quantity.

2. LITERATURE REVIEW

Vehicle routing problem (VRP) is a central issue in the area of transportation planning research. VRP was first introduced by Dantzig and Ramser [1] and has been widely studied by numerous researchers [2, 3]. It can be described as the process of determining optimal routes from a depot to a number of geographically scattered customers, subject to side constraints [4]. A classical version of VRP is capacitated VRP (CVRP), where a vehicle must satisfy certain capacity restrictions. Subsequently, when a distance or total time restriction is also imposed on CVRP, the problem becomes distance-constrained CVRP.

The vehicle routing problem with backhauling (VRPB) considers a vehicle servicing all delivery (*Linehaul*) customers with cargo loaded at the depot, followed by Pickup (*Backhaul*) customer services. Practical applications of this VRP variation are found in grocery industry presented in [5] and [6].

The vehicle routing problem with simultaneous pickup and delivery (VRPSPD) was introduced in 1989 by [7]. It considers both Pickup and delivery service at each customer, while each collected cargo must be returned to the origin depot. This problem is present in milk bottles transporting while empty ones must be returned to the origin depot. A tabu search algorithm, with and without maximum distance constraints, was recently developed in [8] to solve VRP-SPD.

The problem study about the single commodity pickup and delivery starts at the real-world application of the one commodity pickup and delivery traveling salesman problem (1-PDTSP). It was first introduced in Hernández-Pérez and Salazar-González [9] together with an exact approach to solve instances with up to 50 customers. [10] and [11] study the special case of the 1-PDTSP where the delivery and pickup quantities are all equal to one unit, which is known in the literature as the capacitated traveling sales-man problem.

Later, Martinovic, Aleksi [12] introduced the one commodity pickup and delivery vehicle routing problem (1-PDVRP) that deals with multiple vehicles and is the same as the single-commodity traveling salesman problem (1-PDTSP) when the number of vehicles is equal to 1. The main difference between VRPB, VRPSPD, and 1-PDVRP is that in 1-PDVRP cargo picked up from pickup customers can be delivered to delivery customers. In 1-PDVRP, the predefined sequence of servicing customers is not a constraint.

In this paper, we study the one commodity pickup and delivery multi-depot vehicle routing problem (PDMDVRP) which is one of the VRP extensions. In general, the VRP problem and its variations are NP-hard. Therefore, we propose a simulated annealing (SA) algorithm to solve the present problem.

3. PROBLEM DESCRIPTION

This article presents a pickup-and-delivery multi-depot vehicle routing problem (PDMDVRP). A set of depots and customers are introduced. The set of customers are divided into two groups (pickup and delivery customers). One type of product is considered. The product collected from a pickup customer can be supplied to a delivery customer, on the assumption that there is no deterioration in the product. It is assumed that the vehicle has a fixed upper-limit capacity and must start and end the route at the depot. Also, we allow the initial load of the vehicle leaving from the depot to be any quantity.

Parameters:

N : A set of nodes $N = N_0 \cup N_c$

N_0 : A set of potential depots

N_c : A set of customers

F : The number of vehicles available in each depot

K : Set of all vehicles, $|K| = F \cdot |N_0|$

c_{ij} : Traveling cost from node i to node $j, \forall i, j \in N, i \neq j$

Q : Vehicle capacity

T : Maximum duration of a route

M : A big number

d_i : Demand of customer i, $i \in N_c$

p_i : Supply of customer i, $i \in N_c$

Variables:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travel from } i \text{ to } j \\ 0, & \text{Otherwise} \end{cases}$$

$$z_{ij} = \begin{cases} 1, & \text{if depot } j \text{ is assigned to customer } i \\ 0, & \text{Otherwise} \end{cases}$$

$L_{ij} \geq 0$: The load of a vehicle after leaving node i to j

$U_{ij} \geq 0$: The remaining demand after leaving node i to j

$V_{ij} \geq 0$: The remaining supply after leaving node i to j

f_i : The number of vehicle used in each depot i

Objective function:

$$\min Z = \sum_{k \in K} \sum_{i \in N, i \neq j} \sum_{j \in N} c_{ij} \cdot x_{ijk} \quad (1)$$

Subject to:

$$\sum_{k \in K} \sum_{j \in N, i \neq j} x_{ijk} = 1, \quad \forall i \in N_c \quad (2)$$

$$\sum_{j \in N, i \neq j} x_{jik} = \sum_{j \in N, i \neq j} x_{ijk}, \ \forall i \in N, \forall k \in K \quad (3)$$

$$x_{ijk} = 0, \quad \forall i, j \in N_0, \forall k \in K \quad (4)$$

$$\sum_{i \in N_0} \sum_{j \in N_c} x_{ijk} \leq 1, \quad \forall k \in K \quad (5)$$

$$\sum_{j \in N, i \neq j} U_{ji} - \sum_{j \in N, i \neq j} U_{ij} = d_i, \quad \forall i \in N_c \quad (6)$$

$$\sum_{j \in N, i \neq j} V_{ji} - \sum_{j \in N, i \neq j} V_{ij} = p_i, \quad \forall i \in N_c \quad (7)$$

$$\sum_{j \in N_c} U_{ij} = \sum_{j \in N_c} d_i \cdot z_{ji}, \quad \forall i \in N_0 \quad (8)$$

$$\sum_{j \in N_c} V_{ij} = \sum_{j \in N_c} p_i \cdot z_{ji}, \quad \forall i \in N_0 \quad (9)$$

$$L_{ij} = U_{ij} - V_{ij}, \quad \forall i, j \in N, i \neq j \quad (10)$$

$$L_{ij} \leq Q \sum_{k \in K} x_{ijk}, \quad \forall i, j \in N, i \neq j \quad (11)$$

$$L_{ij} \leq d_j \sum_{k \in K} x_{ijk}, \quad \forall i, j \in N, i \neq j \quad (12)$$

$$Q - L_{ij} \geq p_j \sum_{k \in K} x_{ijk}, \ \forall i \in N, j \in N_c, i \neq j \quad (13)$$

$$U_{ij} \geq d_j \sum_{k \in K} x_{ijk}, \quad \forall i \in N, j \in N_c, i \neq j \quad (14)$$

$$V_{ij} \geq p_j \sum_{k \in K} x_{ijk}, \quad \forall i \in N, j \in N_c, i \neq j \quad (15)$$

$$\sum_{j \in N_0} z_{ij} = 1, \quad \forall i \in N_c \quad (16)$$

$$f_i = \sum_{k \in K} \sum_{j \in N_c} x_{ijk}, \ \forall i \in N_0 \quad (17)$$

$$\sum_{k \in K} \sum_{j \in N_c} c_{ij} x_{ijk} \leq T, \ \forall k \in K \quad (18)$$

$$U_{ij} \leq M \sum_{k \in K} x_{ijk}, \ \forall i, j \in N, i \neq j \quad (19)$$

$$V_{ij} \leq M \sum_{k \in K} x_{ijk}, \ \forall i, j \in N, i \neq j \quad (20)$$

$$\sum_{k \in K} x_{ijk} \leq z_{ij}, \quad \forall i \in N_c, \forall j \in N_0 \quad (21)$$

$$\sum_{k \in K} x_{jik} \leq z_{ji}, \quad \forall i \in N_c, \forall j \in N_0 \quad (22)$$

$$\sum_{k \in K} x_{ijk} + z_{ir} + \sum_{m \in N_0, m \neq r} z_{jm} \leq 2,$$
$$\forall i, j \in N_c, i \neq j, \forall r \in N_0 \quad (23)$$

The objective (1) aims to minimize the vehicle traveling cost. Constraint (2) ensures that each customer is served exactly once. In Constraint (3), the number of entering arcs is equal to the number of leaving arcs for each node. Constraint (4) ensures that a vehicle should leave and enter the same depot. Constraint (5) ensures that one vehicle only travels in one route. Constraint (6) and (7) are the constraints for demand and supply, respectively. Constraints (8) and (9) imply that the total demand and supply of customers assigned respectively to a specific depot are satisfied by the vehicles dispatched from the depot. Constraint (10) defines load of a vehicle. Constraint (11) ensures that the load of a vehicle does not exceed the vehicle's capacity. Constraints (12) and (13) guarantee that the load and remaining space are enough to handle the upcoming demand.

Constraint (14) and (15) state the bounds of the remaining demand and supply, respectively. Constraint (16) guarantees that each customer is assigned to one depot. Constraint (17) defines the number of used vehicles for each depot. Constraint (18) ensures that the total traveling cost for each route cannot exceed the maximum duration. Constraints (19)-(23) prohibit infeasible routes.

4. SIMULATED ANNEALING (SA)

Metropolis, Rosenbluth [13] introduced a simple algorithm that can be used to provide an efficient simulation of a collection of atoms in equilibrium at a given temperature by accepting worse solutions in iterations with a small probability called the Metropolis criterion. Later, it was known as a simulated annealing (SA). SA is a local search-based heuristic which is capable of avoiding from being trapped in a local optimum [14]. It has been successfully applied to solve many vehicle routing related problems. For example, Lin, Yu [15] solved the truck and trailer routing problem with SA. Lin and Yu [16] proposed the SA to solve TOPTW. The results show that SA is competitive with other solution approaches which have been applied to solve TOPTW.

In this paper, we applied the SA algorithm for solving PDMDVRP. The proposed SA uses a constructive type of heuristic to generate the initial current solution, and a new solution is then derived from the predefined neighbourhood of the current solution. The neighbourhood structure improves the solution, utilizing three types of common moves: swap, reversion, and insertion. Finally, a local search is performed to refine the solution obtained from the SA algorithm. Figure 1 shows the flowchart of the proposed SA algorithm for PDMDVRP.

4.1 Solution Representation

A solution representation consists of a set of N_0 depots denoted by $\{1, 2, \ldots, N_0\}$, a set of N_c customers denoted by $\{N_{0+1}, N_{0+2}, \ldots, N_{0+NC}, \}$, and N_{dummy} dummy zeros. The first number in a solution representation must be a depot.

4.2 Neighborhood Structures

This research uses a standard SA procedure which randomly selects neighborhood structures: swap, insertion, and inversion, to solve the PDMDVRP. Let N(X) denote the set of neighboring solutions of the current solution X. At each iteration, a new feasible solution Y is selected from N(X) by one of the three types of moves briefly described below.

The swap move randomly selects the i^{th} and the j^{th} locations of X and then exchanges their positions. The insertion move is carried out by randomly selecting the i^{th} location of X and then inserting it into the position immediately before another randomly selected j^{th} location of X. The inversion move is performed by randomly selecting two locations, and then reversing the sequence between the two locations (including the two selected locations). The probabilities of performing the three moves: swap, insertion, and inversion are set to be 1/3, 2/3, and 3/3, respectively. Note that after a move, the tours need to be recalculated as illustrated in the previous section. Thus, a location that was not selected in the original solution may be selected after a move, and vice versa. The new solution is always feasible due to our solution representation scheme.

5. COMPUTATIONAL RESULTS

For testing our solution approach for PDMDVRP, the proposed SA has been implemented in C++ and executed on a PC with a 3.4 GHz processor and 16 GB of RAM, under the Windows 7 operating system.

5.1 Parameter Setting

The proposed SA heuristic requires five parameters I_{iter}, T_0, $MaxT$, $N_{non\text{-}improving}$ and α . I_{iter} denotes the number of iterations the search proceeds at a particular temperature. T_0 represents the initial temperature. $MaxT$ is the maximal allowable computational time used. $N_{non\text{-}improving}$ is the maximum allowable number of temperature reductions during which the best objective function value has not improved. Finally, α is a coefficient used to control the speed of the cooling schedule.

5.2 PDMDVRP Instances

PDMDVRP instances are adopted from Cordeau's dataset of the multi-depot vehicle routing problem. In PDMDVRP instances, the larger problems have 180 delivery customers and 156 pickup customers where travel times equal their corresponding distances. From these instances, smaller problem instances were derived with only the first 25 delivery locations and 20 pickup locations considered. The number of potential depot is from 4 to 10 locations.

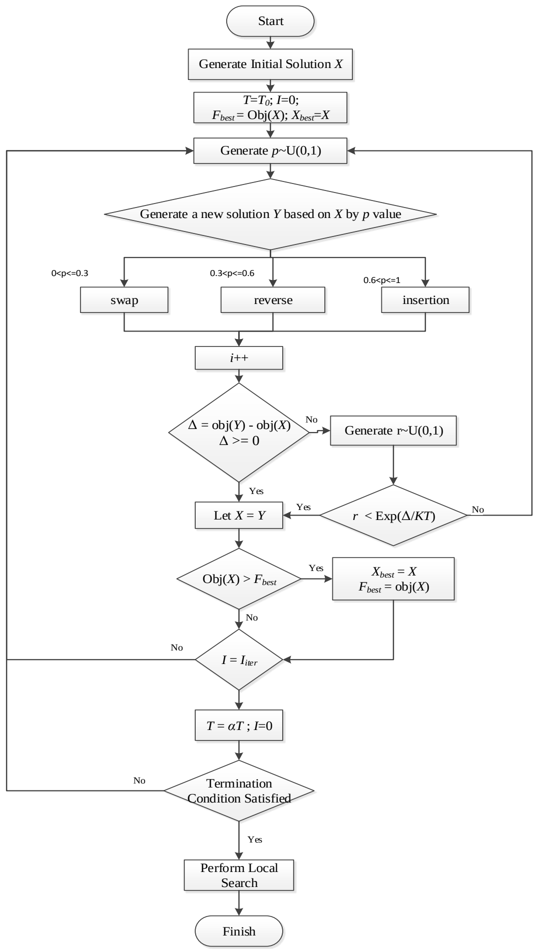

Figure 1: the flowchart of the proposed SA algorithm for PDMDVRP

5.3 Computational Results

This paper uses CPLEX solver to solve the model. If the solver cannot find the optimal solution within 3 hours, it was terminated Table 1 shows the results for PDMDVRP instances. The result shows that CPLEX can only obtain a feasible solution for two instances. The SA produced the better solutions than those obtained by CPLEX. The computational time may depend on various factors, such as the CPU of the machines, the operation system, the compiler, the computer program, and the precision used during the execution of the run. In general, our SA heuristic takes much lesser time than CPLEX for all problem instances.

Table 1 Computational results for PDMDVRP

Instances	**Delivery Customers**	**Pickup Customers**	**Depots**	**AMPL**		**SA**	
				Obj	**Time**	**Obj**	**Time**
p01	25	20	4	471.05	18000	379.53	50.117
p02	25	20	4	435.42	18000	351.74	51.184
p03	38	33	5	N/A	18000	499.69	83.782
p04	50	41	2	N/A	18000	599.18	109.438
p05	50	41	2	N/A	18000	568.03	103.974
p06	50	41	3	N/A	18000	598.4	111.026
p07	50	41	4	N/A	18000	586.44	112.461
p08	125	102	2	N/A	18000	2566.6	405.366
p09	125	102	3	N/A	18000	2476.2	416.146
p10	125	102	4	N/A	18000	2470.9	418.195
p11	125	102	5	N/A	18000	2546.1	414.771
p12	40	34	2	N/A	18000	1069.8	79.577
p13	40	34	2	N/A	18000	1161.5	77.449
p14	40	34	2	N/A	18000	1276.6	80.024
p15	80	69	4	N/A	18000	2127.5	211.016
p16	80	69	4	N/A	18000	2497.2	213.649
p17	80	69	4	N/A	18000	2530.1	220.607
p18	120	119	6	N/A	18000	3456	455.384
p19	120	119	6	N/A	18000	3980.2	443.301
p20	120	119	6	N/A	18000	4077.7	446.503
p21	180	156	9	N/A	18000	5260.8	822.436
p22	180	156	9	N/A	18000	5387.3	809.222
p23	180	156	9	N/A	18000	5568.2	872.974
	Average			**453.237**	**18000**	**2262.428**	**304.7218**

6. CONCLUSION AND FUTURE RESEARCH

This study proposes the pickup and delivery multi-depot vehicle routing problem (PDMDVRP), which aims to minimize the total travel cost of vehicles. In this problem, a single commodity type is collected from a set of pickup customers to be delivered to a set of delivery customers by vehicles dispatched from multiple depots. The simulated annealing (SA) is proposed to solve the problem. The proposed SA is tested on 23 benchmark instances. Computational results show that the proposed SA effectively and efficiently solves PDMDVRP.

Future studies may consider extensions of this problem. We believe that this variant will attract further attention to the pickup and delivery problem. Moreover, for more comparative analysis, it will be very beneficial if other heuristic and metaheuristic approaches are proposed to solve this problem.

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