Determining order-up-to level considering backorder cost per unit short on the base-stock inventory system under intermittent demand

Keisuke Nagasawa†
Faculty of Engineering, Hiroshima University
1-4-1, Kagamiyama Higashi-Hiroshima, Hiroshima 739-8527 Japan.
Tel (+81) 82-424-7703, Email1: nagasa-kei@hiroshima-u.ac.jp

Takashi Irohara
Department of Information and Communication Sciences
Sophia University, 7-1 Kioi-Cho, Chiyoda-Ku, Tokyo 102-8554, Japan
Email: irohara@sophia.ac.jp

Abstract: We analyze a single-echelon single-item base-stock inventory system under intermittent demand. Demand is modelled as a compound Bernoulli (binomial) process. In particular, we focus on the definition of backorder cost. Most previous studies define the backorder cost as a fractional charge per unit short and time. However, there are some cases in which applying this type of backorder cost is not appropriate. Therefore, we consider another type of model for intermittent demand, in which the backorder cost is defined as a fractional charge per unit short, i.e., the backorder cost is decided only by the backorder quantity. In the present study, we propose a new model that decides the optimal order-up-to level by minimizing the total cost, including the backorder cost per unit short, under intermittent demand. We demonstrate the effectiveness of the proposed model and present a sensitivity analysis through numerical experiments.

Key words: inventory control, intermittent demand, base-stock system, backorder cost

1 Introduction

Intermittent demand occurs sporadically and occasionally, with some time periods having no demand at all. Intermittent demand is often observed in spare-part items, certain medicines, expensive products, and so on. There are many intermittent demand stock keeping units (SKUs) compared with relatively few fast-moving or non-sporadic SKUs (Dunsmuir & Snyder (1989) and Williams (1984)). Inventory control for intermittent demand has attracted attention in actual practice in the past few decades. In fact, intermittent demand items are held by suppliers at the retail or wholesale supply chain level. The holding costs of intermittent demand items are considerable. Despite the comparatively low contribution to the total turnover, intermittent demand items may constitute up to 60% of the total investment in stock (Johnston *et al.* (2003)). A number of companies have increased investment in intermittent demand management in recent years (Bacchetti & Saccani (2012)).

However, intermittent demand management is very difficult due to a lack of historical demand data and variable characteristics. In conventional studies, demand is assumed to follow a normal distribution or to be non-intermittent demand (Silver & Bischak (2011) and Silver & Robb (2008)). Therefore, conventional inventory control approaches, in which demand is assumed to follow a normal distribution, are invalid for intermittent demand (Lengu *et al.* (2014)).

In research on inventory control with intermittent demand, demand distribution during lead time is considered in theoretical approaches (Teunter & Sani (2009)). Most intermittent demand models consider a compound demand distribution, in which demand is based on demand size and demand arrival. In fact, Lengu *et al.* (2014) argued that using compound distributions provides a good fit for theoretical intermittent demand models. There are two major models that consider demand arrival. If time is treated as a continuous variable, a Poisson arrival is used and demand intervals follow an exponential distribution. This model has dominated the academic literature due to its comparative simplicity (Babai *et al.* (2011)). If time is treated as a discrete variable, a Bernoulli (binomial) process is often adopted for intermittent demand, resulting in a geometric distribution for demand intervals. This model is easy to apply and fits realistic situations (Strijbosch *et al.* (2000)).

Most studies on the discrete time model do not consider the demand distribution during lead time, although some studies have examined inventory control for intermittent demand using a compound Bernoulli (binomial) process (Dunsmuir & Snyder (1989), Janssen *et al*. (1998) and Strijbosch *et al.* (2000)). The distinction between zero and positive demand has also been used to treat intermittent demand by computing the reorder point *s* using an approximation equation for the service level and expected net inventory level in an (*R*, *s*, *Q*) inventory system with service level restriction (Janssen *et al*. (1998)). A similar methodology has been used for a continuous review (*s*, *Q*) inventory system (Dunsmuir & Snyder (1989) and Strijbosch *et al.* (2000)). A number of recent studies have investigated the demand distribution of a compound Bernoulli (binomial) process. For example, a model considering a demand distribution during lead time and a theoretical method for determining order-up-to levels for intermittent demand items in a periodic review system, which can be applied to both cost- and service-oriented systems, have been proposed (Teunter *et al*. (2010)). A model of demand distribution during lead time under a compound Bernoulli process for an imperfect supply has also been considered (Warsing *et al*. (2013)).

There are three primary backorder (shortage) cost criteria in the field of inventory control modelling: fixed cost "per stockout occasion", fractional charge "per unit short", and fractional charge "per unit short and time" (Axsäter (2000) and Silver *et al*. (1998)). A number of studies have examined the definition of backorder cost (Rosling (2002)). In most studies on inventory control with intermittent demand (including Teunter *et al*. (2010)), the total cost consists of holding and backorder cost "per unit short and time".

However, there are cases in which applying the "per unit short and time" criterion is not appropriate, such as the case in which a discount per unit short is considered as a shortage penalty. As such, we consider another type of model for intermittent demand. We formulate the backorder cost based on the "per unit short" criterion. The difference between the "per unit short and time" criterion and the "per unit short" criterion is described in Fig. 1. When the inventory level before a shipment is positive, there is no difference between the two criteria. However, when the inventory level before a shipment is negative (backorder), the backorder costs are different for these two criteria.

In most studies on inventory control, the shortage cost is modelled using a backorder model It has been argued that a base-stock policy is optimal for the backorder model (Chiang (2006)). In addition, most studies on the inventory management of intermittent demand adopt a base-stock policy, because it is easy to consider and adjust the intermittency characteristic. Thus, in the present study, we adopt the backorder model and base-stock inventory system in order to allow easy comparison with previous studies.

In the present study, we propose a new model that decides the optimal order-up-to level for a base-stock inventory system while minimizing the total cost, including the backorder cost per unit short, under intermittent demand. We demonstrate the effectiveness of the proposed model through numerical experiments.

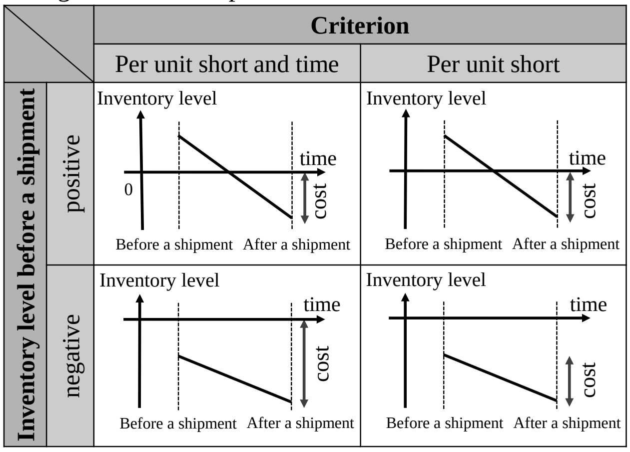

Fig. 1 Difference between backorder costs on two criteria

The remainder of the paper is organized as follows. Section 2 describes the inventory model setting and the notations used in the paper, together with the expected total cost formulation and the method to determine the optimal order-up-to level. In Section 3, we indicate the assumptions and the results of the numerical experiments. We indicate our conclusions and future research in Section 4.

2 Model

2.1 Inventory systems description

We focus on a single-echelon single-item base stock inventory system. Inventory is controlled using a discrete time base-stock inventory system. Each order is triggered immediately upon a demand arrival in order to increase the inventory to the order-up-to level *S*. Each ordered item is delivered after a constant lead time *L*. The review period of the stock is one unit period. The order of events during one period is as follows: receive items, ship items (demand occurs), and review the stock level (an order can be placed). Holding/backorder costs are incurred for each unit holding/backordered. Unfilled demand is backordered.

Demands in successive periods are i.i.d. and are modelled as a compound Bernoulli (binomial) process in which positive demand has a fixed probability; otherwise, the demand is zero. The demand size follows a normal distribution $N(\mu_I,\sigma_I^2)$. The probability density function (p.d.f.) of the lead time demand is shown as a solid line in Fig. 2. The intermittent distribution is the summation of multiplying a normal distribution by the binomial probability. The probability of a demand, as well as the mean and variance of the demand size, must be forecast and updated.

For comparison, we show the demand based on the classical normally distributed model, expressed as $N(L\mu_N, (\sqrt{L}\,\sigma_N)^2)$. Here, μ_N is the average demand size, which is calculated by including no demand, and σ_N is the standard deviation of demand size μ_N. This model is shown by the dashed line in Fig. 2.

The objective is to find the optimal order-up-to level that minimizes the total cost. The optimization problem is formulated as follows.

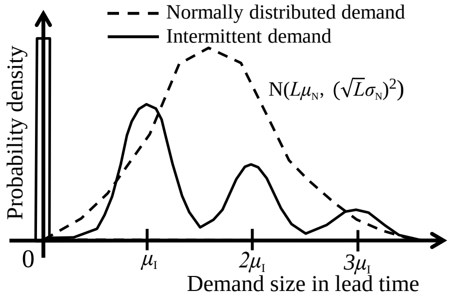

Fig. 2 Probability density of demand during lead time periods

Notation

Decision variable

S : order-up-to level (reorder level: $S-1$)

Parameters

μ_I : average demand size of positive demand for intermittent demand

σ_I : standard deviation of positive demand for intermittent demand

p : probability that the demand occurs ($1/p$: average demand interval, intermittency)

L : lead time

b : backorder cost per unit short

h : holding cost per unit on hand per unit time

Objective

$$\min.\ C_T(S) = b(\omega_1 B_1(S) + \omega_2 B_2(S)) + hH(S) \quad (1)$$

$C_T(S)$: expected total cost per unit time

ω_1 : probability of a positive inventory level before a shipment when shortage occurs

$B_1(S)$: expected backorder quantity per unit time (inventory level before a shipment: positive)

ω_2 : probability of a negative inventory level before a shipment when shortage occurs

$B_2(S)$: expected backorder quantity per unit time (inventory level before a shipment: negative)

$H(S)$: expected inventory level per unit time

Equation (1) indicates the objective of the proposed model, which is to minimize the total cost. The total cost consists of the expected backorder cost and the expected holding cost. We model the two types of backorder cost to consider on the "per unit short" criterion.

2.2 Formulation of the expected total cost and derivation of the optimal order-up-to level

In this section, we formulate the expected total cost, which consists of the expected holding and backorder cost.

2.2.1 Expected inventory level

The expected inventory level can be calculated based on the positive expected value of the p.d.f. of the net inventory level. The p.d.f. of the net inventory level $\varphi(x)$ is as follows:

$$\varphi(x) = \sum_{n=0}^{L}\left(\binom{L}{n} p^n (1-p)^{L-n} \times f_N\left(S - n\mu_I, \left(\sqrt{n}\sigma_I\right)^2\right)\right) \quad (2)$$

where

$$f_N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad \sigma > 0$$

The net inventory level is equal to the order-up-to level S minus the lead time demand. If there is no lead time demand, the net inventory level is S. If there is one lead time demand, the net inventory level is S minus the demand size, following N(μ_I,σ_I^2). If there are n lead time demands, then the net inventory level is S minus the demand size, following N($n\mu_I$,$(\sqrt{n}\ \sigma_I)^2$).

Then, we model the expected value of the positive area in the p.d.f. (2) as the expected inventory level $H(S)$. The equation used to obtain $H(S)$ is as follows:

$$H(S) = \sum_{n=0}^{L}\left(\begin{array}{l}\binom{L}{n} p^n (1-p)^{L-n} \\ \times \int_0^S x \times f_N\left(S - n\mu_I, \left(\sqrt{n}\sigma_I\right)^2\right) dx\end{array}\right) \quad (3)$$

Equation (3) is the expected inventory level, which depends on the holding quantity and the holding time. This holding cost is per unit and per time criterion as well as Teunter *et al.* (2010).

2.2.2 Expected backorder quantity

The backorder costs "per unit short and time" and "per unit short" differ when a shortage occurs at the negative inventory level before a shipment. In order to consider the backorder cost per unit short, we consider the expected backorder quantity for two cases, in which the inventory level before a shipment is positive or negative (hereinafter referred to as the positive and negative cases, respectively). We calculate the event probability ω_i that each case will occur and the expected value of backorder quantity $B_i(S)$ for these two cases.

First, we consider the negative case, because the calculation is easier than that for the case in which the inventory level before a shipment is positive. The expected backorder quantity is also calculated using a distribution of intermittent demand. Here, ω_2 and $B_2(S)$ are as follows:

$$\omega_2 = \sum_{n=1}^{L}\left(\begin{array}{l}\binom{L-1}{n-1} p^n (1-p)^{L-n} \\ \times \int_{-\infty}^0 f_N\left(S-(n-1)\mu_I, \left(\sqrt{n-1}\sigma_I\right)^2\right) dx\end{array}\right) \quad (4)$$

$$B_2(S) = \mu_I \quad (5)$$

The probability ω_2 for the negative case is derived as the summation of the products of the binomial probability of the inventory level after shipping items (demand occurs) and the probability that the inventory level before a shipment is negative, as shown in equation (4). This binomial probability after shipping is derived from the situation that demand occurs in the last period in the lead time. The probability that the inventory level before a shipment is negative is calculated using the p.d.f. of the normal distribution corresponding to the state before the shipment. If the inventory level before the shipment is negative, backorder quantity $B_2(S)$ is equal to the demand size, because the demand is backordered. Equation (5) indicates that the expected demand size is μ_I.

In the positive case, the expected backorder quantity is given by a complex model, although the probability of shortage in the positive case can be obtained in a similar manner to the negative case. Here, ω_1 and $B_1(S)$ are obtained as follows:

$$\omega_1 = \sum_{n=1}^{L} \left(\begin{pmatrix} L-1 \\ n-1 \end{pmatrix} p^n (1-p)^{L-n} \times \int_0^S f_N\left(S-(n-1)\mu_I, \left(\sqrt{n-1}\sigma_I\right)^2 \right) dx \right) \tag{6}$$

$$B_1(S) = \sum_{n=1}^{L} \left(\int_0^\infty x \times f_N\left(\mu_I - \alpha, \left(\sqrt{\sigma_I^2 + \beta} \right)^2 \right) dx \right) \tag{7}$$

where

α : expected value of the truncated normal distribution $f(x; S-(n-1)\mu_I, \sqrt{n-1}\sigma_I, 0, \infty)$

β : variance of the truncated normal distribution $f(x; S-(n-1)\mu_I, \sqrt{n-1}\sigma_I, 0, \infty)$

If the inventory level before a shipment is positive, backorder quantity $B_1(S)$ is the demand size minus the inventory on hand. The demand size follows $N(\mu_I, \sigma_I^2)$. The inventory on hand is calculated by the truncated normal distribution $f(x; S\text{-}(n\text{-}1)\mu_I, \sqrt{n\text{-}1}\sigma_I, 0, \infty)$ with respect to each number of lead time demands. Then, we derive the expected backorder quantity $B_1(S)$ as the expected value of the positive area of the combined normal and truncated normal distributions, as shown in equation (6). The event probability ω_1 is derived as the summation of the products of the binomial probability of the inventory level after shipping items (demand occurs) and the probability that the inventory level before a shipment is positive, as shown in equation (7).

The equations presented herein give the expected backorder costs and the event probabilities for the positive and negative cases described in Fig. 1. These equations can be used to calculate the total expected backorder costs for the positive and negative cases.

2.2.3 Derivation of optimal order-up-to level

The total cost $C_T(S)$ is the summation of the expected holding cost and the expected backorder cost. This total cost $C_T(S)$ includes the normally distributed model, in the case of p=1. So this model can also be regarded as the expansion of the normally distributed model.

The objective is to find the optimal order-up-to level that minimizes the total cost $C_T(S)$. In the total cost $C_T(S)$, the holding cost and backorder cost are in a trade-off relationship for order-up-to level S. In addition, total cost $C_T(S)$ is a convex function in S and the optimal order-up-to-level, S^*, can be determined by using a local search procedure such as Golden Section search (Press *et al*. (1996)).

2.2.4 The backorder rate

We model the two backorder rates for the evaluation value to use numerical experiments.

First, we define the backorder rate for occasion (*BRO*) as the ratio of the number of backorder occasions to all periods. The classical newsboy type model can determine the optimal order-up-to level for fast moving SKUs using this ratio. The order-up-to level is optimal when this ratio equals $h/(b+h)$ (Silver *et al*. (1998) and Silver & Robb (2008)). We can also calculate the theoretical value of the back order rate for occasion using some distribution of intermittent demand. The equation is as follows;

$$BRO = \sum_{n=1}^{L} \left(\begin{pmatrix} L-1 \\ n-1 \end{pmatrix} p^n (1-p)^{L-n} \times \int_{-\infty}^{0} f_N\left(S-n\mu_I, \left(\sqrt{n}\sigma_I\right)^2 \right) dx \right) \tag{8}$$

The backorder rate for occasion can be derived by a simple approach. A backorder occurs only when demand occurs, so we focus on the p.d.f. of the inventory level after shipment. We derive the summation of the products of the p.d.f. of normal distribution and the binomial probability of the inventory level after shipment as the backorder rate.

Next, we define the backorder rate for quantity (*BRQ*) that is the ratio of the expected backorder quantity to all demand size. The equation can be derived by the expected backorder quantity $\omega_1 B_1(S)$ plus $\omega_2 B_2(S)$ divided by the average demand $p\mu_I$.

$$BRQ = \frac{(\omega_1 B_1(S) + \omega_2 B_2(S))}{p\mu_I} \tag{9}$$

3 Numerical experiment

By numerical experiments, we analyze the effectiveness of the proposed model. We confirm the validity of the cost formulation, the feature of intermittent demand model, the cost advantage for the classical models, and the robustness of the proposed model for forecasting error. In numerical experiments, we consider the parameter range. We vary the average demand interval $1/p$, the lead time L and the ratio of the unit holding cost h to the unit backorder cost b (i.e. h/b) in order to analyze its effect on the performance in some experiments. The parameter set used in this experiment is 4000 combinations as in Table 1. The parameters about demand μ_I, σ_I, $1/p$ are known in the numerical experiments.

Table 1 Parameter set of experiment

μ_I	20000
σ_I	2000, 5000
$1/p$	1,2,4,6,8,10,20,30,40,50
L	1,2,4,6,8,10,20,30,40,50
S	[0,190000] (step 10000)
Simulation periods	1 million periods
# of iterations	100

3.1 Comparison of model formulation and simulation

We compare the output values of a simulation and the formulation in order to confirm the validity of the model formulation. The output values are the expected inventory level and the expected backorder level. The parameter set used in this experiment is 4000 combinations as in Table 1.

As an evaluation value, we use the relative error of the output values between the results of the model formulation and the simulation (the relative error = (the value of simulation – the value of formulation) / the value of simulation). Table 2 indicates the statistic of relative error and the rate of the number of combinations whose relative error is within ±0.05 for output values. The relative errors of more than 90 % of all the combinations are within ±0.05 for both output values. From this experiment, we can confirm a certain level of validity of the proposed model.

Table 2 Results of experiment 3.1

	The expected inventory level	The expected backorder quantity
Average	-0.01	-0.03
Max	0.63	0.97
Minimum	-4.04	-22.26
Within ±0.05	98.55%	92.78%

3.2 Numerical analysis of the total cost of the proposed model

In this sub-section, our main objective is to show the difference of the total cost between the intermittent model and the normally distributed model. We analyze the movement of the total cost $C_T(S)$ and the slope of the total cost $C_T(S)$ with respect to the order-up-to level S. The parameter set in this experiment is as shown in Table 3. Figs. 3–6 show the result of substituting L=4, h/b=0.02 in the equation of $C_T(S)$ and the slopes of $C_T(S)$ for different intermittency $1/p$.

As Fig. 3 shows, $C_T(S)$ is convex function in S in the whole parameter set. The optimal order-up-to levels S^* are the minimum points of these graph. As $1/p$ increases, i.e. the degree of intermittency increases, $C_T(S)$ at low order-up-to levels is decreased. This can be explained by the feature of the model that the demand size during lead time decreases as $1/p$ increases. Even if we backorder all items when S=0, the backorder quantity and cost decrease with $1/p$ because of decreasing demand size.

Table 3 Parameter set of experiment 3.2

μ_I	20000
σ_I	2000
$1/p$	1,2,5,7,10,15,21
L	1,2,4,6,8,10,12,14,16,18,20
S	[0,190000] (step 10000)
h/b	[0.002,0.1](step 0.002)

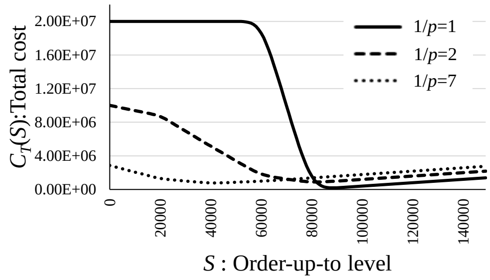

Fig. 3 $C_T(S)$ in order-up-to level S (L=4, h/b=0.02)

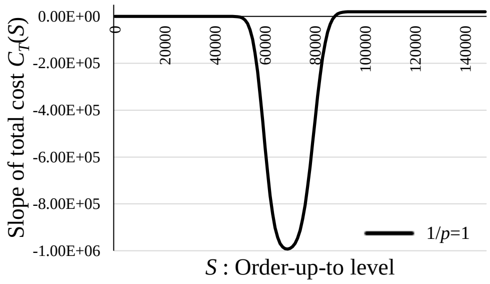

Fig. 4 The slope of $C_T(S)$ in order-up-to level S for $1/p$ = 1 (L=4, h/b=0.02)

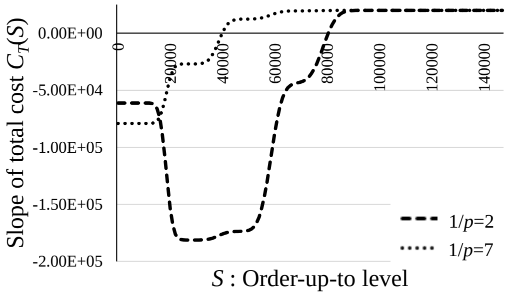

Fig. 5 The slope of $C_T(S)$ in order-up-to level S for $1/p$ = 2 and 7 (L=4, h/b=0.02)

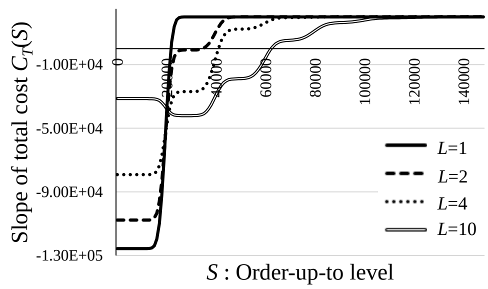

Fig. 6 The slope of $C_T(S)$ in order-up-to level S for L = 1, 2, 4 and 10 ($1/p$=7, h/b=0.02)

When we pay attention to the slope of $C_T(S)$, as shown in Fig. 4 and Fig. 5, there is a great difference between $1/p$=1 and the others. The slope of $1/p$=1 is a simple form with the features of a normal distribution, but intermittent patterns vary smoothly and have a complex change of slope value. When the slope is zero, this order-up-to level is optimal because of the convex property of $C_T(S)$. Fig. 6 shows the slopes of $C_T(S)$ in the case of $1/p$=7, h/b=0.02 for different lead times. We can also see some points of the change in slope. This complex change of slope comes from the wave of the p.d.f. of the intermittent demand. The number of these change points is equal to the number of the lead time, but with a large lead time L these changes are too many and small, making it difficult to see them. This change of the slope is a significant feature of intermittent demand.

Under intermittent demand, the movement of $C_T(S)$ is complex. This complexity change makes finding the optimal order-up-to level difficult. So it is important to consider the intermittency and its effect.

3.3 Numerical analysis of the optimal order-up-to level in the proposed model

We analyze the movement of the optimal point in the formulation of the proposed model for different parameters. The evaluation value is the optimal order-up-to level S^* and the *BRO*. The parameter set in this experiment is as shown in Table 4.

Table 4 Parameter set of experiment 3.3

μ_I	20000
σ_I	2000, 5000
$1/p$	[1,20](step 1)
L	[1,50](step 1)
h/b	[0.001,0.1](step 0.001)

Figs. 7–8 show the results for the optimal point of h/b=0.02 for different intermittency $1/p$ and lead time L. S^* increases in the lead time L as well as the result of no intermittency. This is because the demand during the lead time is larger as the lead time increases. Conversely, S^* decreases in the intermittency $1/p$ due to the decreasing of the demand during the lead time.

In Fig. 8, the *BRO* for intermittent demand increases, although the *BRO* for no intermittent demand is nearly constant for the lead time L. In the "per unit short and time" model, the optimal order-up-to level is when the backorder rate equals $h/(b+h)$ (Silver *et al.* (1998) and Silver & Robb (2008)), which is in no relationship to the lead time L. However, the theoretical optimal *BRO* $h/(b+h)$ does not apply to intermittent demand. This tendency of the *BRO* of the proposed model comes from the difference of the back order cost definition. The theoretical optimal *BRO* $h/(b+h)$ fits into the *BRO* in 1/p=1, because the situation of no intermittency has little effect on the difference of the cost definitions. In the "per unit short and time" model, the holding cost and backorder cost are calculated by the same p.d.f. So we can argue the expected cost on the same condition and derive the optimal *BRO* from the ratio of the unit holding/backorder cost. In the proposed model, the holding cost and backorder cost are calculated from the different p.d.f. So we cannot derive the optimal *BRO* from the ratio of the unit holding/backorder cost. In addition, the backorder cost of the proposed model is lower than the "per unit short and time" model as Fig. 1. As a result, the lower backorder cost makes it possible to have more backorder.

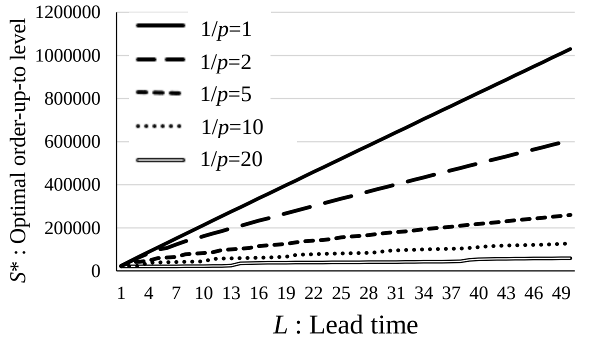

Fig. 7 The optimal order-up-to level S^* during lead time L for $1/p$ = 1, 2, 5, 10 and 20 (h/b=0.02)

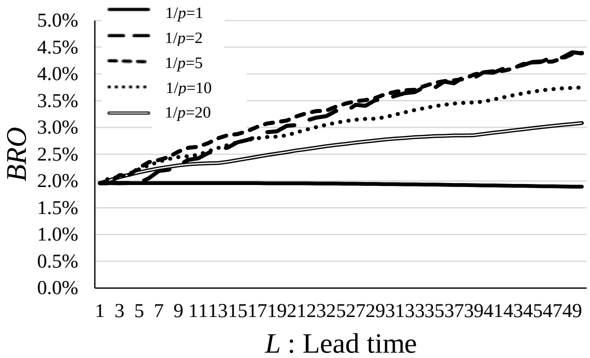

Fig. 8 *BRO* in S^* during lead time L for $1/p$ = 1, 2, 5, 10 and 20 (h/b=0.02)

Figs. 9–10 show the results for the optimal point of L=10 for different intermittency $1/p$ and the ratio of unit holding cost to unit backorder cost h/b. Fig. 9 shows that S^* decreases in h/b. The movement of no intermittency is very slow. However, there are some rapid changes in the results for intermittent demand. These changes occur on the supposition of the amount of demand in the lead time in the optimal situation. This feature is related to the result of 3.2 and is unique to the intermittent demand. Fig. 10 shows also the rise of the optimal backorder level compared to the result of the no intermittency pattern. In the case of $1/p$=20, we can see the convergence in 0.05. This is the upper limit of the backorder rate in $1/p$=20. The upper limit of the backorder rate is p, because even if all the demand is backorder, there is no demand period in the other periods.

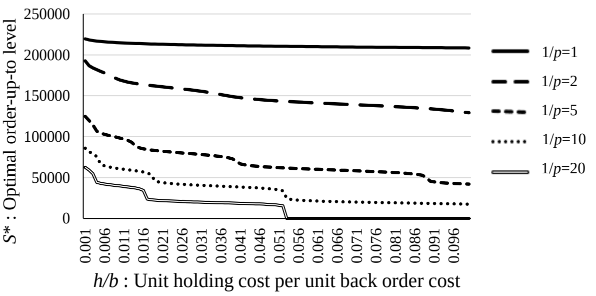

Fig. 9 The optimal order-up-to level S^* in h/b for $1/p$ = 1, 2, 5, 10 and 20 (L=10)

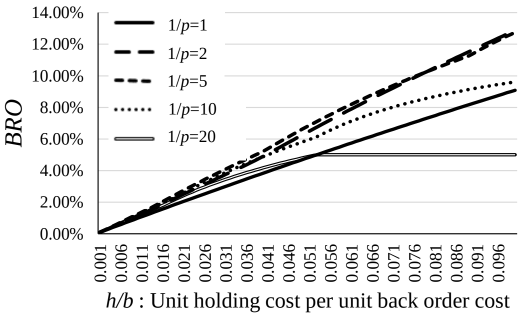

Fig. 10 *BRO* in S^* in h/b for $1/p$ = 1, 2, 5, 10 and 20 (L=10)

From this experiment, we confirm the features of the optimal point of the proposed model. The differences with other models that are argued in this section indicate the necessity of the proposed model.

3.4 Comparison with previous models

Using simulations, we compare three models (the proposed model (PM), Teunter model (Teunter *et al.* (2010)), and the normal model), which decide the order-up-to level. The Teunter model defines the backorder cost on the "per unit short and time" criterion for intermittent demand. The normal model defines the demand during lead time as a normal distribution $N(L\mu_N, L\sigma_N^2)$ and does not consider the intermittency of demand. The parameter set is all the combinations of Table 5.

Table 5 Parameter set of experiments 3.4

μ_I	20000
σ_I	2000
$1/p$	1,2,5,7,10,15,21
L	1,2,4,6,8,10,12,14,16,18,20
h/b	[0.002,0.1] (step 0.002)
Simulation periods	500/p
# of iterations	100

The parameters for demand μ_I, σ_I, $1/p$ are known. The simulation outputs the evaluation values on the definition of PM and the differences of the three models are only the adopted order-up-to levels. We use the relative error of two models as the comparison expression (Relative error = (Result of each model - Result of PM) / Result of each model). We analyze the comparison of PM to the Teunter model and the normal model, respectively.

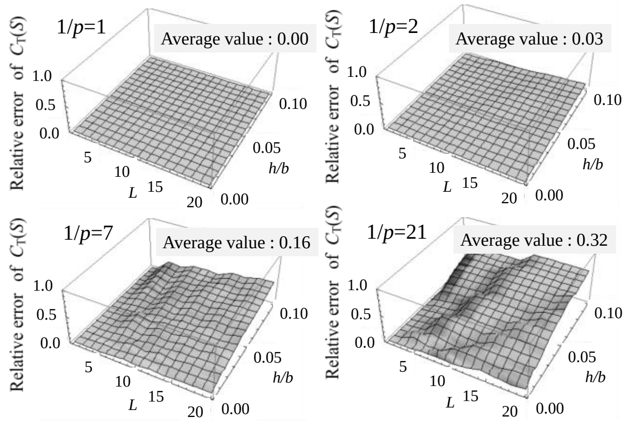

Fig. 11 Relative error of $C_T(S)$ for $1/p$ = 1, 2, 7 and 21 compared to the Teunter model

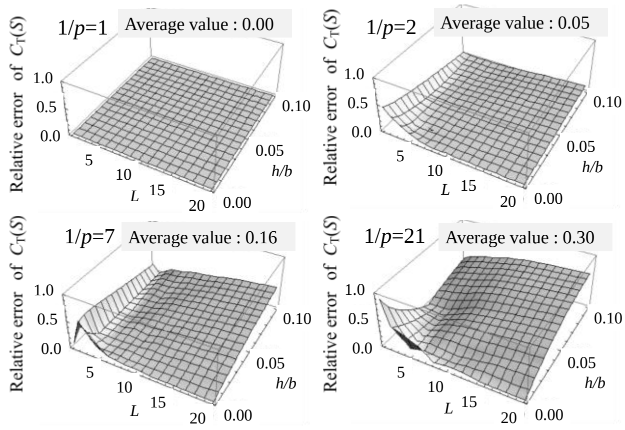

Fig. 12 Relative error of $C_T(S)$ for $1/p$ = 1, 2, 7 and 21 compared to the normal model

In comparison to the Teunter model, as shown in Fig. 11 which is the result of the relative error of $C_T(S)$, PM outperforms in all cases except for L=1 or a part of $1/p$=1 (no intermittency). In L=1, PM and the Teunter model suppose the same p.d.f. during the lead time. In $1/p$=1, the situation of no intermittency has little effect on the difference of the cost definitions. So there are no differences for the two models. In the other parameter set, the order-up-to level of the Teunter model is higher than that of PM. From Fig. 1, the backorder cost of the "per unit short and time" criterion is calculated as higher than the "per unit short" criterion. It is obvious that the higher backorder cost makes the order-up-to level higher.

The features of the difference of $C_T(S)$ between the PM and the Teunter models are complex. We can see that the relative error of the $C_T(S)$ increases in each parameter $1/p$, L, h/b. This result may depend on the evaluation value related to backorder, because the difference of the two models is the backorder cost definition.

In comparison to the normal model, as shown in Fig 12 which is the relative error of $C_T(S)$, PM outperforms in all cases except for $1/p$=1 (no intermittency). The normal model does not consider the intermittency, so it is obvious that there is no difference in $1/p$=1 and that the difference widens as $1/p$ increases.

The features of the difference of $C_T(S)$ between PM and the normal model are also complex. The relative error of $C_T(S)$ increases in L and h/b. In addition, a great difference is found in a very small L and h/b. These losses of the Teunter model occur because the intermittency of demand is not considered.

In small L, the normal model decreases the order-up-to level. However, the probability that demand occurs in every period during the lead time is comparatively high in small L. Then the probability of a high demand size is increased during the lead time. So we need a high order-up-to level to prepare for the high demand. The normal model cannot consider this characteristic and adopts a too small order-up-to level. In addition, if the unit backorder cost rate is high, the loss of too much backorder becomes enlarged.

4 Conclusion

In this research, we developed a formulation for a new model for determining the optimal order-up-to level by minimizing total cost, including the backorder cost per unit short, under intermittent demand. We model the intermittent demand as a compound Bernoulli (binomial) process and normally distributed demand size. In the formulation of backorder cost, we divide the expected backorder quantity into two scenarios, where the inventory level before a shipment is positive or negative, to calculate based on the "per unit short" criterion.

We analyze the effectiveness of the proposed model by numerical experiments. We can confirm the validity of the cost formulation, the features of the intermittent demand model, the cost advantage for the classical models, and the robustness of the proposed model for forecasting error.

As the proposed model is a very limited situation, further research for more flexible models is needed (e.g., review interval, stochastic lead time, lost sales model, etc.). Therefore, continued expansion and more detailed numerical experiments or experiments with real data are required.

Acknowledgement

The authors would like to thank Mr. Yuto Suetsugu, master course student in Prof. Irohara's laboratory of Sophia University, for transforming the mathematical expression and conducting the numerical experiments

REFERENCES

Axsäter, S., 2000. Inventory Control. Kluwer Academic Publishers Group, Dordrecht, The Netherlands

Babai, M. Z., Jemai, Z., & Dallery, Y., 2011. Analysis of order-up-to-level inventory systems with compound Poisson demand. European Journal of Operational Research, 210(3), 552-558.

Bacchetti, A., & Saccani, N., 2012. Spare parts classification and demand forecasting for stock control: Investigating the gap between research and practice. Omega, 40(6), 722-737.

Chiang, C., 2006. Optimal ordering policies for periodic-review systems with replenishment cycles. European Journal of Operational Research, 170(1), 44-56.

Dunsmuir, W. T., & Snyder, R. N., 1989. Control of inventories with intermittent demand. European Journal of Operational Research, 40(1), 16-21.

Janssen, F., Heuts, R., & De Kok, T., 1998. On the (R, s, Q) inventory model when demand is modelled as a compound Bernoulli process. European Journal of Operational Research, 104(3), 423-436.

Johnston, F. R., Boylan, J. E., & Shale, E. A., 2003. An examination of the size of orders from customers, their characterisation and the implications for inventory control of slow moving items. Journal of the Operational Research Society, 54(8), 833-837.

Lengu, D., Syntetos, A. A., & Babai, M. Z., 2014. Spare parts management: Linking distributional assumptions to demand classification. European Journal of Operational Research, 235(3), 624-635.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P., 1996. Numerical recipes in C (2nd ed.). Cambridge: Cambridge University Press.

Rosling, K., 2002. Inventory cost rate functions with nonlinear shortage costs. Operations Research, 50(6), 1007-1017.

Silver, E. A., Pyke, D.F., & Peterson, R., 1998. Inventory management and production planning and scheduling (3rd ed.) New York, John Wiley and Sons, Inc

Silver, E. A., & Robb, D. J., 2008. Some insights regarding the optimal reorder period in periodic review inventory systems. International Journal of Production Economics, 112(1), 354-366.

Silver, E. A., & Bischak, D. P., 2011. The exact fill rate in a periodic review base stock system under normally distributed demand. Omega, 39(3), 346-349.

Strijbosch, L. W. G., Heuts, R. M. J., & Van der Schoot, E. H. M., 2000. A combined forecast-inventory control procedure for spare parts. Journal of the Operational Research Society, 51(10), 1184-1192.

Teunter, R. H., & Sani, B., 2009. Calculating order-up-to levels for products with intermittent demand. International Journal of Production Economics, 118(1), 82-86.

Teunter, R. H., Syntetos, A. A., & Babai, M. Z., 2010. Determining order-up-to levels under periodic review for compound binomial (intermittent) demand. European Journal of Operational Research, 203(3), 619-624.

Warsing, D. P., Wangwatcharakul, W., & King, R. E. 2013. Computing optimal base-stock levels for an inventory system with imperfect supply. Computers and Operations Research, 40(11), 2786-2800.

Williams, T. M., 1984. Stock control with sporadic and slow-moving demand. Journal of the Operational Research Society, 939-948.