Using Differential Evolution in Skull Prosthesis Modelling by Superellipse

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Abstract. This paper presents a method to create the geometric model of skull defects to be applied in skull repair. The method is to generate a graphic that represents the vanished information in the skull when the skull defect is anomalistic and non-symmetric. We propose a geometric shape of skull bone curvature in tomography by using superellipse concept to recover the defect. The superellipse can be adjusted in every tomography slice and the arcs which represent the defect of the skull bone can be modeling in 3D model. The problem is that many similar ellipses can be created by superellipse. This paper finds the suitable solution. This research employs the Differential Evolution (DE) algorithm to find the best solution for every tomographic slice. Once every slice found the best solution, the defect of skull bone can be rebuilt in a 3D model.

Keywords: *Prosthesis, Tomography, Geometric Modeling, Differential Evolution algorithm*

1. INTRODUCTION

Irregularities of the Cranial defects usually occur by injury, birth defect, or head trauma which will result in high risk of brain infections and brain low protection, damage poor postoperative recovery, and cerebral metabolism. In order to restore the integrity of the cranial cavity and maintain a stable physiological intracranial pressure, cranioplasty is commonly performed in clinical by grafting of the deficient regions. Generally, bone grafts which made by Ti6Al4V metal implants or various materials obtained in the initial surgical operation are the optimal bone flap for the cranioplasty. However, an unfitted cranial prosthesis of artificial polymer or metal implants might cause the recrudescent infections. With the development of medical imaging techniques and computer aided system technology, the fitted shape of skull prosthesis could be fabricated effectively and accurately in operation room in a timely fashion. 3D printing (3DP) process provides the promising potential to irregular cranial defects due to its advantages of rapid process and customization shape. Image process of cranial defects is critical to prosthesis design, print, post-proposes and final clinical cranial reconstruction. Within a rapid 2D image process and 3D cranial model construction system, clinical surgeons will be able to implement a customized prosthesis to patients in the shortest surgical time and maximum surgical quality, especially, for emergency cases. This study aims to develop a new algorism by manipulating medical image process which could have automatically define the range, shape, curvature of cranial defects in 2D CT images and 3D anatomy registration. To generate the curve that we needed, we adjust the parameters in superellipse (Gielis, 2003) to estimate the data that represents it curvature. The parameters can be obtained through optimization methods. In this study, we use Differential Evolution algorithm (DE) (Storn & Price, 1997) and superellipse to solve the skull repairing problem. The rest of this paper is organized as follows. Section 2 will review the other skull repair method. Section 3 describes the proposed repair method and procedure. Section 4 presents the experimental studies and discussions. Finally, conclusions will be given in Section 5.

2. Related work

2.1 Past classification method

Several computational methods for prosthesis definition and design could be used to reconstruct the cranial defects in literatures such as mirroring technique (Maravelakis et al., 2008), surface fitting (Chong, Lee, & Kumar, 2006), and deformation algorithm of skull template (Dean & Min, 2003). These methods can be categorized into skull template registration (Liao et al., 2011; Liao et al., 2013) and parametric model fitting (Huang & Shan, 2011; Z. Zhang, Zhang, & Song, 2014). In the skull template registration, the cranial CT images acquired before the defects occur. After the registration, the cranial defects are intrinsically repaired and modeled in the registered images. However, the CT images without cranial defects are usually unavailable in clinical.

The parametric model fitting could repair the defects without reference images. After defining a parametric model, the missing cranial surface is generated by fitting the remaining cranial surface information of the cranium bone. A curve fitting in 2D images owns the same concept with a surface fitting in 3D volumes by using parametric model fitting. The common surface fitting methods include Bezier surface (Chong et al., 2006), radial basis function (Carr, Fright, & Beatson, 1997). The curve and edge of the fitted cranial surface is dependent on their parameter settings and the applied models. Although the missing cranial surface can be obtained in a single 3D fitting, the parameters need to be adjusted to consider the surface shapes and curvatures in different cranium areas, which is a complex problem and individual dependence. 3D fitting without additional constrains may produces abnormal concaves or convex in the fitted cranial surface.

2.2 Application of asymmetric solutions

Evolutionary algorithms (EAs) (Back, Fogel, & Michalewicz, 1997) were inspired by animal's society behaviors and developed many variant meta-heuristic algorithms, such as Particle Swarm Optimization Algorithm (PSO), Genetic Algorithm (GA), and others, in order to solve complex combination optimal problems. Several reviews have used EAs to solve broken skull reconstruct optimal problems (Greboge, Grebogi, Rudek, & Canciglieri Jr, 2011; Junior, Rudek, & Greboge, 2011).

Rudek, Greboge, Coelho, and Canciglieri Jr (2011) used GA to evaluate the parameters of ellipse to overcome the disadvantages of traditional mathematical approaches, for instance suboptimal solution selection, and problem solving inefficiency due to discrete search spaces. This method can generate different types of ellipses to fit the skull. However, junction issues between reconstruction borders have been brought out in their study. Rudek, Canciglieri Jr, and Greboge (2013) have mentioned that the mirror image concept could be adapted into the most cranial reconstruction problems except the defects are in forehead or back of the head(Li, Xie, Ruan, & Wang, 2009; Sengupta, Sengupta, & Ghosh, 2005). In order to reconstruct the asymmetric cranial prosthesis issues, they used PSO (Kennedy, 2010) and superellipse to conquer this asymmetric defect problem.

According to the aforementioned, many researchers solved cranial-repaired problems combined with EAs. Differential Evolution (DE) was proposed by Storn and Price (1997) and is one of the EAs and one of the popular meta-heuristic algorithms. DE is a population-based stochastic search technique that adapts fewer parameters as well as keeps the diversity of the population. This study decided to apply DE methodology since it is more reliable, higher performance, and rapid convergence in solving optimal problems (Hu, Xiong, Su, & Zhang, 2013; Salman, Engelbrecht, & Omran, 2007) than other EAs.

This study took the DE algorithm advantages of fewer parameters, reliability, and fast convergence (Hu et al., 2013; Salman et al., 2007) for solving cranial prosthesis model issues. This investigation proposes to optimal the superellipse parameters by DE methodology in order to create a fitted curvature in each 2D image, furthermore, those fitted curvatures in each 2D image could build a 3D patient-match model for cranial prosthesis.

3. Proposed Method

In order to implement our prosthesis reconstruction strategy, the flow chart in Figure 1 have been established to realize the medical image data process, fitting curve calculation on 2D medical images, and 3D modeling construction.

I. The first step is the CT images acquirement and conversion. In order to filter out the skull bone region in a CT image, some noises such as brain or other soft tissues will need to eliminate in the CT images.

II. The second step is to reconstruct the geometrical model of missing region of cranial bone by using DE algorithm (Storn & Price, 1997). Superellipse formula is adapted to simulate the cranial curve with DE algorithm's rapid search method. Details show in

Section 3.1 and 3.2.

III. The third step, all layers of processed CT images in second step will be integrated and build a 3D visualization of the skull. The cranial prosthesis will be visually validated in this step.

IV. The fourth step is the final stage to print out the physical model of cranial prosthesis. In this study, the printed cranial prosthesis will not be included in the result and discussion.

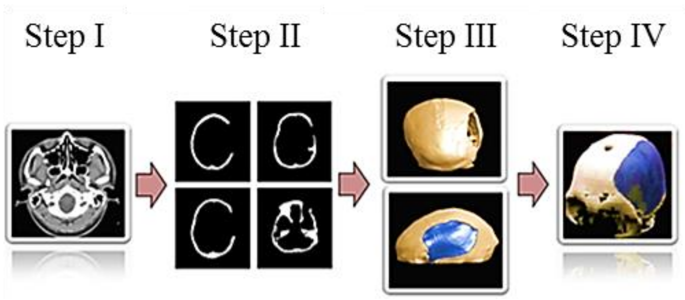


Figure 1 Four main steps of the procedure.

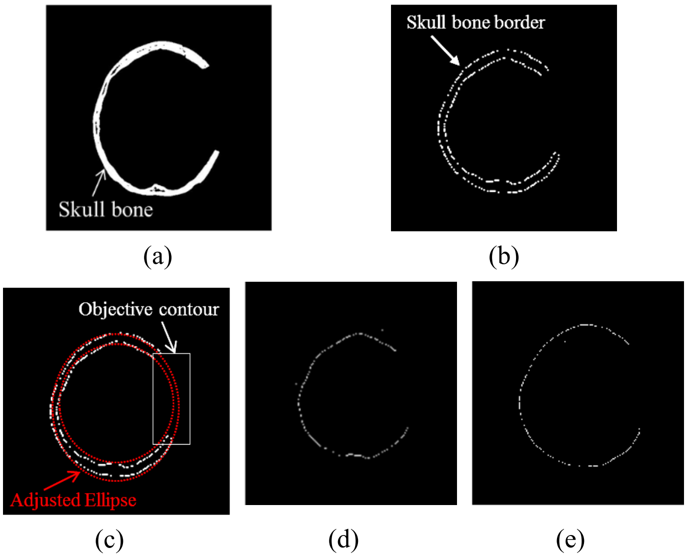


Figure 2 (a) A CT slice of broken skull. (b) The inner border and the outer border of the skull bone (c) Example of fill up the defect skull. (d) The inner border of the skull bone. (e) The outer border of the skull bone.

Figure 2 shows that the concept of superellipse development with DE optimal search process. The method proposed in this study is mainly to solve the problem of repairing damaged cranial skull. Superellipse is used to generate different shapes of curves and compared with remaining skull regions. The white region in Figure 2(a) represents broken skull bone area. In this study, we describe the imperfect skull bone with the inner border and the outer border of the white region as Figure 2(b). This research goal is to use the deformed curves which generated by superellipse (red dotted line) to fill up the breakage as Figure 2(c). We do the inner and outer border experiment separately. The inner border of the skull bone is shown in Figure 2(d). The outer border of the skull bone is shown in Figure 2(e).

3.1 Superellipse definition

From the perspective of skull anatomy, most of them are oval (Rudek et al., 2013). So the bone in each image can be modeled as oval with different shapes. Superellipse can fulfill all requirement of the skull modeling. This mathematical model could create any geometry which is commonly found in nature and create different shapes (Gielis, 2003; Spehr, Gumhold, & Fleming, 2011; X. Zhang & Rosin, 2003). The mathematical formulation of the superellipse is presented in equation (1).

$$r(\varphi) = \frac{1}{\sqrt[n1]{\left(\left(\left|\frac{1}{a}cos(\frac{m}{4}\varphi)\right|\right)^{n2} + \left(\left|\frac{1}{b}sin(\frac{m}{4}\varphi)\right|\right)^{n3}\right)}} \quad (1)$$

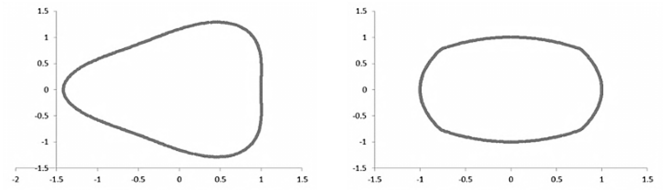
From the equation (1), the parameters, a and b represents the major axis and the minor axis of the ellipse. The m value is the number of fixed arguments on the unitary circle. For example, we obtain a quadrilateral shape when m= 4. The values of $n2$ and $n3$ determined if the shape is inscribed or circumscribed in a unitary circle (Rudek et al., 2013).

In Figure 3, we can observe different polygon's achieved by modified values of superellipse parameters. The polygon in Figure 3(a) has three fixed points (m=3) as a triangle shape and Figure 3(b) has four fixed points (m=4) as a quadrilateral shape, but in these two cases both with rounded border because of modified n values. By the observation, some CT slices in the middle of skull are similar with a normal ellipse. When the shape turns more oval, we can adjust the values of n in the superellipse to fit the skull.

To plot the curve on the CT, we can set up the binary value of a pixel in the current position (i, j) by equation (2) in generating ellipse. The ellipse point $E(i, j)$ can be obtained by polar coordinates.

$$E(i, j) = (r * cos\varphi + x_0, r * sin\varphi + y_0) \quad (2)$$

The concept is adapted to generate elliptical shape which can self-adjust in skull border, shown as Figure 4 (red line). The task is to create the most suitable piece of curved shape for this broken skull by tuning the parameters in superllipse. The optimal parameter set will be found by using DE algorithms in this investigation. Figure 4 shows the example that the parameters of arc have been changed and improved to fit the imperfect skull border in CT slice.



(a) (b)

Figure 3 Examples for different *superellipse* parameters. (a) [a=1, b=1, m=3, $n1$=2, $n2$=4, $n3$=4]; (b) [a=1, b=1, m=4, $n1$=200, $n2$=50, $n3$=50];

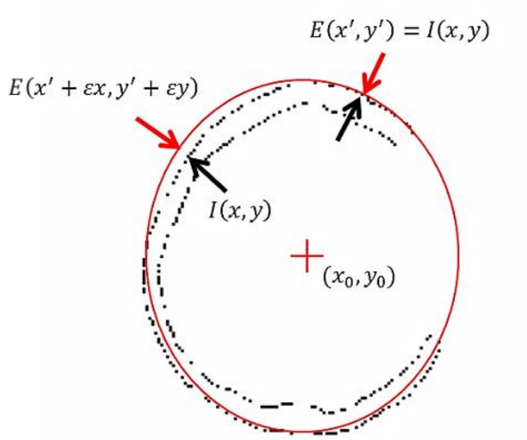


Figure 4 Example of superllipse adjustment in skull border.

3.2 Differential evolution algorithm

As the aforementioned, DE is proposed by Storn and Price (1997) and is a population-based stochastic search technique that is more simply performed with fewer parameters and keeps the diversity of the population. Compares with other EAs (Back et al., 1997), DE is more reliable, performed good, and fast convergence in solving optimal problems (Hu et al., 2013; Salman et al., 2007).

The fitness function $F(X)$ to estimate the ellipse adjustment can be found in (Rudek, Canciglieri Jr, & Greboge, 2012) and it is given by equation (3). Each point $I(i, j)$ in the image I is the position of one pixel and its binary value is 1. The l and c values are respectively the total of rows and columns in the image I.

$$F(X) = \frac{\sum_{i=1}^{l} \sum_{j=c}^{c} [I(i,j) * E(i,j)]}{\sum_{i=1}^{l} \sum_{j=c}^{c} I(i,j)} \quad (3)$$

The r value is the parameter of superellipse generated in equation (1) to estimate the $E(x, y)$ pixel position by evaluation of equation (2). Other parameters, such as a, b, m, $n1$, $n2$, $n3$ from equation (1), center coordinates x_0, y_0 are all generated by the DE algorithm. Besides, the search mechanism of DE in this study includes individual genes' mutation, crossover, and selection operations in evolution process, as the follow pseudo code adapted from (Hegerty, Hung, & Kasprak, 2009).

```
Pseudo-code of skull repair by using Differential Evolution
Begin
  Generate randomly initial populations of solution: a,b,m,n1,n2,n3,x0,y0.
  Calculate the fitness F(X) of the initial populations.
  Repeat
    For each parent, select three populations randomly.
    Create one offspring using the DE mutation and crossover operators.
    Do this a number of times equal to the population size.
    For each member of the next generation
      If offspring is more fit than parent
        Parent is replaced.
    Until a stop condition is satisfied.
End.
```

The necessary parameters mentioned in the pseudo code. Rudek et al. (2011) proposed limited range to restricts the values in the initial population by specifying the lower and upper bounds. The up and low value limit of those parameters can be defined from (Rudek et al., 2011). For example, the up and low limit of 'a' value is between 80 and 200, 'b' value is between 100 and 200, center coordinates 'x_0', 'y_0' in image is between 230 and 270. But the 'm' and '$n1$' '$n2$' '$n3$' are not mentioned in (Rudek et al., 2011). Therefore, according to our analysis of the image in tomography, the up and low bound of the 'm' value is between 1 and 8 and '$n1$' '$n2$' '$n3$' value are between 1 and 32. The unit of measurement is in pixels.

4. Analyses of the Results

A partial white region lacking in lateral region of skull was used to be a testing sample in this case. The DE script was programed in MATLAB. First we extract the skull bone CT image and find out the inner border and the outer border of the cranial skull bone. Then, according to the skull bone border in Figure 1(d)(e), we use DE and superellipse to fulfill the bone border reconstruction. All parameter values of DE are shown as Table 1 suggested in (Storn & Price, 1997). Each algorithm is performed 30 independent runs, 2000 iterations and the particle size of 64. This study employs the experiment at a 64-bit Win7 system computer with i5-2400 CPU and 8G RAM.

Table 1 The parameter values of DE algorithm.

Parameters/DE	DE/rand1
F	0.5
CR	0.9
NP	64
Iteration	2000
Dimension	8

4.1 Experiments Analysis

All parameter values use in fitting the inner border and the outer border of the skull bone are shown as Table 2.

Those numbers in Table 2 represent the average and standard deviation of the experiments. Then we choose a suitable sample from the experiments to discuss. Figure 5 demonstrates a suitable fitting arc curve in all inner border experiments and Figure 6 is a suitable fitting arc curve for outer border experiments. We use yellow color line to represent the solution of inner border and red color line to represent the solution of outer border. Figure 7 shows the combination of the inner border and outer border in a 2D image. We can obtain a solution from Figure 7.The original skull bone show as Figure 2(b). The inner and outer skull bone borders show as Figure 2(d) and Figure 2(e).

Table 2 The parameters of superellipse and fitness function

superellipse/mean(SD)	*Outer border*	*Inner border*
a	170.98 (24.67)	145.52 (36.85)
b	161.07 (29.11)	136.57 (32.08)
m	1.29 (0.99)	1.28 (0.51)
$n1$	17.13 (14.46)	18.52 (13.51)
$n2$	16.67 (14.09)	18.35 (13.43)
$n3$	17.17 (14.49)	18.59 (13.62)
x_0	249.05 (13.56)	257.55 (13.15)
y_0	254.27 (15.55)	244.55 (14.43)
Fitness	0.26 (0.06)	0.25 (0.05)

4.2 Result Discussion

The objective of this research is to search a suitable fitting cranial curve by using superellipse and DE algorithm. However, the suitable fitting curve might not be the optimal solution in our experiment and observations since there are some junction points issues between realistic and fitting curves as shown in Figure 8. In this case, both inner and outer borders are not hundred percent aligned with the contact point of inner and outer borders.

5. Conclusion

Irregularities of the Cranial defects lead to high risk of brain infections and brain low protection, damage cerebral metabolism, and poor postoperative recovery. Cranioplasty is a commonly way to restore the integrity of the cranial cavity and maintain a stable physiological intracranial pressure.

This paper presents a method of prosthesis modeling. We perform a self-adjusted bone curvature optimal algorithm through a mathematical model of superellipse and optimization known as DE algorithm. The experiments validate that the proposed method is possible to build a model of the breakage skull bone. The significant contribution of this work is to know if it is possible to model a piece arc which the information does not exists in the image. We believe that it is more convenient and efficient for medical professionals to help patients who had this kind of problem with this research. We can consider that the proposed method used superellipse and DE is a promising technique to this geometric prosthesis modeling problem. However, the junction problem between bone segments and the arcs are still a problem that needed to be solved in the future and would not cover it in this study.

a	b	m	$n1$	$n2$	$n3$	x_0	y_0	Fitness
130.73	112.51	1.34	31.64	31.77	31.76	264.58	246.00	0.35

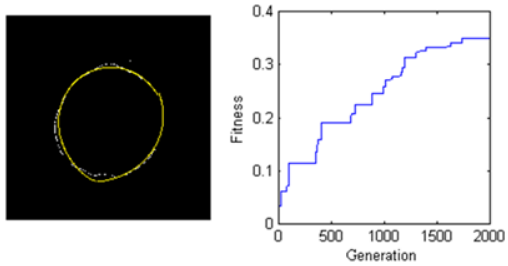


Figure 5 The inner border of the experiment.

a	b	m	$n1$	$n2$	$n3$	x_0	y_0	Fitness
140.20	141.84	1.23	30.58	30.78	30.34	251.33	244.30	0.34

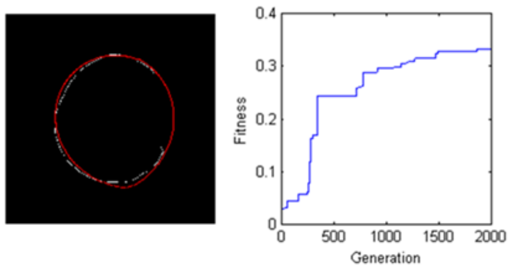


Figure 6 The outer border of the experiment.

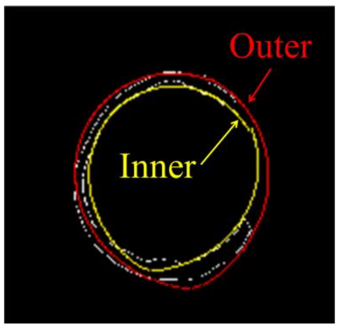


Figure 7 Combination of the inner border and outer border.

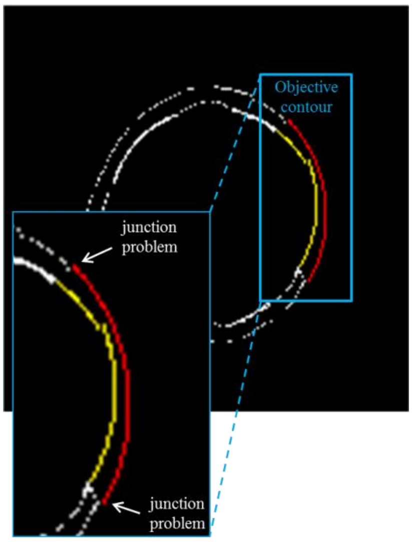


Figure 8 The junction problem for a sampled CT slice.

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