

# A new heuristic method for solving the bi-objective period traveling salesman problem with tour balancing

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**Abstract.** In the single objective period traveling salesman problem (SOPTSP), each of given  $n$  cities has to be visited by the traveling salesman a fixed number of times according to some allowable day combinations over a given  $m$  day planning period with the objective to minimize the total traveling distance over the whole period. The existing solution methods for the SOPTSP always generate a set of unbalanced length of tours that violate most real-world applications, so we expand the SOPTSP to the bi-objective period traveling salesman problem (BOPTSP) by minimizing simultaneously the total traveling time and the difference between the longest and the shortest tours. In this paper, we propose a new heuristic method based on the artificial immune system to solve the BOPTSP with tour balancing, and a test problem taken from literature with 50 cities and 5 day planning period is solved and the  $s$ -metric is applied to evidence the effectiveness of the proposed heuristic method.

**Keywords:** heuristic method, period traveling salesman problem, bi-objective optimization, artificial immune system

## 1. INTRODUCTION

In the single objective period traveling salesman problem (SOPTSP), each of given  $n$  cities has to be visited by the traveling salesman a fixed number of times according to some allowable day combinations over a given  $m$  day planning period with the objective to minimize the total traveling distance over the whole period. For decades, many algorithms were developed for solving the SOPTSP and most of them can find sets of good solutions with only one single objective function. These algorithms cannot work well in the bi-objective problem, such as distance balancing and load balance simultaneously. In recent years, many research works focus on developing algorithms for solving multi-objective problems with considering real-world applications. The vehicle routing problem (Jozefowiez et al., 2007) with the objective to balance each route and minimize the total distance has been studied in literature, and the load balancing problem (Bektas, 2012) with the objective is to balance the workload and distance of each route at the same time has also been addressed.

In this research, we expand the SOPTSP to the bi-objective period traveling salesman problem (BOPTSP) by

minimizing the total distance and the difference between the longest and the shortest tours simultaneously to address the real world applications. We also develop a heuristic method following the concept of the artificial immune system (AIS) for solving the BOPTSP. In Section 2, we describe various proposed methods for solving the bi-objective problem that have been proposed in the OR research. In Section 3, we introduce our new algorithm for solving the BOPTSP based on the concept of artificial immune system. In Section 4, we illustrate the computational result by solving one test problem. In Section 5, we present our conclusions of this paper.

## 2. THE RELATED LITERATURE REVIEW

In this section, we review some methods for solving the SOPTSP, vehicle routing problems (VRP) with considering tour balancing, and the bi-objective optimization problems.

### 2.1 Methods To Solve The SOPTSP Problems

Usually, the single objective problem is to find the optimal solution with only one objective such as the maximum

profit, the cheapest cost, or the shortest distance. For the SOPTSP, the main goal is to minimize the total distance over the whole period. Many heuristics were developed for solving the classical SOPTSP such as Christofides and Beasley (1984) develop an approach for solving the period vehicle routing problem. In the initial step, cities are inserted by its rules, and assign the pattern to cities at the top list to minimize the insert cost. This procedure assigned each day combination and solve each day combination as the TSP. In the final step, the 2-opt heuristic is applied to clean each day route, exchange the pattern and solve twice to improve the solution quality. Chao et al. (1995) proposed an effective heuristic algorithm which is the first research paper to produce an initial solution and then uses five steps method to improve the routes. In addition, they generated many challenging test problems for testing the heuristic, and the new heuristic method can solve these new problems efficiently. Paletta (2002) proposed a new algorithm by selecting a not yet processed city in the iteration process and assign it to a combination. The process repeat until all cities have been chosen, and when the processed cities is equal to the integer parameter, it interrupts to the improvement procedure. The final step is perform under feasible solutions which is the modified improvement procedure. Bertazzi et al. (2004) proposed a heuristic method with inside layer improved procedure for solving period traveling salesman problem. This algorithm is effective for solving 40 test instances with 18 better results.

## 2.2 The VRP Problems With Tour Balancing

Those algorithm methods which we introduce above can work efficiently and had been regarded as the best algorithms for solving SOPTSP to minimize the total distance or cost. But these algorithms do not considering the distance balancing or workload balancing, the computational results might violate the real-life applications. For balancing objective problem such as vehicle routing problem and traveling salesman problem, the objectives of these problems is to balance the workload of tours, the number of customer points, or the total distance over the whole period at the same time, and we illustrate some literatures about the bi-objective VRP. Jozefowicz et al. (2007) proposed a heuristic method which is defined as the Target Aiming Pareto Search for solving the vehicle routine problem with route balancing, and the method combined the Local Search with a genetic algorithm to minimize the total traveled distance and minimize the difference between the longest tour and the shortest tour. In addition, Jozefowicz et al. (2009) propose a meta-heuristic method with two mechanisms to improve its efficiency for solving a bi-objective vehicle routing problem with route balancing and minimizing the total tour length. Lee and Ueng (1999) consider the fairness of drivers, and develop an integer programming model for the bi-objective problem. The two objectives of this model is to minimize the total distance as well as to balance the workload, and

they developed a heuristic method to solve the problems and test by 11 examples.

## 2.3 The Heuristic Methods For The Bi-objective Optimization Problems

Pasia et al. (2007) proposed a population-based local search method for solving the bi-objective vehicle routing problem, and the objectives were minimized the tour length and balance each route at the same time. Abel et al. (2009) proposed an evolutionary algorithm for solving bi-objective vehicle routing problem with time window, and the objectives were minimized the route number and total cost simultaneously. Martínez-Salazar (2014) proposed an algorithm method for solving the bi-objective transportation location routing problem with the objectives to minimize distribution cost and balance workload at the same time. In the BOPTSP, we try to minimize simultaneously the total traveling time and the difference between the longest and the shortest tours. First, we assign an allowable day combination to each city and then solve each tour as a classical TSP during the  $m$  day planning period. The PTSP is defined as an NP-hard optimization problem (Chao, 1995). Since the BOPTSP is another generalization problem of PTSP, it is at least as difficult as the PTSP. The BOPTSP is also an NP-hard optimization problem. The purpose of this research is to develop a heuristic method based on the concept of the artificial immune system for solving bi-objective problem.

## 3. THE PROPOSED HEURISTIC

In this section, we describe the proposed heuristic method for solving the BOPTSP based on basic features of the artificial immune system (AIS). At first we review the AIS related literature and each individual step, and then flow chart of the whole heuristic method is presented.

### 3.1 Artificial Immune System Related Algorithms

During these few years, artificial immune system (AIS) has been applied for developing algorithms in different research fields such as computer defense, optimization problem and other applications. Yoo and Hajela (1999) was the first to apply AIS for solving multi-objective optimization problems (MOOPs). Coello and Cruz Cortés (2002) proposed a multi-objective immune system algorithm (MISA) which is based on the concept of clonal selection theory. In 2005, Coello Coello and Cruz Cortés (2005) also proposed an improved method and find better results. This research extended the single-objective period traveling salesman problem (SOPTSP) to the bi-objective period traveling salesman problem with tour balancing (BOPTSP). The two objectives are:

- (1) Minimize the total distance over the whole period.

- (2) Minimize the difference between the longest and the shortest tours.

In this research, we develop a heuristic method based on the concept of the artificial immune system to find the solution sets along the Pareto frontier. The problem description and method is described in the following sections.

### 3.2 The Problem Description

In this research, the problem example was published by Christofides and Beasley (1984) with 50 cities within 5 day period. This research expand the SOPTSP to the bi-objective period traveling salesman problem (BOPTSP) with the objectives to minimize the total traveling route over the whole period and the difference between the longest and the shortest tour. We define the bi-objective problem as:

Minimize  $f_i(x)$ ,  $i=1, 2$ ,

where each  $f_i(x)$  is an objective function. In this research, different shape represent the frequency the cities required to be visited during the whole period. The triangle denote the cities have to be visited every day. The big circle denote the cities require two visits. The small circle represent the cities require only one visit. In figure 1, we show the test problem.

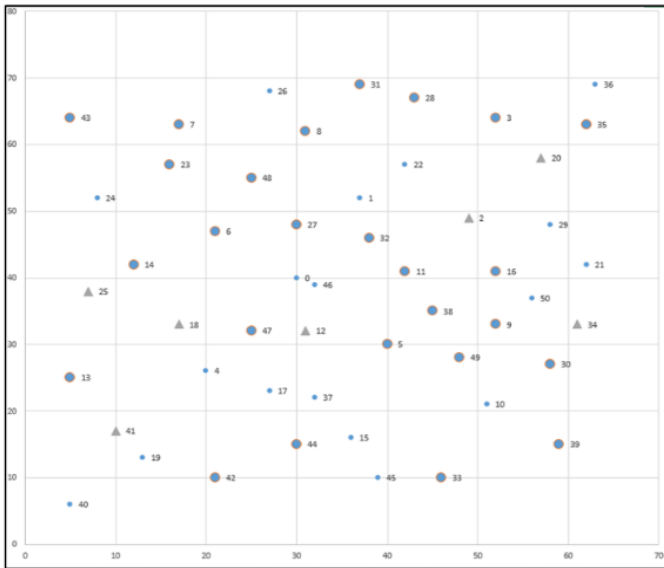


Figure 1. The test problem.

#### 3.2.1 The Initialization Step

In this research, we generated initial solutions by basic tour inserting method. First, we find out cities which have to be served every day, and select the longest city from the home city as the standard city to form a basic tour. Next, we insert

the cities which are near the basic tour to develop a combination by following the nearest selected rule. This method choose the city with the longest length, and assign the near cities to each day combination and repeat the process until all cities are chosen. After generated the initial solutions, we using 2-opt heuristic to optimize every day combination. The initial combinations are illustrated below.

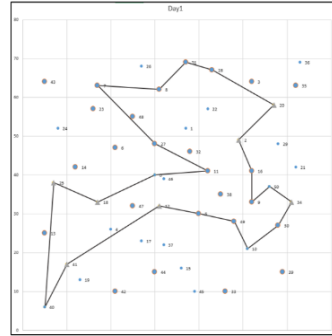


Figure 2(a) TSP tour for Day 1.

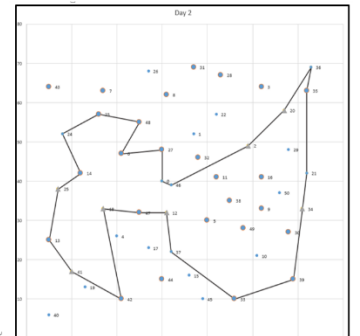


Figure 2(b) TSP tour for Day 2.

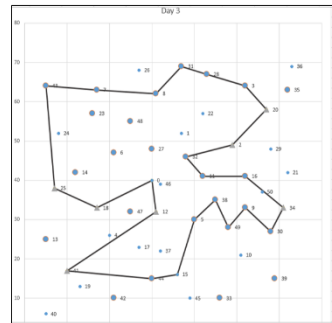


Figure 2(c) TSP tour for Day 3.

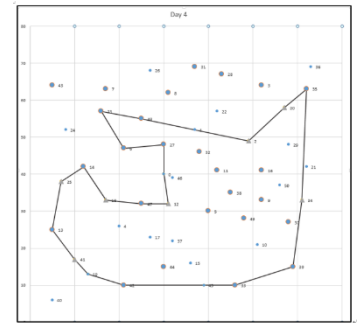


Figure 2(d) TSP tour for Day 4.

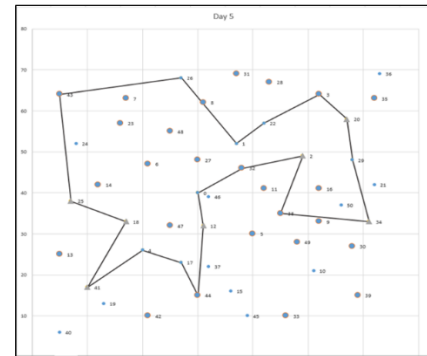


Figure 2(e) TSP tour for Day 5.

#### 3.2.2 The Pareto Frontier Points Selection Step

In this step, we find out the difference between the longest and the shortest tour as the range. Next, we denote the range as the X axis and the total distance as the Y axis and display it to the coordinate.

### 3.2.3 The Mutation Step

On the coordinate, every point along the Pareto frontier has its day combination, and every day combination has five day tours. In this step, we mutate every day tour by inserting one point randomly to another day tour for developing a new day combination. From the coordinate, we select points along the Pareto frontier as our first parent population to generate five offspring generation. Next, we select the dominate points near the Pareto frontier as our second parent population to generate three offspring generation, and very day combination along the Pareto frontier is mutated by this rule to develop their next generations. At this procedure, we try to reassign points to different pattern by using one point or two point movement. For 2-day pattern (day 1, day 3), we move the points to a new pattern which is (day2, day4) to develop two new routes. If the movement reduces the total distance, we make the movement.

### 3.2.4 The Improvement Step

In this step, we try to improve the day combinations by using the 2-opt improvement which is to exchange two arcs in the same route to reduce the total distance. The initial route is 0-1-2-3-4-0 and we exchange 1-2 and 3-4 with 1-3 and 2-4. In this step, we apply the 2-opt improvement to minimize the total distance.

### 3.2.5 The Frontier Points Updating Step

After the new generations were developed, we using 2-opt heuristic to optimize every day tour, and follow the rule to find out the difference between the longest and the shortest tour as the new range. Next, we update the range and total distance on the coordinate for developing a new Pareto frontier.

## 3.3 The Computational Framework

In this section, we propose a bi-objective optimization algorithm based on the AIS concept. The algorithm follows the structure of AIS and uses a randomly selected point to mutate. The main framework is shown in Figure 3.

## 4. COMPUTATIONAL TEST

In this section, our proposed heuristic method is tested by the problem with 50 cities in 5 days period. This test is conducted on a Lenovo computer, which is equipped with 4G of internal memory. The operating system is Windows 10 server and the programming language is C++.

### 4.1 Metrics For Performance Measures

In this bi-objective heuristic method, there are two goals. The first goal is to find the solution along with the Pareto frontier, and the second goal is to mutate the solutions in the obtained dominated front. There are many method to evaluate optimization algorithm. In this research, S metric is used for evaluating the proposed method by evaluating the progress re-

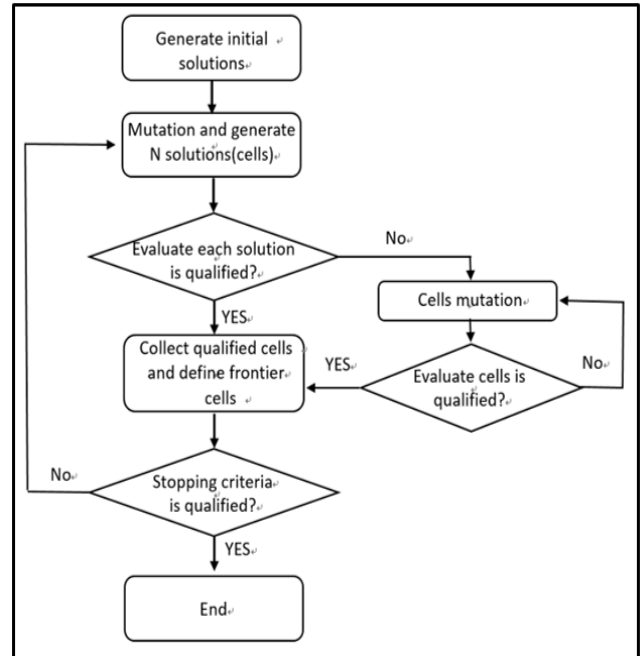


Figure 3. The framework of the proposed algorithm. The region is based on the reference point and each non-dominated points along the frontier.

## 4.2 Computational Results

In order to evaluate this heuristic method, we set a reference point to measure every generation, and the computational result shows that the region is larger from generation 1 to generation 8. The result is listed in Table 1(a) and (b) and the computational region is shown in Figure 5. And the BKS denote the best known solution without tour balance. In figure 4, the black line represent the area of generation 1; the orange line represent the area of generation 2. The gray line represent the area of generation 3; Yellow represent generation 4; Light blue represent generation 5; Green represent generation 6; Deep blue represent generation 7. From table 1, it is obvious to know that the area is larger from generation 1 to Generation 7. Based on the best known solution for this test problem, the route range between the longest route and shortest route is 68, and the range of this computational result is 44.14. In figure 5, we illustrate the computational result for 5 day tours, and it is show that the rout is more balance than the best known solution. In

the real-life optimization problem, this heuristic can develop more reasonable routes to minimize total distance over the period and minimize the range between the longest tour and shortest tour.

Table 1 (a) Area of Generation 1 to Generation 3.

Generation	1	2	3
Area	11188.03	11559.58	12139.82

Table 1 (b) Area of Generation 1 to Generation 3.

4	5	6	7
12434.28	12272.56	14326.05	17665.4

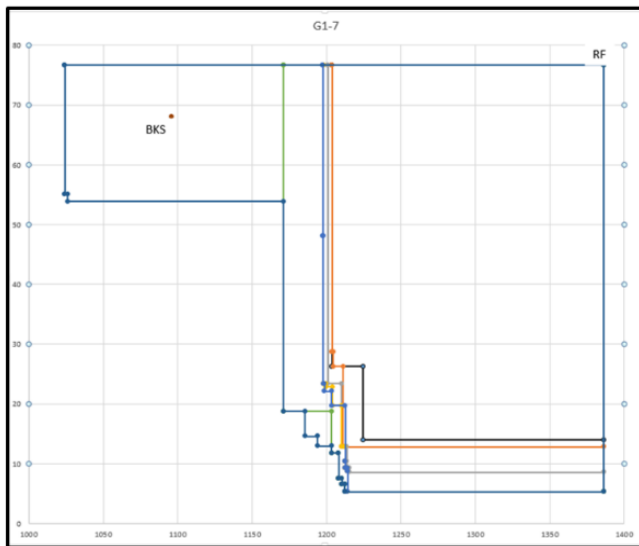


Figure 4. Region of every generation.

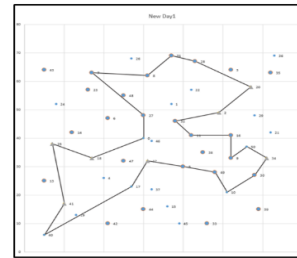


Figure 5 (a) New TSP tour for Day 1.

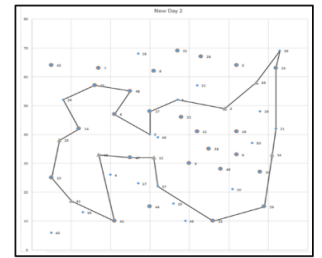


Figure 5 (b) New TSP tour for Day 2.

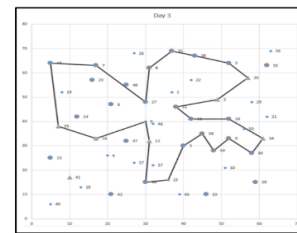


Figure 5 (c) New TSP tour for Day 3.

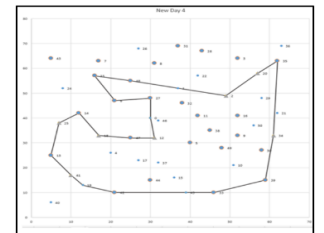


Figure 5 (d) New TSP tour for Day 4.

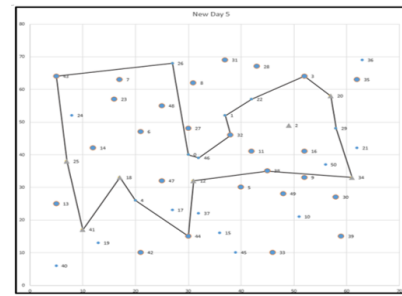


Figure 5 (e) New TSP tour for Day 5.

## 5. CONCLUSIONS AND FURTHER RESEARCH

In this research, we propose a heuristic method based on AIS for solving the BOPTSP. Based on the S metric evaluation result, the region is larger from generation 1 to generation 8 and it is obvious to show that this method can work efficiently for solving the BOPTSP to minimize the total traveling distance over the whole period and the difference between the longest and the shortest tours simultaneously. Next, we will improve this algorithm to be more effective, and solve other PTSP benchmark problems. Particularly, the future work will extend the ability of this proposed algorithm in solving other bi-objective problems such as orienteering problem.

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