Optimal Pricing for Event Tickets in a Matching System

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Abstract. In this paper we study an organizer selling event tickets with a fixed capacity. The organizer provides a matching service in which customers who cannot attend the event can exchange the ticket directly to another customers at face value in the system. We formulate a fixed pricing model to maximize the total expected profit from the selling ticket with matching service. When the customer's valuation for the event follows an exponential distribution, we provide some numerical results to investigate the effect of the matching service on the provider's revenue. The computational experiments reveal that event organizer can increase their revenue by introducing the matching system when (i) the number of potential customer is large enough for the capacity (ii) trade volume of the ticket in the system is relatively high (iii) customer's valuation for the ticket is low.

Keywords: Revenue management; Event; Resale market; Matching system; Fixed pricing

1. INTRODUCTION

A live music and sporting event markets have grown in recent years as expanding the resale market. Resale market gives a profit to consumers who cannot attend the event, because each customer can resale the ticket directly to another consumer. However, the presence of the market leads to generate speculators who purchase ticket for the purpose of earning a premium, and the increase of the speculator s makes consumers who want to attend an event difficult to buy the ticket. In addition, the raising ticket price in the resale market leads to decrease the event organizer's profit from sales of goods or other event tickets. Furthermore, even if the tickets sold out, there is a possibility that seats become vacant if the resale price is too expensive.

In order to prevent resale of tickets, event organizers introduce a face authentication device into event hall. At the time of ticket purchase, customers register their own face image. Then, identity verification is automatically authenticated together with his/her face image data at the event hall. The ticket is issued after the customer is confirm ed his/her identity. This system not only strengthens the ticket resale prevention, but also it is possible to achieve a smooth entry of customers. Although this system is effective as reselling prevention purpose, it has a problem that the customer who purchases the ticket does not allow to give or sell it to other customers when he/she cannot attend the event.

Recently, the event organizer launches a matching service together with the system. The matching service makes it possible that the customer who cannot attend the event can exchange the ticket directly to another consumer at face value, even though the commission fee is charged to both the buyer and seller when the matching is established. However, the system implementation leads to raising the ticket price due to high operating cost, and the supply-demand balance is not reflected to selling price under the matching system. For these reason, some event organizers still allow the presence of the resale market. Thus, in this context, it is required to evaluate the effect of the matching service on the organizer's revenue and customer satisfaction.

However, there is not much literature in operations management that deals with issues regarding event ticket pricing. Balseiro et al. (2011) considered the tournament options in which the identity of the two teams playing in a tournament final is unknown at the time that options are sold. They developed an approach by which an event manager can determine the revenue maximizing prices and amounts of advance tickets and options to sell for a tournament final. Then they showed that, under certain conditions, offering options will increase expected revenue for the event and can increase social welfare. Cui et al. (2014) studied a ticket pricing problem for an event capacity provider that faces resale of tickets in resale market. They derived three pricing strategies, fixed pricing, multi-period pricing, and option pricing, and found how the behavior of optimal prices and revenues depend on the resale transaction costs incurred by the consumers and speculators. They showed that the event capacity providers do not always benefit from restricting resale. We refer to Su (2010) and Cui et al. (2014) for recent references on the ticket resale. Unlike the previous papers, we study whether an event organizer can benefit from the matching service by restricting resale in the resale market. We formulate a fixed pricing model to maximize the total expected revenue from the selling ticket with matching service. Through the numerical study, we show that how the matching service affects to the organizer's optimal price and expected revenue.

The remainder of this paper is organized as follows. In Section 2, we introduce the model under demand uncertainty. In Section 3, we specify the distribution of customer's valuation for event and provide some numerical results.

2. THE MODEL

In this section, we consider a profit maximization problem with matching system. An event organizer sales C units of capacity at a price $p \in [\underline{p}, \overline{p}]$, where $\underline{p}$ and $\overline{p}$ are upper and lower bound of the price, respectively. All of seat are sold at the same price. The selling period is divided into two periods (see Figure 1). In period 1, the organizer launches initial sales of $W(\leq C)$ tickets to the member of fun club. Fixed number of M customers (members) arrivals to purchase advance tickets, and each customer decides either purchase a ticket or not. The number of customer is assumed to be greater than or equal to capacity, that is, $C \leq M$. Let V be the customer's valuation for attending the event. We assume that the customers arriving in different periods have the same valuation. The valuations are random and drawn independently from a distribution with the cumulative distribution function $F(\cdot)$. Thus, $\bar{F}(p) = P(V \geq p)$ represents the probability that the customer purchase the ticket at price p, where $\bar{F}(\cdot) = 1 - F(\cdot)$. Thus, the expected revenue in period 1 is given by

$$R_1(p) = p \ \min\{M\bar{F}(p), W\}. \tag{1}$$

For simplicity the notation, we define the sales in period 1 as $A(p) \equiv \min\{M\bar{F}(p), W\}$.

In period 2, the organizer sells $C - A(p)$ tickets to three types of customers: (i) customers who cannot purchase the ticket due to sold out, (ii) customers who do not buy ticket in period 1, because they may want to wait until some uncertainties in their schedules, purchase the ticket from event organizer, (iii) new customers including the non-member of fun club. Let X be the total number of type (ii) and (iii), and X is a random variable with the cumulative distribution function $G(\cdot)$. We assume that new arrival customers have the same valuation as the customers in period 1. Thus, the number of customers in period 2 is

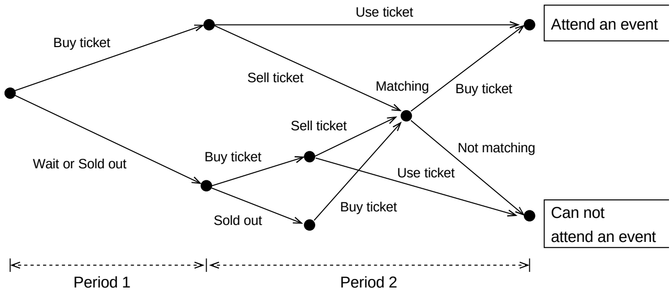

Figure1: Sequence of the events

given by $B(p) = (M\bar{F}(p) - W)^+ + X\bar{F}(p)$. A customer who purchases the ticket in period 1 or period 2 cannot go to the event with probability $\rho \geq 0$, and gets the ticket up on the matching system. We refer to the probability as *resale probability*. Let $Y(p)$ be the number of ticket sold in the matching system, and is given by

$$Y(p) = \rho[A(p) + \min\{B(p), C - A(p)\}]. \tag{2}$$

If the ticket is sold out in period 2, then customers can get the ticket in the matching system if the matching is established. Thus, the number of customers who use the matching system to purchase the ticket is given by $(B(p) - (C - A(p)))^+$, where $a = \max\{a, 0\}$. In addition, the number of tickets traded at the matching system is $\min\{Y(p), (B(p) - (C - A(p)))^+\}$. The refund is paid to the customers who get the ticket up on the system if the trade is done, and a transaction cost is incurred for both of buyer and seller. We denote by τ the commission income from a single ticket if the trade is established at the matching system. Therefore, the expected revenue in period 2 is given by

$$\begin{aligned} R_2(p) = {} & pE_X[\min\{B(p), C - A(p)\}] \\ & + \tau E_X[\min\{Y(p), (B(p) - (C - A(p)))^+\}]. \end{aligned} \tag{3}$$

The first term in equation (3) represents the revenue from selling ticket by organizer in period 2. The second term represents the commission income from the matching system. From equations (1) and equation (3), the total expected revenue for the entire selling season is defined by

$$R(p) = R_1(p) + R_2(p). \tag{4}$$

Since $(a - b)^+ = a - \min\{a, b\}$, we have $(M\bar{F}(p) - W)^+ = M\bar{F}(p) - A(p)$. Thus, we obtain

$$B(p) = (M + X)\bar{F}(p) - A(p) \tag{5}$$

and

$$Y(p) = \rho \min\{(M+X)\bar{F}(p), C\}. \quad (6)$$

By substituting them into equation (3), the total expected revenue (4) can be rewritten as follows;

$$R(p) = pE_X[S(p)] + \tau E_X[\min\{\rho S(p), (M+X)\bar{F}(p) - S(p)\}], \quad (7)$$

where $S(p) = \min\{(M+X)\bar{F}(p), C\}$ represents the total number of sales by the organizer during the entire period. From equation (5), the number of ticket allocated in period 1, W, is not affected to the total expected revenue.

Thus, the objective of event organizer is to find an optimal price so as to maximize the total expected revenue, that is,

$$R^* = R(p^*) \equiv \max_{p \in [\underline{p}, \overline{p}]} R(p). \quad (8)$$

Remark 2.1.
When $\rho = 0$ *or* $\tau = 0$, *the problem is reduced to the one in which the matching service is not served for the customer. We denote the maximal total revenue for this case by* $\tilde{R}^*$.

3. NUMERICAL STUDY

In this section, we specify the distributions of the customer's valuation V and the number of arrival customers X to be an exponentially distributed with parameter $\lambda > 0$ and $\mu > 0$, respectively. Thus, we have $\bar{F}(p) = e^{-\lambda p}$ and $\bar{G}(x) = e^{-\mu x}$. Then, equation (7) can be rewritten as follows;

$$R(p) = e^{-\lambda p}\left[p\left\{M + \frac{1}{\mu}\left(1 - e^{-\mu(Ce^{\lambda p} - M)}\right)\right\} + \frac{\tau}{\mu}\left(e^{-\mu(Ce^{\lambda p} - M)} - e^{-\mu\{(\rho+1)Ce^{\lambda p} - M\}}\right)\right]. \quad (9)$$

Even if the distribution is specified to the exponential distribution, it is hard to show the concavity of the total expected function in (9). Thus, we study the characteristics of the function $R(p)$ in numerically. We begin by setting the parameters for the numerical study and deriving optimal price and maximal expected revenue for various different parameters. We will then show the effect of the introducing the matching system on the total expected revenue and optimal price by comparing the results to those for a case that does not include the matching system. Table 1 lists the base parameters used in the numerical examples.

In Figure 2, we show the relationship between the total expected revenue function $R(p)$ and the price p. $R(p)$ is unimodal in the range of $p \in [40, \infty]$.

Table 1: Parameters

M	C	ρ	λ	μ	τ
1000	500	0.1	0.01	0.01	10

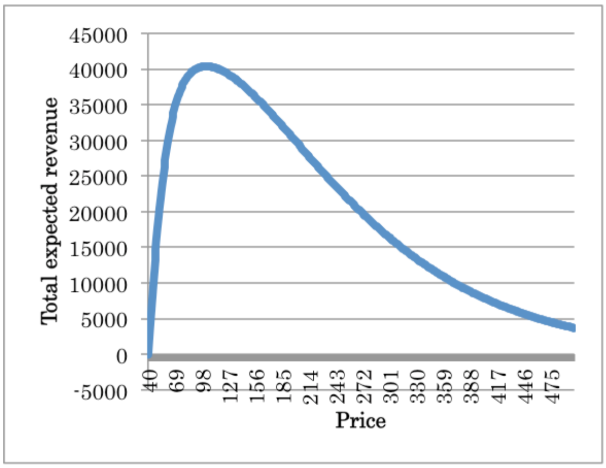

Figure 2: Total expected revenue with respect to the selling price

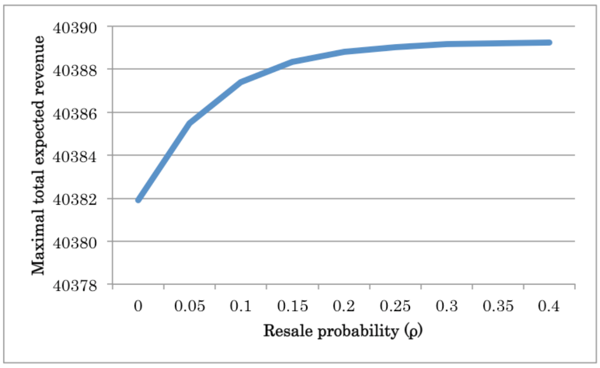

Figure 3: Maximum total expected revenue with respect to resale probability

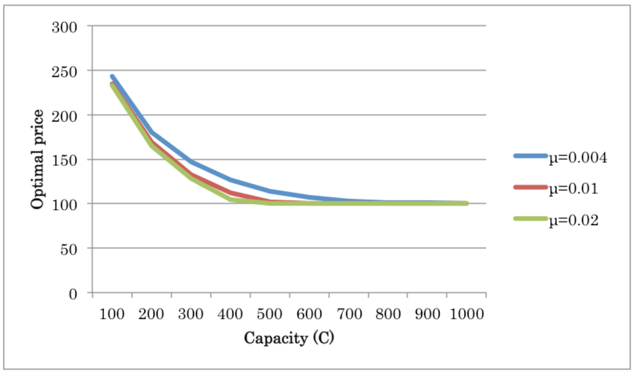

Figure 4: Optimal price for different values of parameter μ with respect to capacity

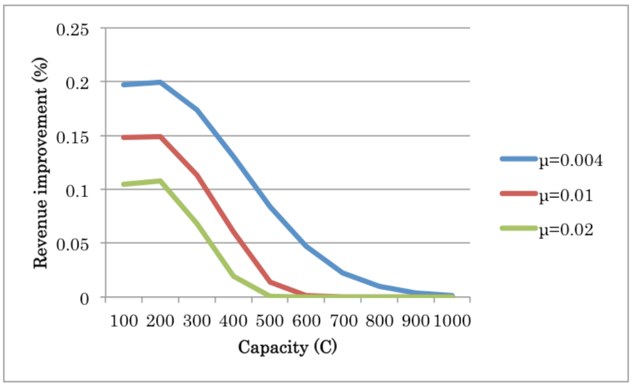

Figure 5: Revenue improvement for different values of parameter μ with respect to capacity

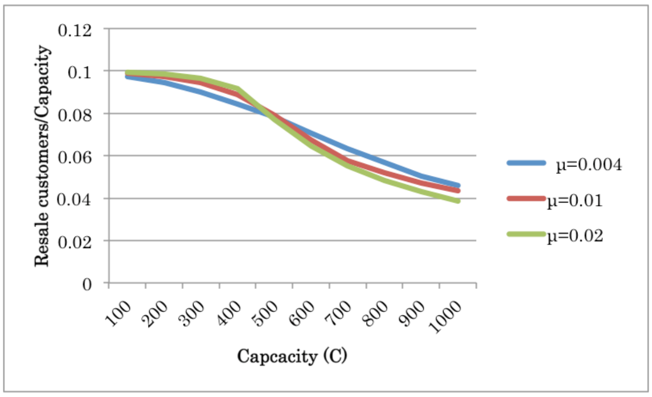

Figure 6: Fraction of number of resale customer to the capacity with different values of parameter μ

Figure 3 illustrates the effect of the resale probabi lity ρ on the maximal total expected revenue. We see that the revenue increases as the number of resale tick ets in the matching system increases. However, the ch ange of the resale probability does not have much effe ct on the total revenue.

In Figure 4 and Figure 5, we show the effect of capacity size on the optimal price and the percentage of benefit of introducing the matching system, respectively. The percentage of benefit compared to the benefit of non-introducing the system is calculated as $100 \times \left(R^{*} - \tilde{R}^{*}\right)/\tilde{R}^{*}$. Since the number of potential customer is $M = 1000$, the optimal price is high when the capacity size is small. In addition, the large value of μ implies the low number of new arrival customers in period 2. Thus, the optimal price also increases as the number of arrivals in period 2 increases. From Figure 5, we see that the matching service is more effective when the capacity size is small. As shown in Figure 6, the fraction of number of resale customers to the capacity is high for small capacity. Thus, the revenue from transaction fee through matching service represents a large percentage of total rev enue when

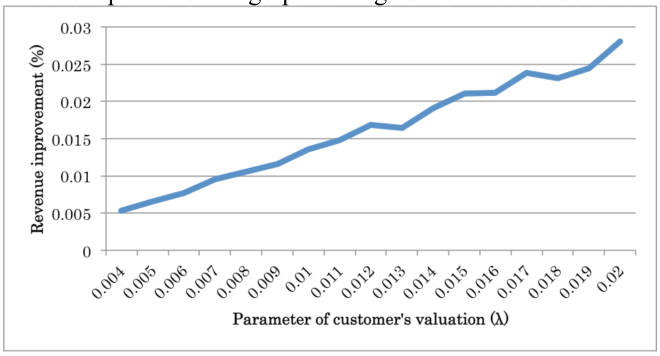

Figure 7: Revenue improvement with respect to the pa rameter of customer's valuation λ

enue when the capacity size is small.

Finally, Figure 7 shows the impact of a change in the parameter of customer's valuation on the benefit derived from matching service. The large value of the parameter λ implies that the low value of customer's valuation for the ticket. Thus, the matching service is effective in the total revenue when the customer's valuation is low.

4. CONCLUSIONS

This paper presents a pricing model for event ticket with matching service. We show numerically that the organizer makes a profit from the service mainly due to the commission income. Although, in this paper, we have only considered organizer's revenue, it would be interesting to also investigating how the system affects the consumer's welfare. Since the customer can buy the ticket in period 2 at face value by using the matching service, the welfare would be higher than that for the case where customers only purchase the ticket from the resale market. Such a result would be helpful to show the effectiveness of matching service.

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