Heterogeneous Fleet Vehicle Routing Problem in Delivering Industrial Gas

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Abstract. This paper is about the Vehicle Routing Problem (VRP) which has been concerned for years by many researchers and entrepreneurs. Three common extensions of VRP, Heterogeneous Fleet Capacitated Vehicle Routing Problem (HFCVRP), Heterogeneous Fleet Vehicle Routing Problem with Time Window (HFVRPTW) and Heterogeneous Fleet Split Delivery Vehicle Routing Problem (HFSDVRP) are studied on the case of a Vietnamese industrial gas and welding electrode company – SOVIGAZ. The objective is to improve the distribution system of the company by minimizing the total travelling distance and vehicle acquisition or set-up cost. Then the problem is solved by an exact method – mix integer linear programming with the support of an optimization solving tool, CPLEX. The most appropriate model to the company delivery system will be chosen under Multiple Attribute Decision Making (MADM) method.

Keywords: Capacitated Vehicle Routing Problem, Vehicle Routing Problem with Time Window, Split Delivery Vehicle Routing Problem, Heterogeneous Fleet, Multiple Attribute Decision Making.

1. INTRODUCTION

Transportation scheduling is very important to logistics field. Logistics is defined as a study of effective processes for delivery and disposition of commodities and personnel. A company with an effective planning and scheduling on transportation system will reduce its transportation cost and delivery time. Hence, customers' satisfaction and logistics performance will be enhanced.

SOVIGAZ is chosen to be a case study of applying Vehicle Routing Problem in industrial gas distribution. It was established in 1974 by merging Oxygen Acetylene Extreme Orient Society and Vietnamese Society of Industrials Gas. This company is leading in supplying medical gas, industrial gas and other chemicals for manufacturing in Vietnam..

Up to now, SOVIGAZ has approximately more than 200 customers, where a major part constitutes of hospitals, and manufacturing sites which require industrial gas for various kinds of activities such as, production, shipbuilding and so on. In average, the company has to handle nearly 100 customers per day. Currently, the resources of vehicles for delivering gas are still abundant, so that there is no pressure on the company for using its vehicles effectively. However, in the future, when the number of customers increases, the vehicles must be used in a smarter way to satisfy the system. This study will focus on improving the current transportation system of the company through minimizing transportation distance, vehicles cost and the number of trucks used.

Three models which are considered to be suitable for the company delivery system include HFCVRP, HFVRPTW and HFSDVRP. Models output will be evaluated under Multiple Attribute Decision Making method. Then, the most appropriate model is chosen to apply in real situation of the company.

2. LITERATURE REVIEW

Capacitated vehicle routing problem (CVRP) is popularly studied by many researchers and applied in real cases of transportation scheduling. CVRP with distance constraints was researched by TakwaTlili, Sami Faiz, SaoussenKrichen, 2013. Distance-constrained CVRP was formulated as an integer-programming problem in order to minimize the vehicles' traveled distances subject to system requirements.

R. Tavakkoli-Moghaddam, N. Safaei, Y. Gholipour, 2005 considered an extended type of the CVRP in which the cost is only dependent on the type and capacity of available vehicles. This paper is aim to minimize the heterogeneous fleet cost and maximize the capacity utilization. For solving the problem, a hybrid simulated annealing (SA) based on the nearest neighborhood was introduced.

Another variant of VRP which is cumulative CVRP was declared by LiangjunKe, Zuren Feng, 2012. The objective is to minimize the cumulative time such as the total arrival time at the customers under the constraints of capacity limitations. Two-phase metaheuristic was proposed to deal with the problem.

Nancy L. Nihan, Edward K. Morlok, 1975 generated a set of good transportation alternatives during early and intermediate stages of transportation planning. A linear programming model of a multi-modal transportation system was developed. The goal is to minimize total annual transportation costs with three classes of cost relationships which are road capital and operating costs, and costs of common carriers with fixed and variable vehicle size.

A research of Anand Subramanian, PucaHuachiVazPenna, Eduardo Uchoa, Luiz Satoru Ochi, 2011 is about heterogeneous fleet VRP with distinct capacities and costs. The paper was discussed to determine the best fleet composition as well as the set of routes that minimize the sum of fixed and travel costs. Then, a hybrid algorithm which is composed by an Iterated Local Search (ILS) based heuristic and a Set Partitioning (SP) formulation was proposed to solve the problem.

Not only that, the fleet size and mix problem for capacitated arc routing was introduced by Gunduz ULUSOY, 1985. The objective of minimizing the total distance travelled was replaced by minimizing both fixed and variable costs of used multiple vehicles. The solution process comprises of four phases repetitively until finding the optimal one.

Moreover, Wen Lea Pearn, 1987 did a research oncapacitated arc routing problem to find approximate solutions for minimizing the total routing cost. This kind of problem is a capacitated variation of the arc routing problems in which there is a capacity constraint associated with each vehicle. Due to the computational complexity of the problem, recent research has focused on developing and testing heuristic algorithms which solve the CARP approximately.

Another paper of heterogeneous fixed fleet VRP in which there are different types of vehicles and a given number of vehicles of each type was studied by Jose Brandao, 2011. The objective is to minimize the total costs, satisfying customers' requirements and visiting each customer exactly once. Tabu search algorithm was proposed and tested on several benchmark problems.

For supporting CVRP, Imdat Kara, Gilbert Laporte, TolgaBektas, 2003 did a research on subtour elimination constraints. Therefore, all vehicles can start and end their routes at the depot with a satisfaction of meeting all demands.

In Chapter 7, Vehicle Routing Problem with Time Windows from a book edited by Paolo Toth Daniele Vigo talks about how to solve a problem relating to capacity and time constraints. This chapter not only provides few basic mathematic models of minimizing total transportation cost but also satisfies deliveries and demands known in advance. Model is mentioned, multi-commodity network flow model, might be helpful to solve such a problem with time window and capacity constraints.

For more research on vehicle routing problem (VRP), Manolis N. Kritikos, George loannou, 2013 proposed a solution approach for a practical problem in real life involving VRP with time windows, vehicles in different capacities and some overloads are permitted. The objective is to minimize total distance traveled by vehicles, the fixed costs of using vehicles and the capacity violations of all vehicles in the final schedule. A mathematical model was set up with relaxed capacity constraints and solved by a new heuristic method.

A research is about variation of the CVRP, the Split Delivery Vehicle Routing Problem, where a customer can be visited more than once by different vehicles. Leonardo Berbotto, Sergio Garcia, Francisco J. Nogales, 2011 presented two mathematical formulations for this kind of problem which is called Split Delivery Vehicle Routing Problem with Stop Nodes: a vehicle flow formulation and a commodity flow formulation. Then, a heuristic approach, Tabu Search is used for solving, and a comparison of its performance with and without the stop nodes.

3. MATHEMATICS

A. Heterogeneous Fleet Capacitated Vehicle Routing Problem

The problem is defined as follows: Let G(V,A) be a complete graph, where $V = \{0,1,\ldots,i,\ldots n+1\}$, is the node set (Node i=0 and node i=n+1 represent a depot and the others correspond to the customers) and $A = \{(i,j) : i,j \in V, i \neq j\}$ is the arc set.

There are n nodes of customers whose demand is $d_i$to visit with a support of K vehicles that areinitially placed at the depot. Then, goods are delivered to a set $D = V \backslash \{0,n+1\}$ of customers and return to the depot after finishing.

Assumptions:

• Each vehicle must start and end its route at the depot.
• Each customer is visited only once by a single vehicle.
• The mean velocity of travel for all vehicles is constant.
• Heterogeneous fleet of vehicles with different capacities is applied. Moreover, the cost of each type of vehicle is fixed.
• Split delivery is not allowed.

Notations:

k: vehicle type

i,j: vertex

(i,j): arc

Parameters:

n: number of customers

K: number of available vehicles

c_{ij}: travelling distance from vertices i to vertices j

f_k: a fixed acquisition cost is incurred for each of vehicle in the routes

C_k: capacity of vehicle k

d_i: demand of customer i

M: large constant

Decision variables:

$$x_{ijk} = \begin{cases} 1 \text{ if } arc(i,j) \in A \text{ is traversed by vehicle } k \\ 0 \text{ otherwise} \end{cases}$$

$$z_k = \begin{cases} 1 \text{ if vehicle } k \text{ is used} \\ 0 \text{ otherwise} \end{cases}$$

v_{ik} is an integer variable associating with each customer i and vehicle k

$$\text{Min} \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} \sum_{k=1}^{K} c_{ij} \times x_{ijk} + \sum_{k=1}^{K} f_k \times z_k \quad (A.1)$$

Subject to

$$\sum_{k=1}^{K} \sum_{j=1}^{n+1} x_{ijk} = 1 \quad \forall i = 1,\dots,n,\ i \neq j \quad (A.2)$$

$$\sum_{i=1}^{n} x_{in+1k} = z_k \quad \forall k \quad (A.3)$$

$$\sum_{j=1}^{n} x_{0jk} = z_k \quad \forall k \quad (A.4)$$

$$\sum_{i=0}^{n} x_{ijk} = \sum_{i=1}^{n+1} x_{ijk} \quad \forall k,\ \forall j = 1,\dots,n \quad (A.5)$$

$$\sum_{i=1}^{n} d_i \left(\sum_{j=1}^{n+1} x_{ijk}\right) \leq C_k \times z_k \quad \forall k \quad (A.6)$$

$$z_k \leq M \times \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} x_{ijk} \quad \forall k \quad (A.7)$$

$$v_{ik} - v_{jk} + C_k \times x_{ijk} \leq C_k - d_j \times z_k \quad \forall i,j = 1,\dots,n, \forall k, i \neq j \quad (A.8)$$

$$d_i \times z_k \leq v_{ik} \leq C_k \quad \forall i,j = 1,\dots,n, \forall k, i \neq j \quad (A.9)$$

$$x_{ijk} \in \{0,1\} \quad \forall i,j,k \quad (A.10)$$

$$z_k \in \{0,1\} \quad \forall k \quad (A.11)$$

Equation (A.1) is the objective functions to minimize the total travelling distances of trucks and the total vehicle acquisition or set-up cost.

Constraint (A.2) ensures that only one vehicle can enter and depart from every customer. Constraints (A.3) and (A.4) imply that every vehicle has to directly leave from and return to the depot. Constraint (A.5) is the typical flow conversation equation that ensures the continuity of each vehicle route. Then, constraint (A.6) states that the total load of each vehicle must not exceed its capacity. Constraint (A.7) makes sure that no customers are serviced by inactive vehicles. Moreover, constraints (A.8) and (A.9) are Capacity-cut constraint (CCC) ensures that the sub-tours are eliminated. Finally, constraints (A.10) and (A.11) imply integrality for the x_{ijk} and z_k variables.

B. Heterogeneous FleetVehicle Routing Problem with Time Window

The problem is defined as follows: Let G(V,A) be a complete graph, where V = {0,1,⋯,i,⋯n+1}, is the node set (Node i=0 and node i=n+1 represent a depot and the others correspond to the customers) and A = {(i,j) : i,j∈ V, i≠ j} is the arc set.

There are n nodes of customers whose demand is d_i to visit with a support of K vehicles that are initially placed at the depot. Then, goods are delivered to a set D = V\{0,n+1} of customers and return to the depot after finishing.

More than that, all customers have time window $[a_i,b_i]$ in which their orders must arrive.

Assumptions

• Each vehicle must start and end its route at the depot.
• Each customer is visited only once by a single vehicle.
• The mean velocity of travel for all vehicles is constant.
• Heterogeneous fleet of vehicles with different capacities is applied. Moreover, the cost of each type of vehicle is fixed.
• Split delivery is not allowed.
•Time window constraints of all customers cannot be violated.
• All vehicles leave the depot at time 0.

Notations:

k: vehicle type

i,j: vertex

(i,j): arc

Parameters:

n: number of customers

K: number of available vehicles

s_i: service time of customer i

t_{ij}: travelling time to travel from vertices i to vertices j

$[a_i,b_i]$: time windows of customer i to be served

E: the earliest possible departure from the depot

L: the latest possible arrival at the depot

c_{ij}: travelling distance from vertices i to vertices j

f_k: a fixed acquisition cost is incurred for each of vehicle in the routes

C_k: capacity of vehicle k

d_i: demand of customer i

M: large constant

Decision variables:

$$x_{ijk} = \begin{cases} 1 \text{ if } arc(i,j) \in A \text{ is traversed by vehicle } k \\ 0 \text{ otherwise} \end{cases}$$

$$z_k = \begin{cases} 1 \text{ if vehicle } k \text{ is used} \\ 0 \text{ otherwise} \end{cases}$$

T_{ik}=

$$\begin{cases} \text{time since vehicle } k \text{ begins to serve customer } i \\ 0 \text{ if vehicle } k \text{ does not visit customer ie} \end{cases}$$

$$\text{Min} \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} \sum_{k=1}^{K} c_{ij} \times x_{ijk} + \sum_{k=1}^{K} f_k \times z_k \qquad \text{(B.1)}$$

Subject to

$$\sum_{k=1}^{K} \sum_{j=1}^{n+1} x_{ijk} = 1 \qquad \forall i = 1, \dots, n,\ i \neq j \qquad \text{(B.2)}$$

$$\sum_{i=1}^{n} x_{in+1k} = z_k \qquad \forall k \qquad \text{(B.3)}$$

$$\sum_{j=1}^{n} x_{0jk} = z_k \qquad \forall k \qquad \text{(B.4)}$$

$$\sum_{i=0}^{n} x_{ijk} = \sum_{i=1}^{n+1} x_{ijk} \qquad \forall k,\ \forall j = 1, \dots, n \qquad \text{(B.5)}$$

$$\sum_{i=1}^{n} d_i \left(\sum_{j=1}^{n+1} x_{ijk} \right) \leq C_k \times z_k \qquad \forall k \qquad \text{(B.6)}$$

$$z_k \leq M \times \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} x_{ijk} \qquad \forall k \qquad \text{(B.7)}$$

$$a_i \sum_{j=1}^{n+1} x_{ijk} \leq T_{ik} \leq b_i \sum_{j=1}^{n+1} x_{ijk} \ \forall i, j = 1, \dots, n, \forall k, i \neq j \qquad \text{(B.8)}$$

$$(T_{ik} + s_i + t_{ij} - T_{jk}) \leq (1 - x_{ijk}) \times M \quad \forall i = 1, \dots, n, \forall j = 1, \dots, n+1, \forall k \qquad \text{(B.9)}$$

$$(E \times z_k + t_{0i} - T_{ik}) \leq (1 - x_{0ik}) \times M \quad \forall i = 1, \dots, n, \forall k \qquad \text{(B.10)}$$

$$(T_{ik} - L \times z_k) \leq (1 - x_{in+1k}) \times M \quad \forall i = 1, \dots, n, \forall k \qquad \text{(B.11)}$$

$$x_{ijk} \in \{0,1\} \forall i, j, k \qquad \text{(B.12)}$$

$$z_k \in \{0,1\} \forall k \qquad \text{(B.13)}$$

$$T_{ik} \geq 0 \forall i = 0, \dots, n+1, \forall k \qquad \text{(B.14)}$$

Equation (B.1) is the objective functions to minimize the total travelling distances of trucks and the total vehicle acquisition or set-up cost.

Constraint (B.2) ensures that only one vehicle can enter and depart from every customer. Constraints (B.3) and (B.4) imply that every vehicle has to directly leave from and return to the depot. Constraint (B.5) is the typical flow conversation equation that ensures the continuity of each vehicle route. Then, constraint (B.6) states that the total load of each vehicle must not exceed its capacity. Constraint (B.7) makes sure that no customers are serviced by inactive vehicles. Moreover, constraint (B.8) is assurance of vehicle routing feasibility in which vehicles come at the right time window of customers. Constraint (B.9) states schedule feasibility with respect to time considerations. Next, constraint (B.10) implies the first node that vehicle comes. Constraint (B.11) talks about the arrival limit in time of vehicle, vehicle k has to visit the last customer of the last route before the time L of depot. Finally, constraints (B.12), (B.13) and (B.14) imply integrality for the x_{ijk} and z_k variables and non-negativity for the T_{ik} variable.

C. Heterogeneous Fleet Split Delivery Vehicle Routing Problem

The problem is defined as follows: Let G(V,A) be a complete graph, where V = {0,1,…,i,…n+1}, is the node set (Node i=0 and node i=n+1 represent a depot and the others correspond to the customers) and A = {(i,j) : i,j ∈ V, i ≠ j} is the arc set.

There are n nodes of customers whose demand is d_i to visit with a support of K vehicles that are initially placed at the depot. Then, goods are delivered to a set D = V\{0,n+1} of customers and return to the depot after finishing.

Assumptions

(B.4)

- Each vehicle must start and end its route at the depot.
- Each customer is visited more than once by available vehicles.
- The mean velocity of travel for all vehicles is constant.
- Heterogeneous fleet of vehicles with different capacities is applied. Moreover, the cost of each type of vehicle is fixed.
- Split delivery is allowed.

Notations:

k: vehicle type

i,j: vertex

(i,j): arc

Parameters:

n: number of customers

K: number of available vehicles

c_{ij}: travelling distance from vertices i to vertices j

f_k: a fixed acquisition cost is incurred for each of vehicle in the routes

C_k: capacity of vehicle k

d_i: demand of customer i

M: large constant

Decision variables:

$$x_{ijk} = \begin{cases} 1 & \text{if arc}(i,j) \in A \text{ is traversed by vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$z_k = \begin{cases} 1 & \text{if vehicle } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

y_{ik}: delivered quantity to customer i by vehicle k

$$\text{Min} \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} \sum_{k=1}^{K} c_{ij} \times x_{ijk} + \sum_{k=1}^{K} f_k \times z_k \quad \text{(C.1)}$$

Subject to

$$\sum_{k=1}^{K} \sum_{j=1}^{n+1} x_{ijk} \geq 1 \quad \forall i = 1, \dots, n, \ i \neq j \quad \text{(C.2)}$$

$$\sum_{i=1}^{n} x_{in+1k} = z_k \quad \forall k \quad \text{(C.3)}$$

$$\sum_{j=1}^{n} x_{0jk} = z_k \quad \forall k \quad \text{(C.4)}$$

$$\sum_{i=0}^{n} x_{ijk} = \sum_{i=1}^{n+1} x_{ijk} \quad \forall k, \ \forall j = 1, \dots, n \quad \text{(C.5)}$$

$$\sum_{i=1}^{n} d_i \left(\sum_{j=1}^{n+1} x_{ijk} \right) \leq C_k \times z_k \quad \forall k \quad \text{(C.6)}$$

$$z_k \leq M \times \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} x_{ijk} \quad \forall k \quad \text{(C.7)}$$

$$\sum_{i=0}^{n+1} \sum_{j=0}^{n+1} x_{ijk} \leq |D| - 1 \quad \forall D \subseteq 0, \dots, n+1, \forall k \quad \text{(C.8)}$$

$$y_{jk} \leq d_i \times \sum_{j=0}^{n+1} x_{ijk} \ \forall i = 1, \dots, n, \forall k \quad \text{(C.9)}$$

$$\sum_{i=1}^{n} y_{ik} \leq C_k \forall k \quad \text{(C.10)}$$

$$\sum_{k=1}^{K} y_{ik} = d_i \forall i = 1, \dots, n \quad \text{(C.11)}$$

$$x_{ijk} \in \{0,1\} \forall i, j, k \quad \text{(C.12)}$$

$$z_k \in \{0,1\} \forall k \quad \text{(C.13)}$$

$$y_{ik} \geq 0 \forall i = 1, \dots, n, \forall k \quad \text{(C.14)}$$

Equation (C.1) is the objective functions to minimize the total travelling distances of trucks and the total vehicle acquisition or set-up cost.

Constraint (C.2) ensures that at least a vehicle can enter and depart from every customer. Constraints (C.3) and (C.4) imply that every vehicle has to directly leave from and return to the depot. Constraint (C.5) is the typical flow conversation equation that ensures the continuity of each vehicle route. Then, constraint (C.6) states that the total load of each vehicle must not exceed its capacity. Constraint (C.7) makes sure that no customers are serviced by inactive vehicles. Moreover, constraint (C.8) is sub-tours elimination constraint. Constraint (C.9) states the delivered quantity of each vehicle does not exceed the demand of each customer. Next, constraint (C.10) implies that limit the vehicle load by its capacity. Constraint (C.11) talks about satisfaction of the entire demand. Finally, constraints (C.12), (C.13) and (C.14) imply integrality for the x_{ijk}, z_k and y_{ik} variables.

4. RESULT AND DISCUSSION

The corresponding data and final results are given from Table 1-9.

For choosing the best option among qualified solutions based on a specific set of criterion, MADM approach a potential candidate. This method is used to support the decision maker in finding one that best suits their goal under multiple attributes. Currently, there are many ways to rank solutions by MADM, one of the well-known method is Analytic Hierarchy Process (AHP).

AHP was developed by Thomas L. Saaty in 1970s, is a process of structuring a decision problem, representing and quantifying its elements, relating those elements to desired goals, and evaluating alternative (Alt) solutions. The procedure of using AHP is following described:

Step 1: List of goals, criteria and alternatives

For each criterion, step two to step five are performed

Step 2: Develop alternative pairwise comparison matrix

Rate the relative importance between each pair among alternatives. The matrix presents numerical ratings comparing between horizontal – first alternative and vertical – second alternative.

Ratings are given as follows:

Extremely preferred	-	9
Very strongly preferred	-	7
Strongly preferred	-	5
Moderately preferred	-	3
Equally preferred	-	1

Step 3: Develop a normalized matrix

In this step, we divide each number in each column of the pairwise comparison matrix by its sum of that column.

Step 4: Develop the priority vector

In the normalized matrix, the average of each row is calculated. The values form a priority vector of alternatives with respect to a particular criterion. The sum value of this vector is equal to one.

Step 5: Calculate a consistency ratio (CR)

The consistency of the input subjective in the pairwise comparison matrix can be measured by calculating a consistency ratio. A consistency ratio of less than 0.1 is good. In the other hand, it is greater than 0.1, which means the input subjective should be re-evaluated.

Step 6: Develop a priority matrix

After steps two through five have been performed for all criteria, the results of step four are summarized in a priority matrix by listing the decision alternatives horizontally and the criteria vertically. The column entries are the priority vectors for each criterion.

Step 7: Develop a criteria pairwise development matrix

This is done in the same manner as that used to construct alternative pairwise comparison matrices by using subjective ratings (step 2). Similarly, normalize the matrix (step 3) and develop a criteria priority vector (step 4).

Step 8: Develop an overall priority vector

Multiply the criteria priority vector (from step 7) by the priority matrix (from step 6).

Criteria considered in this research are:

- The total travelling distances of trucks: The criterion refers to sum of all travelling distances by available vehicles to satisfy customers demand. The longer the total distance is, the higher the cost of transportation is. Therefore, it is reasonably good to have the total travelling distances of trucks as low as possible.

- The total vehicle acquisition or set-up cost: This criterion mostly considers about fixed daily cost which relates to entrance ticket fee and trucks preparation. The higher the total vehicle acquisition or set-up cost is, the less the profit is received. Hence, the total vehicle acquisition or set-up cost needs to be as low as possible.

- Customer satisfaction level: The criterion measures customer satisfaction of qualified order based on model's result. The evaluation is on scale of ten. The higher the customer satisfaction is, the more the prestige of company gets. Thus, the customer satisfaction level should be as high as possible.

- Computation duration: This criterion is necessary for the applicability of the model. It is hard to implement a good model's result which takes long solving duration into a real case because of the limitation of making decision time. Therefore, it is good to have computation duration as short as possible.

To find the most suitable model for the company delivery system, three potential alternatives representing for three models of VRP are proposed. And the results are described as following:

Table 1: Result summary of alternative one

Alternative one (Heterogeneous Fleet Capacitated Vehicle Routing Problem Model)	
The total travelling distances of trucks	268 (km)
The total vehicle acquisition or set-up cost	1820 (thousand VND)
Customer satisfaction level	8/10
Computation duration	14 (minutes)

Table 2: Result summary of alternative two

Alternative two (Heterogeneous Fleet Vehicle Routing Problem with Time Window Model)	
The total travelling distances of trucks	269 (km)
The total vehicle acquisition or set-up cost	1800 (thousand VND)
Customer satisfaction level	9.5/10
Computation duration	10.3 (minutes)

Table 3: Result summary of alternative three

Alternative three (Heterogeneous Fleet Split Delivery Vehicle Routing Problem Model)	
The total travelling distances of trucks	278 (km)
The total vehicle acquisition or set-up cost	1740 (thousand VND)
Customer satisfaction level	7/10
Computation duration	11.5 (minutes)

The complete run of three models give three feasible alternatives. In each alternative, one or two of its criteria has superior result than others. The specific analysis is given as follows:

Criterion (C) one (The total travelling distances of trucks): The alternative one has the best result (268 km) because of the smallest travelling distances. This criterion helps decrease transportation expense such as fuel price and travelling time consuming of all trucks. Although result of alternative one is not much smaller than alternative two's result (269 km), it is 10 km less than alternative three's outcome.

Criterion two (The total vehicle acquisition or set-up cost): Similarly, the alternative three has the optimal result with the minimum vehicle cost. It can help the company save on fixed daily cost of entrance ticket fee and trucks preparation. The alternative two has the second best solution with only exceed 60 thousand VND (with the total 1800 thousand VND) more than the alternative three. The alternative one has much higher vehicle cost than the optimal result of total 1820 thousand VND.

Criterion three (Customer satisfaction level): Although the alternative two does not have the optimal result in minimizing total travelling distances and total vehicle acquisition or set-up cost, it has significantly optimal result in customer satisfaction level (9.5/10). It is opposite with the alternative one (8/10) and the alternative three (7/10).

Criterion four (Computation duration): the alternative two has the best solution of computation duration (10.3 minutes). The alternative two does not run much faster than the alternative three (11.5 minutes) but it save nearly 4 minutes model running than the alternative one (14 minutes).

Next, in order to rank the solution, the AHP analysis is applied as below:

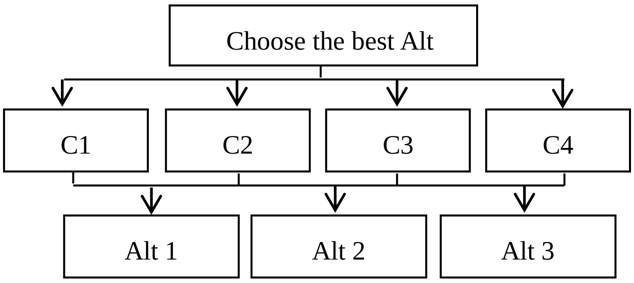

Figure 1: AHP analysis

Table 4: Pairwise comparison matrix for criteria

	C1	C2	C3	C4	Priority Vector
C1	1	1/3	1/5	3	0.122
C2	3	1	1/3	5	0.263
C3	5	3	1	7	0.558
C4	1/3	1/5	1/7	1	0.057

Table 5: Pairwise comparison matrix for first criterion

Criterion one	Alt1	Alt2	Alt3	Priority Vector	CR
Alt1	1	3	9	0.649	0.069
Alt 2	1/3	1	7	0.295	
Alt 3	1/9	1/7	1	0.057	

Table 6: Pairwise comparison matrix for the second criterion

Criterion two	Alt 1	Alt 2	Alt 3	Priority Vector	CR
Alt 1	1	1/3	1/9	0.068	0.072
Alt 2	3	1	1/7	0.155	
Alt 3	9	7	1	0.776	

Table 7: Pairwise comparison matrix for the third criterion

Criterion three	Alt 1	Alt 2	Alt 3	Priority Vector	CR
Alt1	1	1/5	3	0.193	0.058
Alt 2	5	1	7	0.724	
Alt 3	1/3	1/7	1	0.083	

Table 8: Pairwise comparison matrix for the fourth criterion

Criterion four	Alt 1	Alt 2	Alt 3	Priority Vector	CR
Alt 1	1	1/7	1/5	0.074	0.054
Alt 2	7	1	3	0.643	
Alt 3	5	1/3	1	0.283	

Table 9: Overall priority vector

Priority for Criteria	0.122	0.263	0.558	0.057	
	C1	C2	C3	C4	**Overall priority**
Alt 1	0.649	0.068	0.193	0.074	0.209
Alt 2	0.295	0.155	0.724	0.643	**0.517**
Alt 3	0.057	0.776	0.083	0.283	0.273

The rank is Alternative 2 > Alternative 3 > Alternative 1

Therefore, the alternative 2 or HFVRPTW model is chosen to be applied for the company delivery system.

5. CONCLUSION

In this research, three practical models of VRP which are HFCVRP, HFVRPTW and HFSDVRP are solved by Mix Integer Linear Programming. Results got from those three were generated as three alternatives. After that, they were evaluated under four criteria, namely, the total travelling distances of trucks, the total vehicle acquisition or set-up cost, customer satisfaction level and computation duration. Among those, total distances, total vehicle cost, and computation duration can be achieved by solving mathematical model and customer satisfaction level is determined by the company through customer comment for each model. Then, the best alternative is selected, based on the rank of them. As a result, alternative two which is HFVRPTW model result is recommended to be implemented.

The HFVRPTW model allows once visit at each customer with time window delivery system and multiple customers serve. By applying Mix Integer Linear Programming model into real transportation scheduling system of the company, the research has shown a reasonable result compared to the current schedule.

In reality, this paper has shown optimistic result for the company. In comparison with the current schedule, the proposed schedule yields about 15% improvement in total travelling distance of all trucks. More than that, customers are satisfied with the exact delivery amount and appropriate period which belongs to their working time frame. Additionally, necessary used trucks are almost utilized. SOVIGAZ still schedule trucks manually so it takes much time to fulfill large scale of demand. With the proposed model in this thesis, the approximate ten minutes of computational duration is acceptable. Furthermore, only few steps to get the optimal solution, it can be a strong supportive tool for SOVIGAZ delivery system.

Heuristic algorithms should be studied find optimal solution faster and appropriate to the case study.

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