Using Confidence Intervals of Index Cpp to Construct an Evaluation System for Manufacturers

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Abstract. Choosing manufacturers is an important decision for enterprises, and the most critical factor for evaluating manufacturers is quality. The process capability index is an overall measurement tool, easy to understand for businesses to evaluate the product or the quality of the product provided by suppliers or manufacturers and enhancement their quality performance. The index Cpp not only reflects yield and process loss, but also analyzes precision and accuracy and process monitoring, which is more suitable for practical application. The index Cpp is converted to the function of δ and γ, the joint confidence interval δ and γ of $(1-\alpha)$% and the index Cpp (δ, γ), are served to be constraints and the objective function to establish a mathematical programming model to calculate confidence intervals of the index Cpp. Next, confidence intervals of the index Cpp of each manufacturer can be determined; an evaluation system for manufacturer is then established on the basis of EXCEL. This proposed evaluation system, established by the confidence interval of the index Cpp and a comparison model, is easy to understand and saves a complicated statistical process. In addition, precision and accuracy of individual manufacturers can be corrected and improved. Therefore, this proposed evaluation system is a more efficient system for real world application.

Keywords: process incapability indices, confidence intervals, joint confidence intervals, mathematical programming, manufacturer evaluation.

1. INTRODUCTION

Efficiency and accuracy are tremendously emphasized in the modern business world. Therefore, regardless of the service or the manufacturing industry, precise monitoring during the production process is required to have a market share. According to Dickson's study (1966), among the 23 supplier evaluation criteria suggested, the most important one is quality. Weber et al. (1991) then offered an in-depth discussion based on Dickson's research and investigated the frequency of using these 23 criteria; the conclusion was that quality is still the most critical factor. Wilson (1994) also conducted a study on supplier evaluation and concluded that the significance of quality is rising, whereas the emphasis on price is decreasing. In addition, manufacturers are having a higher demand for service. Thus, quality is the combination of meeting customers' requirements for specifications and integrating designs and manufacturing capabilities between manufacturers and designers. In fact, process capability indices are a means for evaluating and monitoring process performance and quality of the product. That is, these indices can evaluate whether the functions of product meet the specific requirement after excluding assignable causes from the process. They still can monitor the process and enhance the quality of the product in conformity with customer requirements. So PCI is a simple and easy to understand for overall process evaluation for enterprises.

The process capability index *PCI* is based on upper and lower specification limits and targets, *LSL*, *USL*, *T*, demanded by customers to manufacture specification compliance products. This measurement is calculated by the process mean μ and the correlation between the standard deviation σ and process specifications. Enterprises may judge the stability and yield of the process by the *PCI*. Juran (1974) first proposed the process capability index C_p, which was defined as the ratio of *LSL* and *USL* to the actual standard deviation of the process. Consequently, an estimate $\hat{\sigma}$ is usually used to replace σ. Kane (1986) presented the index C_{pk} and the one-sided specification index (C_{pu}, C_{pl}) to reflect process movement on average. When the process flows evenly beyond the midpoint between *USL* and *LSL*, the applicability of the index C_p should be considered. C_p and C_{pk} are not taken into consideration the difference between the process mean and the target value. Therefore, Chan et al. (1988) proposed the index C_{pm} based on Taguchi's loss function. C_p, C_{pk}, and C_{pm} are defined and expressed as follows:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma} \tag{1}$$

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma} \tag{2}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} \tag{3}$$

USL and *LSL* refer to the upper and lower specification limits, respectively, μ as the process mean, σ as the process standard deviation, *T* as the target value, $\sigma^2 + (\mu - T)^2$ as the expected value of Taguchi's loss function, $m = (USL + LSL) / 2$ as the midpoint of the specification interval and $d = (USL - LSL) / 2$ as half of the length of the specification interval.

Greenwich and Jahr-Schaffrath (1995) proposed the process incapability index C_{pp}. The index C_{pp} is converted from the index C_{pm}, i.e., it is the inverse square of the index C_{pm}, which is expressed as follows:

$$C_{pp} = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2 \tag{4}$$

where $D = d / 3$, $d = (USL - LSL) / 2$.

Philps et al. (1994) proposed the idea of loss ratio, which means $LR = 1/C_{pm} = (CR + TR)^{1/2}$. Consequently, the equation can be expressed as follows:

$$C_{pp} = \left(\frac{1}{C_{pm}}\right)^2 = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2 = C_{ia} + C_{ip} \tag{5}$$

where $C_{ia} = \left(\frac{\mu - T}{D}\right)^2$ as the inaccuracy index for evaluation of process accuracy;

$C_{ip} = \left(\frac{\sigma}{D}\right)^2$ as the imprecision index for evaluation of process precision.

As a result, in addition to process capability evaluation using C_{pp}, indices C_{ia} and C_{ip} can further analyze the accuracy and precision of the process for an understanding of different factors as well as to point a specific direction for correction. Actually, the index C_{pp} reveals process yield and process loss simultaneously. That is, when $C_{pp} = C$ with $Yield\% = 2 - 2\Phi(3 / \sqrt{C})$ and $|\mu - T| / d \leq \sqrt{C} / 3$ (a smaller *C*), the ratio of the process mean deviating from the target value becomes lower and develops a higher process yield. Pearn et al. (1999) indicated that the index C_{pp} containing C_{ia} and C_{ip} is superior to the index C_{pm}. Accuracy and precision analysis are more suitable for practical applications, which is a more efficient and convenient tool for quality monitoring for enterprises.

Chou et al. (1990) claimed that the use of point estimation might misjudge process capability due to sampling error, whereas interval estimation not only considers the standard error of point estimation, but it also obtains the precision of interval estimation by the confidence level. In addition, the lower limit of confidence level can be adopted to determine process capability to reduce misjudgment of process level. Hence, many statistical quality control scholars have proposed statistical inference to derive the properties and confidence intervals of process capability indices. For instance, Pearn et al. (1999) suggested statistical testing for examination of C_{pp} in conformity with customer requirements; Chen (1998) presented the estimator of C_{pp} and applied the method for process capability monitoring proposed by Spiring (1995) for further discussions. To sum up, it is found that the confidence intervals of the indices induced by the scholars are quite similar, and only a limited number of scholars were interested in inducing confidence intervals related to the index C_{pp} because the probability density distribution function of the process capability index C_{pp} is much more complicated than C_p, C_{pk}, or even C_{pm}. In fact, the index C_{pp} can be transformed into the function of $\delta = (\mu - T) / D$ and $\gamma = \sigma / D$. Although it is quite complex and difficult to infer the real probability density function of C_{pp}, nevertheless, the joint confidence interval, $(1-\alpha)$, between δ and γ can be obtained easily. The index $C_{pp}(\delta, \gamma)$ is served as the objective function and the joint confidence interval of $(1-\alpha)$ between δ and γ is used as a restraint in this research to establish a mathematical programming model for calculating the real confidence interval of the index C_{pp}.

Chen et al. (2001) and Chen et al. (2002) claimed the process capability index was an efficient and convenient tool for process capability evaluation. Therefore, plenty of scholars developed evaluation models on the basis of this index, such as Chen and Chen (2006); Chen, Chen, and Li (2005); Chen and Huang (2006); Chen and Chen (2004); Huang and Chen (2003); and Chen, Huang, and Hung (2002). Degraeve et al. (2000) indicated that supplier evaluation criteria should be regulated specifically to enhance manufacturing performance of enterprises; Das and Narasimhan (2000) proposed that evaluation results of suppliers could affect the cost, quality, transportation, and marketing performance of a business. Thus, selection of manufacturers is a critical purchase decision for enterprises. As a result, many scholars have devoted their efforts toward developing supplier decision models. For instance, Narasimhan (1983) and Nydick and Hill (1992) used AHP to evaluate the quality, price, service, and delivery of suppliers; Timmerman (1986) utilized the cost ratio approach for supplier evaluation criterion; others include Weber and Current (1993) and Tompson (1990). Here, a supplier evaluation system based on the confidence interval of the index C_{pp} will be developed for judgment of the process capability. In addition, a comparison model is also established. In this way, enterprises are not only capable of choosing manufacturers with better process capabilities but also selecting better manufacturers for backup to prevent problems, such as insufficient capacity or out of stock resulting from other external factors. As a result, the manufacturer evaluation system developed by the confidence interval in this research can meet customer requirements for product quality and proves to be more efficient in manufacturer process monitoring, which is a better solution for real evaluation of quality manufacturer.

2. Solving the confidence intervals of C_{pp}

As aforesaid, the index C_{pp} may reflect process yield and loss, and further analysis for process improvement can be conducted by the indices of precision and accuracy. Hence, it is considered to be a reliable index in practice. Actually, the index C_{pp} can be converted to the function of $\delta = (\mu - T) / D$ and $\gamma = \sigma / D$ and δ and γ can be regarded as process parameters of relative specifications. That is to say, every coordinate (δ, γ) represents one process. Consequently, the index C_{pp} is redefined as follows:

$$C_{pp} = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2 = \delta^2 + \gamma^2 = Z^2 \qquad (6)$$

wherein $Z^2 = \delta^2 + \gamma^2$. Apparently, it is very difficult to infer the probability density distribution function of $\hat{C}_{pp}$ leading to a complicated calculation of its confidence interval. However, it is easier to infer the joint confidence interval of δ and γ. On the basis of Boole's inequality, the joint confidence interval of δ and γ can be inferred Thus, Cartesian product is obtained as follows:

$$S(X) = [m_1, \quad m_2] \times [n_1, \quad n_2]$$

wherein $$m_1 = \hat{\delta} - \frac{\hat{\gamma}}{\sqrt{n}} t_{n-1,\alpha/4} \qquad (7)$$

$$m_2 = \hat{\delta} + \frac{\hat{\gamma}}{\sqrt{n}} t_{n-1,\alpha/4} \qquad (8)$$

$$n_1 = \sqrt{\frac{(n-1)\hat{\gamma}^2}{\chi^2_{n-1,\alpha/4}}} \qquad (9)$$

$$n_2 = \sqrt{\frac{(n-1)\hat{\gamma}^2}{\chi^2_{n-1,1-\alpha/4}}} \qquad (10)$$

The set $S(X)$ is $(1-\alpha)$% joint confidence interval of δ and γ. In other words, the probability of the process capability index $C_{pp}(\delta, \gamma)$ within $S(X)$ is up to $100(1-\alpha)$%. Therefore, to locate $M = \max\{C_{pp} | (\delta, \quad \gamma) \in S(X)\}$ and $N = \min\{C_{pp} | (\delta, \quad \gamma) \in S(X)\}$ in the set $S(X)$ is to determine the upper and lower limits of the confidence interval for index C_{pp}, which is $P(N \le C_{pp} \le M | (\delta, \quad \gamma) \in S(X)) = 1 - \alpha$.

Next, a mathematical programming approach will be adopted to calculate the $100(1-\alpha)$% confidence interval of the index C_{pp}. It is known from equation (1) that a corresponding square relationship exists between Z and C_{pp}. When Z is the maximum, the index C_{pp} is also maximum, which is the upper limit of the confidence interval; in contrast, the minimum Z leads to the minimum of the index C_{pp}. As a result, the mathematical programming model of upper and lower limits of the confidence interval for the index C_{pp} is expressed as follows:

$$\max\ (\min)\quad Z=\sqrt{\delta^2+\gamma^2}$$
$$s.t.\quad m_1 \le \delta \le m_2$$
$$0 \le n_1 \le \gamma \le n_2$$

Restraints in the model above are the same, and targets are set for the maximum and minimum. The optimum of joint confidence intervals of δ and γ under different situations will be calculated. First, the relationship between the target and the restraint in Fig. 2.1 is observed. Two curves represent the objective functions and the square stands for the restraint constituted by the joint confidence interval of δ and γ. That is to say, the feasible region is $F = \{(\delta, \gamma) \mid m_1 \le \delta \le m_2, 0 \le n_1 \le \gamma \le n_2\}$. When $\gamma = n_2$, the maximum feasible region will be $F_U = \{(\delta, \gamma) \mid m_1 \le \delta \le m_2, \gamma = n_2\}$. Likewise, when $\gamma = n_1$, the minimum feasible region will be $F_L = \{(\delta, \gamma) \mid m_1 \le \delta \le m_2, \gamma = n_1\}$ If the heading should run into more than one line, the run-over should be flushed left.

As stated above, when the upper limit of C_{pp} confidence interval is calculated, the maximum Z is obtained, which means $F_U = \{(\delta, \gamma) \mid m_1 \le \delta \le m_2, \gamma = n_2\}$. Consequently, the optimum would not be affected no matter if γ axis were concluded. Therefore, when γ axis serves as the centerline of the feasible region, the optimum will fall on both ends of the joint confidence interval of δ and γ, which are (m_1, n_2) and (m_2, n_2), respectively. When the feasible region moves to the left or the right on the basis of the γ axis, the optimum will be (m_1, n_2) and (m_2, n_2), respectively. The above statement is expressed as follows:

Situation 1: $m_1 + m_2 = 0$ with the optimum of (m_1, n_2) and (m_2, n_2), the upper confidence limit of index C_{pp} is $m_1^2+n_2^2$ and $m_2^2+n_2^2$;

Situation 2: $m_1 + m_2 > 0$ with the optimum of (m_2, n_2), the upper confidence limit of index C_{pp} is $m_2^2+n_2^2$;

Situation 3: $m_1 + m_2 < 0$ with the optimum of (m_1, n_2), the upper confidence limit of index C_{pp} is $m_1^2+n_2^2$.

Likewise, when the lower limit of C_{pp} confidence interval is calculated, the minimum Z is obtained and the feasible region is $F_L = \{(\delta, \gamma) \mid m_1 \le \delta \le m_2, \gamma = n_1\}$. As a result, if the feasible region contains γ axis, the minimum will definitely fall on the coordinate with δ as zero, leading to the optimum as $(0, n_1)$. When the feasible region does not include the γ axis and falls on the left or right side of γ axis, the optimum will be (m_2, n_1) and (m_1, n_1), respectively. The statement is expressed as follows:

Situation 1: $m_1 \le 0 \le m_2$ with the optimum of $(0, n_1)$, the lower confidence limit of index C_{pp} is n_1^2;

Situation 2: $m_1 > 0$ with the optimum of (m_1, n_1), the lower confidence limit of index C_{pp} is $m_1^2+n_1^2$;

Situation 3: $m_2 < 0$ with the optimum of (m_2, n_1), the lower confidence limit of index C_{pp} is $m_2^2+n_1^2$.

Refer to the appendixes for detailed explanations of proving the upper and lower limits of confidence for index C_{pp}. Thus, $\hat{\delta}$ and $\hat{\gamma}$ can be calculated by *USL*, *LSL*, target value *T*, and sampling data, which is to say m_1, m_2, n_1, and n_2 can be computed via equations (2) through (5). Next, situations of m_1 and m_2 will be judged by comparing Tables 2.1 and 2.2, and confidence intervals of the index C_{pp} will be calculated. The manufacturer evaluation system will be established by applying the confidence intervals of the index C_{pp} in section 3. Process data provided by manufacturers shall be used to compute the confidence intervals of the process capability index, and a model of comparing the confidence intervals of C_{pp} is provided for completion of manufacturer evaluation.

3. Establishment of a manufacturer evaluation system

As stated earlier, the index C_{pp} is converted to the functions of δ and γ, and a mathematical programming model is constructed. Confidence intervals of the index C_{pp} for joint confidence intervals between δ and γ under different situations are computed. A manufacturer evaluation model based on the confidence intervals of C_{pp} will be established.

To maintain generality, k manufacturers for evaluation are hypothesized here. First, joint confidence intervals between δ_i and γ_i $[(m_{1i}, m_{2i}) \times (n_{1i}, n_{2i})]$ are computed by the sampling data provided by manufacturers via equations (2) through (5). Next, situations of m_{1i} and m_{2i} are determined by comparing the upper and lower confidence limits of C_{pp} for the confidence level (L_i, U_i) of C_{ppi} for each manufacturer. i stands for the *ith* manufacturer with $i=1, 2, \ldots, k$. For convenience, symbols are defined as follows:

1. $B_i = U_i - L_i$, which means the length of the confidence interval of C_{ppi} for the *ith* manufacturer.
2. $C_i = (L_i+U_i) / 2$, which means the midpoint of the confidence interval of C_{ppi} for the *ith* manufacturer.
3. $C_{(1)} = \min(C_1, C_2, \ldots, C_k)$, which means the minimum of the midpoint of the confidence interval of C_{ppi} for the *ith* manufacturer.
4. $length(A_{ij})$ refers to the overlapping length between the confidence interval of C_{ppi} for the *ith* manufacturer and that for the target manufacturer *j*, which is $length\{(L_i, U_i) \cap (L_j, U_j)\}$.
5. Suppose $C_{(1)} = C_j$, then $J_i = length(A_{ij}) / \min\{B_i,$

B_j} is the determination index of manufacturer *i*.

On the basis of the above definitions, the steps for establishing a comparison model for manufacturer evaluation are described as follows:

Step 1：Compute B_i, C_i and $C_{(1)}$ of the *ith* manufacturer.
Step 2：Select the manufacturer corresponding to $C_{(1)}$ as the target. For convenience of expression, the *jth* manufacturer is presumed to be the target manufacturer here, which means $C_{(1)} = C_{(j)}$.
Step 3：Calculate $length(A_{ij})$.
Step 4：Compute the determination index $J_i = length(A_{ij}) / min\{B_i, B_j\}$ with $0 \leq J_i \leq 1$.
Step 5：Use J_i as the determination index for manufacturer evaluation, and priority will be given to greater values.

4. Conclusions

The process capability index C_{pp} not only can reflect the process yield and loss simultaneously, but also further analyzes and determines by use of the inaccuracy index and the imprecision index. It is more efficient for quality monitoring for enterprises. A mathematical programming model is derived here to compute the confidence intervals of the index C_{pp} and convert the index into the functions of $\delta = (\mu - T) / D$ and $\gamma = \sigma / D$. A mathematical programming model is constructed using the joint confidence intervals of $(1-\alpha)$ between δ and γ as the restraint and $C_{pp}(\delta, \gamma)$ as the objective function for computations of different joint confidence intervals of $(1-\alpha)$ between δ and γ. Thus, the confidence interval of the index C_{pp} is calculated successfully. The confidence intervals of the index C_{pp} are then used to set up a manufacturer evaluation system. A comparison model under various situations is established, and the determination index J_i is developed as the comparison criterion. Finally, the EXCEL program is utilized to set up the manufacturer evaluation system in compliance with the comparison model developed. An example of TFT–LCD array manufacturers is given to illustrate application of this system. The best manufacturers can be selected rapidly via this system, and backup manufacturers will be available to cope with insufficient supply to prevent a greater loss. Furthermore, the guidance of this proposed evaluation method can be used to improve even the worst quality level of manufacturers. This effective system is easy to comprehend and convenient for practical application.

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