The Shortage Study of EOQ Model with Defective and Reworked Items

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Abstract. The main purpose of this paper is to determine the value of economic order quantity (EOQ) with two types of imperfect items, defective and reworked ones, under 100% inspection. Where defective items are treated as scraps and reworked ones will be sent back to supplier to get repaired. A mathematical model is built on the joint distribution of defective and reworked units which follow a multivariate hyper-geometric distribution given that above two items come from some uniform distribution separately. We extend the study to shortage cases that may occur due to the lack of good items to fulfill demand rate during an order cycle, therefore, it is assumed that manufacturer has to pay additional penalty when the shortage occurs. The expected total cost per unit time is then calculated over its corresponding probabilities as well as the expected total cost. Finally, a numerical computation is provided to illustrate the conclusive outcome of proposed model.

Keywords: EOQ model, imperfect items, shortage, expected total cost per unit time

1. INTRODUCTION

Ever since the EOQ model was introduced in early of twenty century, it has been widely used in many industrial spheres. The function of EOQ model is to determine the optimal order size that minimizes the total cost per unit time. However, a number of unrealistic assumptions of this model have led many researchers to make efforts in the studies of modifications of EOQ model. For instance, the classical EOQ model that assumed all items are of perfect quality with no shortage allowance is getting harder to be satisfied in common practice. Hence, the traditional EOQ model is considered to be conflicted with the real-life situations by current methods and the weaknesses of this model encourage researchers to improve the EOQ model so that it could be utilized under reasonable applications.

There are various recent studies on the field, Sphicas (2006) examined EOQ model with backorder cost under algebraic approach. Toews, Pentico and Drake (2011) tried to redefine holding, backorder and lost sales costs to allow the percentage demand of backorder increasing linearly as the time until delivery decreases. Nasr, Maddah and Salameh (2013) considered a correlated random binomial supply in their EOQ model.

Salameh and Jaber (2000) mentioned the probability of items with imperfect quality through inspection process in each cycle. They identified that the imperfect items could be sold by the end of the screening process. Goyal and Cárdenas-Barrón (2002) presented a simple approach for determining the economic production quantity of an item with imperfect quality. They showed that the nearoptimal results are obtained by using the simple approach. The model suggested in their note is easier to implement. Chang (2004) developed EOQ model having imperfect quality without shortages in which demand and defective rate are taken as triangular fuzzy numbers.

Wee, Yu and Chen (2007) extended the work of Salameh and Jaber (2000) to develop an inventory model with imperfect quality allowing shortages, taking defective rate as a random variable. Al-Salamah and Alsawafy (2011) discussed a new EOQ model by exploring the probabilities of defective and reworkable items under uniform distribution. However, they do not allow shortage to occur throughout a cycle. At last, Öztürk, Eroglu and Lee (2015) offered a mathematical model which determines the economic order quantity and back-order quantity for a single item in a 100% inspection of ordering process to separate good, reworkable, or scrap items. For more detailed literatures, we refer readers to (Khan et al., 2011; Rezaei and Salimi, 2012).

The present study is built upon Al-Salamah and Alsawafy's (2011) model with imperfect quality of defective and reworkable items. Al-Salamah and Alsawafy's (2011) model does not allow shortages to occur and each lot receives 100% screening to separate good, defective and reworkable items. They assumed that defective items can not fulfill customer's demand. At the end of screening process, reworkable items will be returned to the supplier to get repaired so that they can be sold later and defective items will also be sold to the secondary market.

To guarantee no shortage to occur, it needs to satisfy the following inequality

$$x - (\max\{P_S\} + \max\{P_R\})\ y \geq d, \tag{1}$$

However this inequality is too ideal to apply, we then introduce our mathematical model and solution methodology as well as the notations including x, y, d, P_S, and P_R appeared in the previous equation in the next sections.

2. MATHEMATICAL MODEL

2.1 Notations for the Model

To begin our development of proposed model, necessary notations will firstly be defined to form the equations. These notations are enlisted as follow.

K	fixed cost of placing an order
c	purchasing cost per item
e	screening cost per item
y	the order size for each cycle
d	demand per unit time
P_S	percentage of defective items in a cycle
$f_1(p_S)$	probability density function of P_S
P_R	percentage of reworkable items in a cycle
$f_2(p_R)$	probability density function of P_R
h	holding cost per item
G	the number of good items in the current cycle
N	the number of defective items in the current cycle
R	the number of reworkable items in the current cycle
x	the number of screening items per unit time
w	the number of reworkable items able to be repaired per unit time
s	shortage cost (penalty cost) per item
t	screening time, $t = y/x$
q	the number of screening times for the entire screening process

To accommodate all situation including shortage, it is compulsory to compute every single possibility in each screening lot through multivariate hypergeometric distribution of good, defective and reworkable item combinations over order size and screening rate combinations. Moreover, to rebuild a reasonable model, it is necessary to define t (screening time) which is restricted to

$$t = \frac{y}{x} = \left[\frac{y}{x}\right] + \left(\frac{y}{x} - \left[\frac{y}{x}\right]\right) = m + r$$

where $m = \left[\frac{y}{x}\right]$ denotes the nearest integer that is smaller than the value of $\frac{y}{x}$ and $0 \leq r < 1$.

Let q represent the number of screening times for the entire inspection process, hence,

$$q = \begin{cases} m+1, & if\ r \neq 0 \\ m\ \ , & if\ r = 0 \end{cases} \tag{2}$$

Assume that the random variables of good and defective items for the i^{th} lot are G_i, and N_i, $i = 1,2, \ldots, q-1$. The sum of all these values must be equal to the good items G and defective items N in the whole lot which are $\sum_{i=1}^{q} g_i = G$ and $\sum_{i=1}^{q} n_i = N$ while the corresponding observations are $g_1, g_2, \ldots, g_{q-1}$ and $n_1, n_2, \ldots, n_{q-1}$. As to the last lot, g_q and n_q can be directly computed from $g_q = G - \sum_{i=1}^{q-1} g_i$ and $n_q = G - \sum_{i=1}^{q-1} n_i$. In real problem, all the good, defective and reworkable items are considered as a whole unit, therefore G, N and R are integers.

2.2 Model Development

To formulate our distribution, we set g_i and n_i as number of good items and defective items found on the i^{th} unit time. Hence the multivariate hypergeometric distribution given at fixed values of P_S and P_R is retrieved as the following:

$$\Pr\left(G_1 = g_1, N_1 = n_1 \mid p_S, p_R\right) = \frac{\binom{G}{g_1}\binom{N}{n_1}\binom{R}{x-g_1-n_1}}{\binom{y}{x}},$$

$g_1 = 0,1,2,...,x$, $n_1 = 0,1,2,...,N$ (3)

$$\Pr\left(G_2 = g_2, N_2 = n_2 \mid p_S, p_R, G_1 = g_1, N_1 = n_1\right) = \frac{\binom{G-g_1}{g_2}\binom{N-n_1}{n_2}\binom{R-(x-g_1-n_1)}{(x-g_2-n_2)}}{\binom{y-x}{x}}$$

, $g_2=0,1,2,...,x$, $n_2=0,1,2,..,N$ (4)

Hence the probability for the last random screening lot (q-1) will be:

$\Pr(G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} |$

$p_S, p_R, G_1 = g_1, N_1 = n_1, \dots, G_{q-2} = g_{q-2}, N_{q-2} = n_{q-2}) =$

$$\frac{\binom{G-g_1-\cdots-g_{q-2}}{g_{q-1}}\binom{N-n_1-\cdots-n_{q-2}}{n_{q-1}}\binom{R-((q-2)x-\sum_{i=1}^{q-2}(g_i+n_i))}{(x-g_{q-1}-n_{q-1})}}{\binom{y-(q-2)x}{x}},$$

$g_{q-1} = 0,1,2,\dots,x$, $n_{q-1} = 0,1,2,..,N$

(5)

Then we proceed to compute the value of probability density function of $\Pr(g_1, n_1, \dots, g_{q-1}, n_{q-1}, |P_s, P_R)$ which is assumed to be jointly multivariate hypergeometric random variable under fixed P_S and P_R as follow：

$$\Pr\left(G_1 = g_1, N_1 = n_1, \dots, G_{n-1} = g_{n-1}, N_{q-1} = n_{q-1} \middle| p_S, p_R\right)$$
$$= \Pr(G_1 = g_1, N_1 = n_1 | p_S, p_R)$$
$$\times \Pr(G_2 = g_2, N_2 = n_2 | p_S, p_R, G_1 = g_1, N_1 = n_1) \times \dots \times$$
$$\Pr(G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} | p_S, p_R, G_1 = g_1, N_1 = n_1, G_2 = g_2, N_2 = n_2, \dots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1})$$

(6)

Thus,

$$\Pr\left(G_1 = g_1, N_1 = n_1, \dots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \mid p_{S,} p_R\right)$$

$$= \frac{\binom{G}{g_1, g_2, \dots, g_{q-1}}\binom{N}{n_1, n_2, \dots, n_{q-1}}\binom{R}{x-g_1-n_1, x-g_2-n_1, \dots, x-g_{q-1}-n_{q-1}}}{\binom{y}{x, x, \dots, x}},$$

(7)

Therefore the expected total cost per unit time (*ETCPUT*) when P_S and P_R are fixed can be defined as follow:

$$ETCPUT(G_1 = g_1, N_1 = n_1, \dots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \mid p_S, p_R)$$

$$= \sum\sum\dots\sum \Pr\left(G_1 = g_1, N_1 = n_1, \dots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \mid p_S, p_R\right) \times$$

$$TCPUT\ (G_1 = g_1, N_1 = n_1, \dots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \mid p_S, p_R)$$

(8)

The computation of equation (8) will be discussed in the next section. P_S and P_R are continuous uniform distributed with p.d.f. described as follow:

$$f_1(p_S) =$$

$$\begin{cases} \frac{1}{\beta_1-\alpha_1}, for & \alpha_1 \le p_S \le \beta_1 \\ 0 \quad , otherwise \end{cases} \tag{9}$$

and

$$f_2(p_R) =$$

$$\begin{cases} \frac{1}{\beta_2-\alpha_2}, for & \alpha_2 \le p_R \le \beta_2 \\ 0 \quad , otherwise \end{cases}$$

(10)

The expected total cost per unit time for the value of order size (y) is defined as:

$$ETCPUT(Y) = \int_{\alpha_1}^{\beta_1}\int_{\alpha_2}^{\beta_2} f_1(p_S)\, f_2(p_R) \times$$

$$ETCPUT(G_1 = g_1, N_1 = n_1, \ldots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \mid p_S, p_R)$$

$$\times\, \mathrm{d}p_R\, \mathrm{d}p_S \tag{11}$$

Since it is difficult to determine the exact computat -ion result for the integral function of P_S and P_R, thus we use the approximation method as follow:

$$ETCPUT(Y) \cong \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \frac{\beta_1-\alpha_1}{n_1}\frac{\beta_2-\alpha_2}{n_2}\ f_1(p_S) f_2(p_R) \times$$

$$ETCPUT\left(G_1 = g_1, N_1 = n_1, \ldots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \middle| p_{S_i}, p_{R_j}\right)$$

$$\cong \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \frac{\beta_1-\alpha_1}{n_1}\frac{\beta_2-\alpha_2}{n_2}\ f_1(p_S) f_2(p_R) \times$$

$$ETCPUT\left(G_1 = g_1, N_1 = n_1, \ldots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \middle| p_{S_i}, p_{R_j}\right)$$

$$= \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \frac{\beta_1-\alpha_1}{n_1}\frac{\beta_2-\alpha_2}{n_2}\frac{1}{\beta_1-\alpha_1}\ \frac{1}{\beta_2-\alpha_2} \times$$

$$ETCPUT\left(G_1 = g_1, N_1 = n_1, \ldots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \middle| p_{S_i}, p_{R_j}\right)$$

$$= \frac{1}{n_1 n_2} \times$$

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} ETCPUT\left(G_1 = g_1, N_1 = n_1, \ldots, G_{q-1} = g_{q-1}, N_{q-1} = n_{q-1} \middle| p_{S_i}, p_{R_j}\right) \tag{12}$$

where $p_{S_i} = \alpha_1 + \frac{(\beta_1-\alpha_1)i}{n_1}$ and $p_{R_j} = \alpha_2 + \frac{(\beta_2-\alpha_2)j}{n_2}$.

For our simulation, we define $n_1 = 10$ and $n_2 = 10$ to divide each P_S and P_R into 10 parts of equal length to approximate the above integral computation. Within these divisions, it is assumed that the approximation will be sensitive enough to represent *ETCPUT* value.

2.3 TCPUT Computation

To calculate the total cost of inventory, we first examine inventory level for four different cases of non-shortage and shortage cases in our model:

1. Inventory level of EOQ for non-shortage case,
2. Inventory level of EOQ for shortage case during inspection,
3. Inventory level of EOQ for shortage case during rework time,
4. Inventory level of EOQ for shortage case during inspection and rework time.

For discussion, We then choose case 4 to be an example and assume that $q = 2$, thus $m = 1$. Remind that

$$\mathbf{N} = \overline{P_S \cdot y},$$

$$R = \overline{(P_S + P_R)y} - N.$$

Moreover, $\overline{P_S \cdot y}$ and $\overline{(P_S + P_R)y}$ are all integers since the number of good, defective and reworkable items must be a rounded number. If they are not integers, they will be rounded to the nearest integer values for total cost computation.

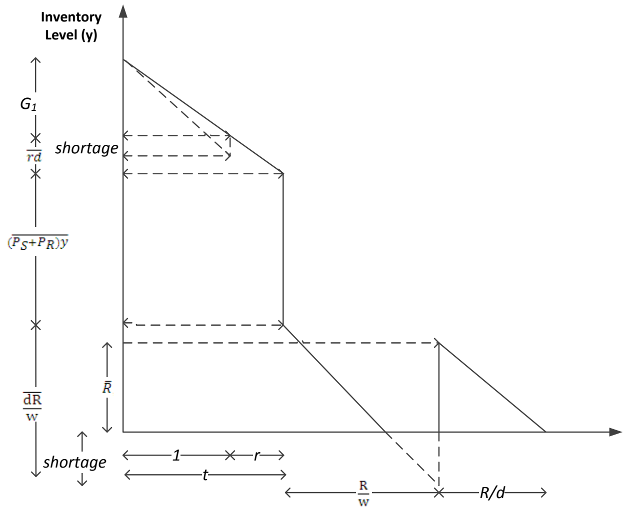

Figure 2.1 Inventory level of EOQ for shortage case during inspection and rework time

Figure 2.1 represents shortage case occurring in the first screening and rework time when random inspection of purchased items is conducted. Note that G_1 is the number of good items at the first screening time. Thus, the total cost of inventory per unit time (TCPUT) will be

$$TCPUT = \{K + cy + ey + h[\frac{(y + (y - G_1))}{2} + \frac{((y-G_1) + (y - G_1 - \overline{rd}))r}{2} + \frac{(y - G_1 - \overline{rd} - \overline{(P_S + P_R)y})^2}{2d} + \frac{R^2}{2d}] + \left((d - G_1) + \left(\frac{\overline{dR}}{w} - (y - G_1 - \overline{rd} - \overline{(P_S + P_R)y})\right)\right)s\} \;/\; \{t + \frac{R}{w} + \frac{R}{d}\} \quad (13)$$

3. NUMERATION RESULTS FOR ETCPUT

Since inventory cost computation needs to consider a random proportions of good, defective and reworkable items, it becomes obviously that probability of shortage to occur is unavoidable. Therefore, in order to build a more reliable scheme to estimate expected total cost per unit time corresponding more closely to real situation, we utilize compatible programming tools to execute the syntax or calculation logic and generate desired result based on defined parameter at a quick time. This program is specially designed to handle the complexity of computing total cost with the variation of good, defective and reworkable items, order quantity, inspection rate, demand rate as well as other related costs at each unit time along with its corresponding probability.

To illustrate the computation result generated from the simulation, we observe and retain the outcome as graphical function of order quantity and respective *ETCPUT* based on below parameters:

Order size, y	$y \in [15,359]$
Fixed cost, K	=\$50/cycle
Variable cost, c	= \$ 3/unit
Screening cost, e	= \$ 0.5/unit
Penalty cost, s	= \$ 4.5/unit
Holding cost, h	= 0.9/unit
Demand, d	= 100/ unit time
Screening rate, x	= 120/unit time
Rework rate	= 30/unit time

Given the defective rate, P_S~U(0, 0.05) and the reworkable rate, P_R~U(0, 0.05), the solution procedure is applied to generate results for *ETCPUT*(y). Hence, the optimal value of y (y*) = 126 and the optimal value of ETCPUT(y*) = 480.56196.

Interestingly we observed a pattern of the expected total cost value that will drop as number of order quantity increases to a certain point and turn over to increase gradually as number of order increases within a range of order quantity. The behavior of *ETCPUT*(y) is displayed in Figure 3.1.

In this study, a mathematical model is developed to determine an optimum inventory with two random imperfect percentages P_S and P_R within a complete screening process. Defective and reworkable proportions (P_S, P_R) are random variables with uniform distributions and each screening lot is assumed to follow a multivariate hypergeometric distribution.

We point out that the new proposed model is sufficient enough to accommodate shortage consideration as they are carefully observed. The optimal ordering policy is depicted to illustrate the solution procedure under known demand circumstances.

We believe that several types of manufacturers can be benefited from our study, especially those who typically require a longer time to complete their inspection process such as automobile or electronic assembly lines. These fields of industry usually need to perform a more comprehensive testing within each component, part or engine performance. Due to these reasons, a limited number of tested items may not always fulfill the demand from customer, therefore our model can be applied to cope with the situation and generate a more efficient solution in terms of inventory costs.

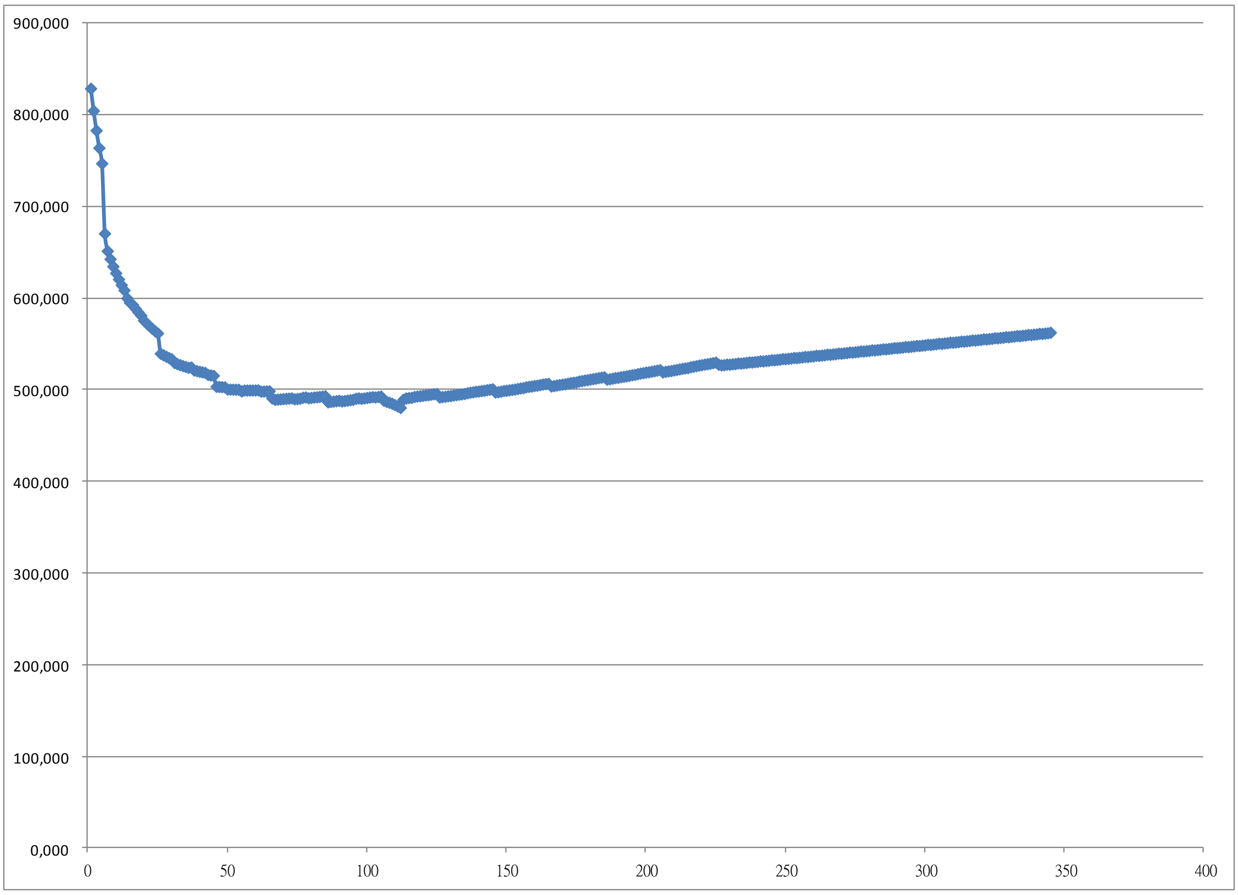

Figure 3.1 ETCPUT(y) for y=15 to y=359.

REFERENCES

Al-Salamah, M., and Alsawafy, O. (2011) Economic order quantity for items with two types of imperfect quality. *An International Journal of Optimization and Control: Theories & Applications*, 2,73-82.

Chang, H.C. (2004) An application of fuzzy sets theory to the EOQ model with imperfect quality items. *Computers and Operations Research*, 31,2079-2092.

Goyal, S.K., and Gardenas-Barron, L.E. (2002) Note on: Economic production quantity model for items with imperfect quality–a practical approach. *International Journal of Production Economics*, 77,85-87.

Khan, M., Jaber, M.Y., Guiffrida, A.L., and Zolfaghari, S. (2011) A review of the extensions of a modified EOQ model for imperfect quality items. *International Journal of Production Economics*, 132,1–12.

Nasr, W.W., Maddah, B., and Salameh, M.K. (2013) EOQ with a correlated binomial supply. *International Journal of Production Economics*, 144,248–255.

Öztürk, H., Eroglu, A., and Lee, G.M. (2015) An economic order quantity model for lots containing defective items with rework option. *International Journal of Industrial Engineering : Theory Applications and Practice*, 22, 683-704.

Rezaei, J., and Salimi, N. (2012) Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. *International Journal of Production Economics*, 137, 11–18.

Salameh, M.K., and Jaber, M.Y., (2000) Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 64,59-64.

Sphicas, G.P. (2006) EOQ and EPQ with linear and fixed backorder costs: Two cases identified and models analyzed without calculus. *International Journal of Production Economics*, 100,59–64.

Toews, C., Pentico, D.W., and Drake, M.J. (2011) The deterministic EOQ and EPQ with partial backordering at a rate that is linearly dependent on the time to delivery. *International Journal of Production Economics*, 131,643–649.

Wee, H.M., Yu, F., and Chen, M.C. (2007) Optimal Inventory Model for Items with Imperfect quality and shortage backordering. *Omega*, 35,7-11.