Relationship between Fluctuation Stock and Safety Stock

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Abstract. Inventory is classified into cycle stock and safety stock in periodic review systems. Cycle stock is defined as inventory that absorbs differences between supply and demand frequencies. Safety stock is defined as inventory that absorbs various differences between supply and demand and is a mixture of inventories which have various purposes. We previously defined Fluctuation stock as inventory that absorbs the difference in terms of time and quantity between supply and demand on the basis of fluctuations in demand, and the method for calculating is established. Fluctuation stock can be calculated using four factors. Fluctuating demand is one of the factors and should be expected. In traditional inventory systems, inventory shortage is equal to the excess of actual demand over expected demand. Safety stock is needed to avoid such shortages. In the inventory system, even if actual demand exceeds expected demand, inventory shortages still may not occur. This means that less safety stock is needed. We refer to the upper limit at which an inventory shortage does not occur as the 'acceptable amount'. In this paper, we have derived a method for calculating the acceptable amount. This will serve as a stepping stone to establishing a method for calculating uncertain inventory.

Keywords: acceptable amount, fluctuation stock, safety stock

1. INTRODUCTION

We previously proposed a new approach to establishing a method for calculating inventory (Yamazaki, 2016). We defined an inventory that has a certain holding purpose and used the proposed approach to establish a method for calculating that inventory. That inventory is referred to as 'fluctuation stock'. Here we clarify the relationship between fluctuation stock and safety stock and demonstrate the effectiveness of the proposed approach.

2. LITERATURE REVIEW

Previously inventory is classified into cycle stock and safety stock in periodic review systems. Cycle stock is defined as inventory that absorbs differences between supply and demand frequencies. Safety stock is defined as inventory that absorbs various differences between supply and demand, and is a mixture of inventories which have various purposes.

In this inventory system, previous research have discussed the inventory policies. There are various streams of research that are related to inventory policies for periodic review systems. Here we outline four of them.

The first stream of research focused on a disappointed model, letting the disappointed demand for a product flow to other products or other companies in (Lippman and McCardle, 1997; Parlar, 1988; Avsar and Gursoy, 2002; Netessine and Rudi, 2003; Rudi, and Wang, 2006; Ahn and Olsen, 2007; Olsen and Parker, 2008).

The second stream of research focused on a replacement model in which there are two demand sources: demand for new products and demand for failed product replacement in (Cohen, Nahmias, and Pierskalla, 1980; Baker and Urban, 1988; Kelle and Silver, 1989; Yuan and Cheung, 1998; Khmelnitsky and Gerchak, 2002; Feinberg and Lewis, 2005; Decroix et al., 2005; Decroix, 2006; Khawam, Hausman, and Cheng, 2007; Huang, Kulkarni, and Swaminathan, 2007; Huang, Kulkarni, and Swaminathan, 2008; Zhou, Tao, and Chao, 2011).

The third stream of research focused on emergency models in which the supplier has two replenishment modes: regular lead time with lower cost and shorter lead time with higher cost in (Neuts, 1964; Fukuda, 1964; Rosenshine and Obee, 1976; Whittemore and Saunders, 1977; Blumenfeld, Hall, and Jordan, 1985; Chiang and Gutierrez, 1996; Chiang and Gutierrez, 1998; Alfredsson and Verrijdt, 1999; Tagaras and Vlachos, 2001; Teunter and Vlachos, 2001; Decroix, 2006; Chartniyom et al., 2007).

The fourth stream of research led to the development of a perishable inventory model in which a product deteriorates with age in (Nahmis and Pierskalla, 1973; Fries, 1975; Nahmis, 1975; Weiss, 1980; Kalpakam and Arivarignan, 1988; Liu, 1990; Moorthy, Narasimhulu, and Basha, 1992; Lian and Liu, 1999; Liu and Lian, 1999; Tekin, Gurler, and Berk, 2001; Nahmias, Perry, and Stadje, 2004; Avinadav and Arponen, 2009; Avinadav, Herbon, and Spiegel, 2013).

Other streams have focused on a substitutability model, a random yield model, a duopoly model, etc. Moreover, an optimal policy has been proposed for whether the set-up cost is included or not, the production system is multi stage or single stage, the production capacity is limited or infinite, the order sizes are discrete or continuous, the planning horizon is given or infinite and a disappointed order is back ordered or is met in next period. However, they have common development, making the models complex and realistic.

3. PROPOSED APPROACH

We describe the process of establishing a method for calculating fluctuation stock by first explaining the approach we previously proposed (Yamazaki, 2016). A method for calculating an inventory quantity without deficiency or excess can be established by using this approach. Specifically, a situation without deficiency or excess is when the minimum on-hand inventory during a certain period is zero. To the best of our knowledge, there have been no reports presenting a method for calculating inventory without deficiency or excess. Most reports have presented methods for calculating an inventory that minimizes cost.

Here we clarify the difference between the two approaches. Since actual situations are complex and vary greatly, it is difficult to derive a general method for calculating an inventory that achieves cost minimization. Thus, previous work generally derived a method by using an inventory model consisting of various factors derived from an actual situation.

The objective of modelization is the simplification of a complex situation. The important thing is that the factors, which are the components of an inventory model, are arbitrarily derived. Thus, there is a wide variety of inventory models. While a large number of them have been reported, they have not absolutely approached the actual situation nonetheless. This is partly because the objective of previous studies has been to develop a method for calculating inventory by using an inventory model. Indeed, the focus of those studies was whether the method achieved cost minimization under the inventory model. The relationships among the factors and their adequacy in the inventory model were not a concern. A sufficient discussion of an inventory model has not been held until now. This is one of the reasons that those methods are less likely to be applied to an actual manufacturing situation.

Our approach to establishing a method for calculating inventory without deficiency or excess does not treat an inventory model. Instead, it treats an inventory system. We define an inventory system as the thing formed by extracting a piece from an actual situation (Yamazaki 2016). That is, an aggregate of inventory systems indicates an actual situation. Previous research has treated an inventory model as something that is made. However, an inventory system exists in actual situations, so an inventory system is not a thing that is made but is a thing that is clarified. It is clarified by identifying the factors composing the system and the relationships among them. Once an inventory system has been clarified, a method for calculating inventory without deficiency or excess can be established. The objective of previous research was to derive the optimal policy by using an inventory model, but establishing a method for calculating inventory requires clarifying the inventory system. One inventory system has been clarified using this approach, and a method for calculating inventory with one purpose has been established (Yamazaki, 2016). Methods for calculating inventory can be established one by one as inventory systems are clarified one by one. Eventually, actual inventory problems will be solved.

There is another difference between the previous and proposed approaches. Moreover, a category of inventory has not been discussed yet. Most research efforts have proposed an optimal policy for periodic review systems.

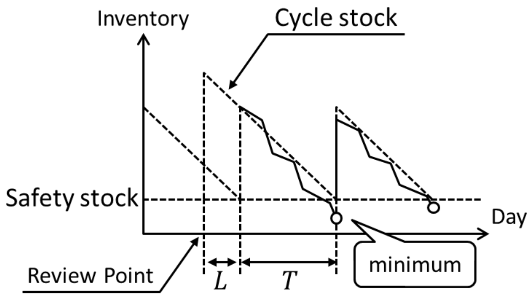

Figure 1: Inventory classified into cycle stock and safety stock in periodic review systems.

Figure 1 is used to explain periodic review systems. Inventory is classified into cycle stock and safety stock in this figure. Cycle stock is defined as inventory that absorbs differences between supply and demand frequencies. Safety stock is defined as inventory that absorbs various differences between supply and demand. A method for calculating cycle stock without deficiency or excess has been established, but a method for calculating safety stock has not been established. The method for calculating fluctuation stock without deficiency or excess was established by using the proposed approach. Fluctuation stock had been included in safety stock.

Many sorts of inventory with various holding purposes are probably still included in safety stock. This is one reason a method for calculating safety stock without deficiency or excess has not been established. The objective of previous research was to develop a method for calculating safety stock that minimizes cost. That is, previous research has considered messy safety stock. One inventory that had been included in safety stock was defined, and then a method for calculating this inventory was established. The two approaches differ in this regard as well.

4. FLUCTUATION STOCK

Fluctuation stock has been comprised in safety stock as described above. Fluctuation stock was defined as inventory that absorbs the difference in terms of time and quantity between supply and demand. Specifically, the difference between supply and demand in terms of time is represented by the difference between the lead time from when an order is received to its due date ('order lead time') and the lead time from production to inventory ('replenishment lead time'). The difference in terms of quantity between supply and demand is represented by the difference between demand per day and the limited production capacity.

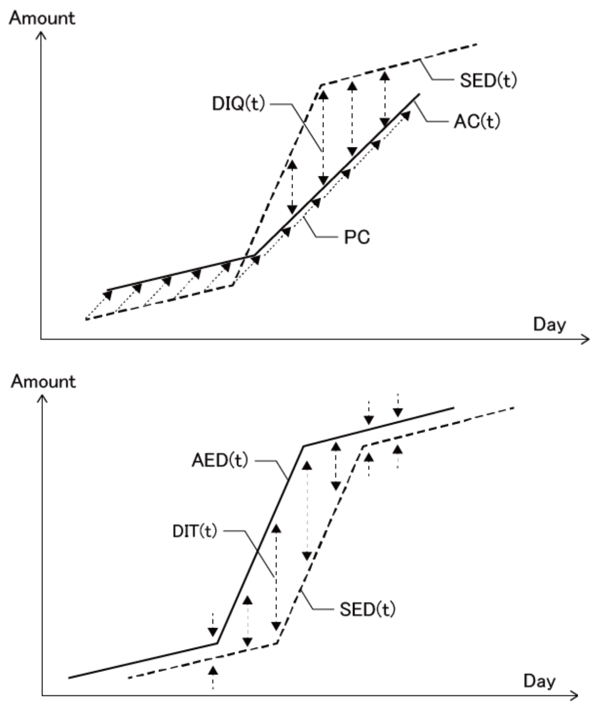

Figure 2: Method for calculating difference between demand and supply in terms of time and quantity (Yamazaki, 2016).

4.1 Calculation Method

The calculation method described in Yamazaki (2016) is summarized here.

Let $I(t)$ be the on-hand inventory available at the beginning of the tth day, and let $ED(t)$ be the expected demand needed for delivery to a customer on the tth day. We assume that demand is expected each month. Thus, fluctuation stock is calculated each month, meaning that the periodic review period is one month. Let $ED(t)$ be the actual demand received at the beginning of the tth day. It is directly satisfied from on-hand inventory. Replenishment order $O(t)$ to reach the target level is released for processing each day.

Let DL be the difference between replenishment lead time and order lead time. It is calculated by subtracting order lead time from replenishment lead time. DL is negative if the replenishment lead time is less than the order lead time. Let $AED(t)$ be the accumulation of $ED(t)$, and let $SED(t)$ be equal to $AED(t-DL)$. The difference in terms of time (see Figure 2) is calculated using

$$DIT(t) = \max\{AED(t) - SED(t), 0\}. \quad (1)$$

Let PC be the daily production capacity, and let $AC(t)$ be the accumulation of PC. The latter is calculated by adding PC to the smaller of $SED(t-1)$ and $AC(t-1)$. The difference in terms of quantity (see Figure 2) is calculated using

$$DIQ(t) = \max\{SED(t) - AC(t), 0\}. \quad (2)$$

If DL is negative, $DIT(t)$ is always zero, but $DIQ(t)$ is not always zero. That is, the proposed calculation method shows that there is a need to hold inventory if the production capacity is low and the demand fluctuation is large, even if the replenishment lead time is less than the order lead time.

Fluctuation stock is calculated using

$$FS = \max_{t=1,2,\cdots,T}\{DIT(t) + DIQ(t)\}, \quad (3)$$

where there are T days in the periodic review period.

Let $AO(t)$ be the accumulation of $O(t)$, and let TI(t) be the tentative inventory. The latter is calculated by sub tracting the accumulation of actual demand, $AAD(t)$, fro m the accumulation of replenishment orders, $AO(t-1)$. The latter is the smaller of subtracting TI(t) from FS an d subtracting it from PC.

$$O(t) = \min[FS - \{AO(t-1) - AAD(t)\}, PC]. \quad (4)$$

4.2 A Difference from Previous Inventory System

In traditional inventory system, which consists of cycle stock and safety stock, if actual demand is larger than expected demand, inventory shortage occurs. The excess of actual demand over expected demand is equal to the shortage. Safety stock is needed to avoid the shortage. In the inventory system proposed in Yamazaki (2016), the excess of actual demand over expected demand is not equal to inventory shortage. Even if actual demand is larger than expected demand, inventory shortage may not occur. This means that less safety stock is needed to avoid inventory shortage than in traditional inventory system.

The following explanation comes from Yamazaki (2016). It has been slightly altered to facilitate understanding.

This is the numerical example presented here, we assume (i) there are 20 working days in a month, (ii) expected demand is 20 during the first 10 days and 100 during the last 10 days, (iii) production capacity is 80 per day and (iv) replenishment lead time is three days longer than order lead time.

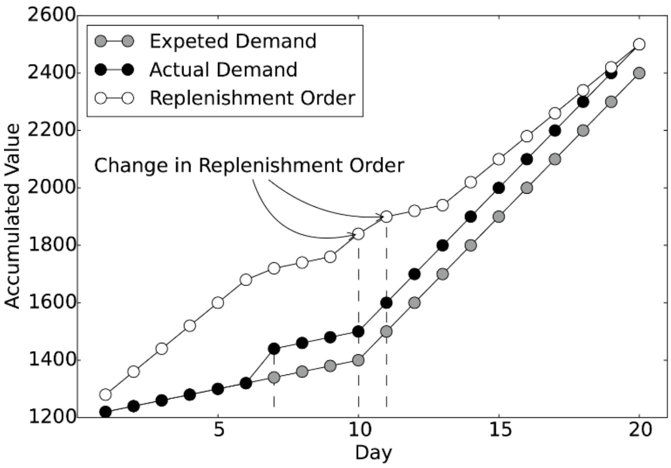

Figure 3 : Results of actual demand exceeding expected demand on seventh day (Yamazaki, 2016).

Fluctuation stock is calculated using expected demand as a variable number, but daily replenishment order is calculated using actual demand. Here, actual demand is not equal to expected demand. Expected demand is 20 for the seventh day, and actual demand is 120 (see Figure 3). In the case here, the minimum on-hand inventory is zero on the twentieth day. That is, an inventory shortage does not occur although actual demand exceeds expected demand. In traditional inventory systems, inventory shortage is equal to the excess of actual demand over expected demand. Safety stock is needed to avoid such shortages, and previous research has proposed an optimal policy for safety stock. In the inventory system proposed here, shortages do not occur even if actual demand exceeds expected demand. This means that less safety stock is needed. The optimal policy for safety stock should thus be set using the proposed system.

We refer to the upper limit at which an inventory shortage does not occur as the ‘acceptable amount’. The proposed inventory system differs from traditional ones in terms of the existence of an acceptable amount.

5. ACCEPTABLE AMOUNT

5.1 Calculation Method

The existence of an acceptable amount is one of the features of an inventory system including fluctuation stock. Here we clarify the method for calculating an acceptable amount.

Let $FI(t,k)$ be the on-hand inventory available at the beginning of the $(t+k)$th day in case the replenishment order is equal to PC since the $(t+DL)$th day. If DL is positive, $FI(t,k)$ is calculated using

$$FI(t,k) = AO(t+k-1) - AED(t+k-1), \qquad (5)$$

where $1 \le k \le DL$, and

$$FI(t,k) = AO(t+DL-1) + PC * k - AED(t+DL+k-1), \qquad (6)$$

where $DL < k \le T$. If DL is negative. $FI(t,k)$ is calculated using

$$FI(t,k) = AO(t+DL-1) + PC * (k-DL) - AED(t+k-1). \qquad (7)$$

Let $AA(t)$ be the acceptable amount.

$$AA(t) = \min_k\{FI(t,k)\} \qquad (8)$$

5.2 Numerical Example

Here, an acceptable amount is calculated for the cond itions described in Section 4.2. Figure 4 shows the expe cted demand and acceptable amount.

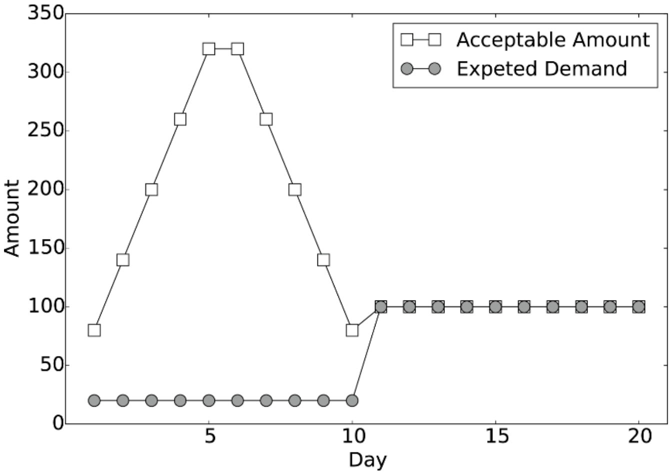

Figure 4: Relationship between expected demand and acceptable amount

The acceptable amount greatly exceeds the expected demand during the first 10 days, when expected demand is 20. In contrast, the acceptable amount is equal to exp ected demand during the last 10 days, when expected de mand is 100. Actual demand rarely exceeds the acceptab le amount since the acceptable amount is relatively large during the first ten days. Thus, safety stock is not need ed even if actual demand is larger than the expected de mand during the first ten days. In contrast, the actual de mand may sometimes exceed the acceptable amount, whi ch is equal to the expected demand during the last ten days. Thus, safety stock is needed to avoid inventory sh ortage, and it is equal to the excess of actual demand o ver expected demand during the last ten days. Safety sto ck is not needed if the actual demand is not likely to e xceed the expected demand during the last ten days whe ther or not actual demand exceeds expected demand duri ng the first ten days.

6. CONCLUSION

After most types of inventory systems have been clarified through the proposed approach, an inventory system that includes inventory for absorbing uncertainty will still need to be clarified. We refer to this inventory as ‘uncertain inventory’. An alternative approach is thus needed to establish a method for calculating uncertain inventory. The acceptable amount is a key component of such an approach.

In this paper, we have derived a method for calculating the acceptable amount. This method can be used to clarify the relationship between the expected demand and the acceptable amount. This will serve as a stepping stone to establishing a method for calculating uncertain inventory.

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