Dynamic Optimization of Shared Capacity under Demand Uncertainties and Price Differentiation

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Abstract. This research studies capacity planning problems with price differentiation and demand uncertainties. In response to the concept of sharing economy, capacity sharing is now prevailing in service and manufacturing sectors. While flexible capacity is shared by different products or customers, different pricing strategies are oftentimes applied to different customer or product groups. As a result, price differentiation of shared capacity is prevailing. Although price differentiation is common, the average selling price (ASP) of products is still widely used in capacity planning processes to reduce the complexity of capacity planning processes. Under the ASP model, distinct price information is omitted and capacity planning fails to reflect all changes in market conditions. Therefore, it is crucial to develop a multiple-price model that explicitly considers price differentiation and demand uncertainties.

In this research, demand uncertainties are considered using Markov decision processes over a finite planning horizon. The objective is to maximize expected profit and to improve robustness of capacity strategies. According to our numerical examples, the resulting profit from multiple-price model is higher than that of the ASP model. For systems with high capacity expansion costs and higher product price differentiation, the proposed multiple-price model can improve overall profit by up to 500%.

Keyword: Sharing Economy, Capacity Planning, Demand Uncertainties, Price Differentiation

1. INTRODUCTION

This research discusses the capacity planning under product price discrimination and stochastic demand growth. It is motivated by sharing economy. The concept that companies use the identical capacity to manufacture different products is like product price discrimination.

Consumers' purchasing behaviors are highly affected by the economic conditions. However, the economic conditions fluctuate over time. Although capacity is adjusted to meet future demand, capacity excess or capacity shortage would result in extra cost and shrink the profit. On the other hand, companies use the product price discrimination to seek maximized profit. But the prices of products are usually represented as average selling price (ASP) in companies' capacity planning processes. Although ASP simplifies the complexity of the problem, the information of each product about profit is omitted, especially in the case of insufficient capacity. When the capacity is insufficient, the use of ASP in capacity planning cannot accurately identify approximate revenue since the ASP model dilutes the information about the composition of products' prices and production quantities.

The objective of this study is to maximize the expected profit and the robustness of capacity plan under stochastic demand. The decision variable is the optimal quantity of capacity expansion in each period given some scenario and accumulated capacity level. The scenario transition in each period follows the Markov process. The dynamic capacity planning model connects demand growth and capacity planning over time. The use of multi-price helps us obtain much more expected profit and achieve higher utilization of capacity. Instead of using ASP, the capacity planning model adopting multiple-price as the price factor is showed to have better profit performance and higher capacity utilization.

2. RELATED LITERATURE

The demand of products usually varies with times. For the deterministic demand growth case, demand can be predicted in advance to simplify the problem. Luss (1982) defined three demand growth types: linear demand growth, exponential demand growth and exponential demand with saturation rate. The demand forms a cyclic pattern, so the forecast demand data can be used in the capacity planning models. Kogan and Herbon (2008) used a decomposition approach to solve a single product capacity planning model with periodic demand variation. Sankul et al. (2016) develops a capacity planning and facility selecting model to improve overall capacity planning of organization

In the literature about capacity planning, mathematical programming approaches like stochastic programming or linear programming are widely used to help find the optimal solutions. Bienstock and Shapiro (1988) use stochastic programming to solve an optimization problem of resource acquisition. Derek and Fernando (2016) shows the impact of trading on market concentration, prices, and consumer surplus.

Capacity planning is usually a long-term issue. Dynamic programming is suitable to solve problems of this type. Rather than expanding capacity only, an expanding work by Millen (1974), Erlenkotter (1977) proposed a set of policies with three decisions: capacity expansion, inventory and import, to a strategic capacity planning problem. Botterud and Korpås (2007) considered a power generation investment case and formulated the optimal capacity investment model by stochastic dynamic programming. The uncertainty in demand was considered and real option was also used in this research.

In our research, we present a model in which the demand is affected by time and price simultaneously. The use of Markov properties would be more realistic to characterize the real demand scenario evolution. The ability of a firm to apply price discrimination gets little attention. This research mainly discusses the capacity planning under stochastic demand growth and multi-price products, aiming to reach the maximized total expected profit in a finite planning horizon.

3. PROBLEM DESCRIPTION

Because of the existence of flexible capacity, different products commonly share the same capacity and are sold at different prices to maximize revenue. Price differentiation as well as capacity flexibility enables manufacturers to yield much more profit by offering various products. Since the capacity is limited, the arrangement of capacity for different types of product is very crucial.

In reality, companies usually use the ASP to reduce complexity of the original multiple-price problem. However, the lack of consideration about the future capacity level gives rise to some problems like the deviation between the estimated ASP and the real ASP. In addition, the simplified ASP model neglects the relationship between multiple-price products and their corresponding demands.

This study assumes that the demand scenario transition from period to period follows Markov process. Thus, the dynamic programming is hired to find the optimal capacity expansion quantity in each period to meet uncertain demand. Machine transference, outsourcing and inventory are not considered in our model, and demand is fully fulfilled by capacity.

4. DYNAMIC CAPACITY PLANNING MODEL

The decision variable of the model is the capacity expansion quantity. The objective is to maximize profit. The optimal capacity expansion quantity is obtained by the dynamic programming technique. Basic elements for the dynamic programming model are defined in the following paragraphs.

The state($s_t(c_t, d_{j,t}(\cdot))$) involves the current capacity level(c_t) and the demand($d_{j,t}(\cdot)$) in different economic environment j($j \in \{\mathrm{H, M, L}\}$, H:High, M:Medium, L:Low). Different capacity expansion actions(a_t) are made under different states. The cost of capacity expansion(EC) should be paid in the current period(t), while the purchased capacity appears in the next period.

Every decision has an effect on the state in the next period. That is to say, the state of the next period $s_{t+1}(c_{t+1}, d_{j',t+1}(\cdot))$ is affected by the state of this period $s_t(c_t, d_{j,t}(\cdot))$ and the action a_t. Thus, the transition probability is defined as below:

$$P(s_{t+1}|s_t, a_t) = P\left(c_{t+1}, d_{j,t+1}(\cdot)\middle|c_t, d_{j,t}(\cdot), a_t\right)$$
$$= P(c_{t+1}|c_t, a_t) \times P\left(d_{j,t+1}(\cdot)\middle|d_{j,t}(\cdot)\right)$$

Since the demand shift is independent of the capacity level, the transitional probability can be decomposed into two parts, transitional probability about capacity change and about demand shift. The left part talks about capacity. If the capacity level in the next period is equal to the current capacity plus the capacity expansion quantity, $c_{t+1} = c_t + a_t$, then

$P(c_{t+1}|c_t, a_t) = 1$; otherwise, $P(c_{t+1}|c_t, a_t) = 0$. The right part is about demand shift. We use the one-step transition probability matrix like below to describe the relationship of consecutive market states.

$$\begin{array}{c} \\ H \\ M \\ L \end{array} \begin{array}{c} \quad H \qquad\qquad\qquad M \qquad\qquad\qquad L \\ \begin{bmatrix} P(d_{H,t+1}|d_{H,t}) & P(d_{M,t+1}|d_{H,t}) & P(d_{L,t+1}|d_{H,t}) \\ P(d_{H,t+1}|d_{M,t}) & P(d_{M,t+1}|d_{M,t}) & P(d_{L,t+1}|d_{M,t}) \\ P(d_{H,t+1}|d_{L,t}) & P(d_{M,t+1}|d_{L,t}) & P(d_{L,t+1}|d_{L,t}) \end{bmatrix} \end{array}$$

The vertical axis is the current market state, while the horizontal axis is the market state in the next period. Each element in the matrix indicates the transition probability under some state combination. Note the sum of elements in each column should be one.

The production quantity is the minimum of demand quantity and capacity. The highest price(p_1) product is produced first, $Q_{1,t} = min\left(d_{j,t}(p_1), c_t\right)$, then the second highest price(p_2) product, $Q_{2,t} = min\left(d_{j,t}(p_2) - Q_{1,t}, c_t - Q_{1,t}\right)$. Note that the production quantity should be nonnegative.

The reward(r_t) of each period depends on the state and the expansion decision in each period. In the multiple-price model, the reward function in the capacity planning process is $r_t(s_t, a_t) = \sum_i\left(p_{i,t} \times Q_{i,t}\right) - EC \times a_t$. The revenue from selling products minus the capacity procurement cost constitutes the reward function at that period.

We also build a comparison model that uses ASP in the capacity planning process. In contrast, the reward function for the ASP model is $r_{t,ASP}(s_t, a_t) = ASP \times \sum_i Q_{i,t} - EC \times a_t$.

The primary difference between the ASP model and the multiple-price model is how they estimate reward. In the multiple-price model, we sum the products of the price and the corresponding demand quantity for each item to obtain the current selling revenue. In contrast, the ASP model uses a simpler approach. The product of the estimated ASP and total demand quantity is regarded as the current selling revenue.

Due to the uncertain economic climate fluctuation, it's almost impossible to predict future demands precisely. Based on the assumption, they estimate the expected demand quantity of each product. According to the demand forecasts and price information, the ASP can be acquired.

In the future period *n* and certain economic environment, product *i*'s approximate demand $Q_{i,n}$ is $d_{j=M,n}\left(p_{i,n}\right)$. The revenue contributed by each product is $Q_{i,n} \times p_{i,n}$. Thus, the ASP in period *n* is

$$ASP_n = \frac{\sum_i p_{i,n} \times Q_{i,n}}{\sum_i Q_{i,n}}$$

Using the similar method, the ASP for different periods can be calculated. Hence, the original multiple-price problem can be reduced to an ASP capacity planning form. Nevertheless, due to the moderate demand forecast assumed in the ASP model, the actual demand uncertainty characteristic is ignored. The gap between the ASP model and the practical problem might lead to the overinvestment problem mentioned in the previous section.

We use $v_t^*(s_t)$ to stand for the maximized total expected profit for state s_t from time *t* to the end of the planning horizon. The optimality equation maximizes the sum of current reward and expected future accumulated reward. The optimality equation is defined as

$$v_t^*(s_t) = \max_{a_t}\{r_t(s_t, a_t) + \textstyle\sum_i P(s_{t+1}|s_t, a_t) \times v_{t+1}^*(s_{t+1})\}.$$

Iterative algorithm is widely used to solve dynamic programming models. Since there is a one period lead time for capacity expansion, capacity expansion in the last period(T) will never be optimal. Thus, $a_T = 0$. Therefore, the optimality equation in the last period is equal to the reward function in the last period. The implementation procedure of the iterative algorithm is shown below:

Step1: When *t*=T, calculate $v_T^*(s_T)$ for each state s_T. $v_T^*(s_T)$ is the revenue obtained by only selling products without purchasing new capacity in the last period. The terminal condition is

$$v_T^*(s_T) = \max_{a_T}\{r_T(s_T, a_T)\} = r_T(s_T, 0)$$

Step2: Given $v_{t+1}^*(s_{t+1})$ for all s_{t+1}, $v_t^*(s_t)$ can be defined iteratively by the optimality equation for each capacity level s_t. In the optimality equation, an optimal capacity expansion quantity to maximizes the sum of current reward (current revenue minus capacity expansion cost) and expected future profit for every s_t.

$$v_t^*(s_t) = \max_{a_t}\left\{r_t(s_t, a_t) + \sum_i P(s_{t+1}|s_t, a_t) \times v_{t+1}^*(s_{t+1})\right\}$$

Step3: Repeat step 2 until $t = 0$.

In the backward induction algorithm, $v_t^*(s_t)$ would be the optimal expected total profit under each initial capacity level within the planning horizon. The optimality equation generates not only the optimal expected profit but also the optimal control policies by:

$$a_t^*(s_t) = \underset{a_t}{argmax}\left\{r_t(s_t, a_t) + \sum_i P(s_{t+1}|s_t, a_t) \times v_{t+1}^*(s_{t+1})\right\}$$

5. NUMERICAL EXAMPLE

The main purpose of this section is to investigate investment behaviors of the two models and verify the performance of the multiple-price model.

To realize the investment actions within the planning horizon, we generate 100 sets of random demand scenarios for simulation. Each set of demand scenarios is a 20-period demand data, which represents the possible demands in the future. During each simulation, the investment action at each period is made according to the decision policy proposed by the dynamic programming algorithm. Actions and revenues for the two models in each period are recorded for comparison.

At the beginning of the planning horizon, the ASP model is more active than multiple-price model. Since ASP_t, for any set of discriminated price, the average price must be greater than the lowest price. Therefore, the marginal value of adding capacity is certainly higher in the ASP model. That is why the ASP model invests more in capacity expansion.

In the later periods, in the face of real demand and capacity shortage, the multiple-price model is able to grab more profit by taking advantage of the price discrimination. The production priority would be given to the product with higher price. In contrast, the ASP model doesn't take into account this factor, so the expected revenue obtained in the ASP model will be smaller than that in the multiple-price model. Besides, if the expected revenue growth gained by the capacity enhancement can't offset the investment cost in the following stages, it's not profitable to engage in capacity expansion. Based on the two reasons, the ASP model would stop increasing the capacity level earlier.

6. CONCLUSION

There exist three significant differences between the multiple-price model and the ASP model. First, at the earlier stages, the ASP model would be more aggressive in capacity expansion because the marginal loss caused by capacity shortage is relatively high. Second, the ASP model would stop its capacity expansion action earlier as a result of the lack of profit advantages from price discrimination. Third, the overall capacity investment decisions become more conservative as the capacity level increases, planning horizon rolls, or economic environment goes down. Since most mainstream capacity planning manners adopt the ASP as the decision parameter, we provide following advices based on our observations. First, to avoid excessive capacity enhancement at the earlier stages, the capacity investment quantities could be slightly less than those provided by the ASP model. Second, the ASP model usually ends capacity expansion too early, so it's encouraged to extend the capacity investments under prosperous market state. In addition, taking advantage of demand curves to estimate the long-term revenue, enterprises can easily establish the optimal pricing policies, thereby maximizing their future profit.

In this research, we only consider the single capacity expansion problem. Future research can extend to the topics about additional decisions like multiple capacity or technology acquisition, making the capacity decision model more realistic. Besides, service level and utilization rate are not employed as evaluation criterion. If the objective is to minimize idle capacity, idling cost should be considered to prevent the occurrence of idle machines. On the other hand, if managers are concerned about service level, the constraints to avoid production shortage or the shortage penalties can be incorporated into the decision model.

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