

# Applying Fuzzy DEMATEL with Yager's t-norms for Green Supply Chain Management

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**Abstract.** This study extends the traditional fuzzy DEMATEL. Fuzzy DEMATEL operations usually adopt traditional  $\alpha$ -cut arithmetic. However, traditional fuzzy arithmetic exhibits the fuzziness accumulation phenomenon, which will make the fuzzy model/method more complex. This research develops the fuzzy DEMATEL with Yager's t-norms operations as a substitute for the fuzzy DEMATEL technology. The novel fuzzy DEMATEL with Yager's t-norms has been examined the green supply chain management practices, and compare with others decision models. This fuzzy DEMATEL with Yager's t-norms provides more credible information/results and analysis.

**Keywords:** Fuzzy DEMATEL, Yager's t-norms, green supply chain management

## 1. INTRODUCTION

Decision Making Trial and Evaluation Laboratory (DEMATEL) (Gabus and Fontela, 1973; Fontela and Gabus, 1976) is a popular method in decision making methods. The aim of DEMATEL solve complicated world problems regarding issues such as race, hunger, environmental protection, energy, etc (Fontela and Gabus, 1976). Nowadays because of the fuzziness of judgment, some researches combine DEMATEL method with fuzzy set theory to solve the complex system problems. In the fuzzy DEMATEL mechanism, fuzzy arithmetic is one main technique for calculating the total relation fuzzy matrix (Horng et al., 2013; Yeh and Huang, 2014). Past researches have usually adopted  $\alpha$ -cut arithmetic. However,  $\alpha$ -cut arithmetic realizes that the fuzziness of the model complicates calculation, and was fuzzier due to the fuzziness accumulation phenomenon of the  $\alpha$ -cut arithmetic (Chang et al., 2006). In this paper, the novel fuzzy DEMATEL with Yager's t-norms operation is developed in order improve on the shortcomings fuzzy DEMATEL. The novel fuzzy DEMATEL with Yager's t-norms operations will provide more credible information/results and analysis across quadrants in order

to evaluate cause and effect relationships. Furthermore, the parameterized Yager's t-norms can obtain different fuzzy intervals with setting parameter.

## 2. FUZZY DEMATEL WITH YAGER'S T-NORMS

In decision-making system, people usually make judgment according to their experience and expertise. This is human centric activity and certainly is processed in uncertain environments. Traditional/crisp DEMATEL may be not effectively suitable for solving group or multi-criteria decision-making in uncertain environments. Therefore, the DEMATEL should be needed to build an extended DEMATEL method by applying fuzzy theory. To deal with human assessments, the preferences of decision makers are employed to fuzzy numbers by adopting fuzzy linguistic scale. Firstly, the parameterized Yager's t-norms operations  $T_p$  can be form as following (Keresztfalvi, 1993):

$$T_p(x, y) = \max\{0, 1 - \sqrt[p]{(1-x)^p + (1-y)^p}\} \quad (1)$$

$$x, y \in [0, 1], p \geq 1$$

Let  $A=(a_1, a_2, a_3)$  and  $B=(b_1, b_2, b_3)$ . The addition is

extended by Yager's t-norms can be defined as:

$$A +_{T_p} B = (a_2 + b_2 - (\sqrt[q]{(a_2 - a_1)^q + (b_2 - b_1)^q}),$$

$$a_2 + b_2, a_2 + b_2 + (\sqrt[q]{(a_3 - a_2)^q + (b_3 - b_2)^q})). \quad (2)$$

where  $q \geq 1$  is such that  $1/p + 1/q = 1$ .

The multiplication is extended by Yager's t-norms can be defined as:

$$A \times_{T_p} B = (a_2 \times b_2 - (\sqrt[q]{b_2 \times (a_2 - a_1)^q + a_2 \times (b_2 - b_1)^q}),$$

$$a_2 \times b_2, a_2 \times b_2 + (\sqrt[q]{b_2 \times (a_3 - a_2)^q + a_2 \times (b_3 - b_2)^q})). \quad (3)$$

In this section, the fuzzy DEMATEL with Yager's t-norms operations will be briefly introduced as following:

**Step 1.** (Average the assessment) Identifying decision goal-gathering the relevant information evaluates the advantages and disadvantages. This is necessary to from expert committers for group knowledge to achieve the goals. It must acquire and average the assessment of experts' preferences using (Lin and Wu, 2004).

**Step 2.** (Initial direct-relation fuzzy matrix) Based on average the assessment, the initial direct-relation fuzzy matrix  $Z$  is produced as following:

$$Z = \begin{bmatrix} 0 & z_{12} & \cdots & z_{1n} \\ z_{21} & 0 & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & 0 \end{bmatrix} \quad (4)$$

In the matrix,  $z_{ij} = (l_{ij}, m_{ij}, r_{ij}) \forall i = 1, \dots, n, j = 1, \dots, n$ . and  $z_{ij} = (0, 0, 0) \forall i = j$ .

**Step 3.** (Normalizing) By normalizing initial direct-relation fuzzy matrix, the normalizing direct-relation fuzzy matrix  $X$  can be obtained as following:

$$X^{T_p} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{bmatrix}^{T_p} \quad (5)$$

where

$$x_{ij} = \frac{z_{ij}}{\max_{1 \leq i \leq n} \sum_{j=1}^n z_{ij}} = \left( \frac{l_{ij}}{\max_{1 \leq i \leq n} \sum_{j=1}^n z_{ij}}, \right.$$

$$\left. \frac{m_{ij}}{\max_{1 \leq i \leq n} \sum_{j=1}^n z_{ij}}, \frac{u_{ij}}{\max_{1 \leq i \leq n} \sum_{j=1}^n z_{ij}} \right) \quad (6)$$

**Step 4.** (Total relation fuzzy matrix) The total relation fuzzy matrix can be defined and computed as following (Lin and Wu, 2004):

$$T^{T_p} = \lim_{k \rightarrow \infty} (X_1 +_{T_p} X_2 +_{T_p} \cdots +_{T_p} X_k)$$

Then

$$T = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{nn} \end{bmatrix} \quad (7)$$

**Step 5.** (Producing a causal diagram) The sum of rows and the sum of columns are respectively denotes as vectors  $D$  and  $R$  within the total relation fuzzy matrix.

$$D = \left[ \sum_{i=1}^n t_{ij} \right]_{1 \times n}^{T_p} \quad (8)$$

$$R = \left[ \sum_{j=1}^n t_{ij} \right]_{1 \times n}^{T_p} \quad (9)$$

Then  $D_i + R_i$  and  $D_i - R_i$  are calculated by Yager's t-norms operations  $\forall q=1$  to  $\infty$ . To finalize the procedure, all calculated  $D_i + R_i$  and  $D_i - R_i$  are defuzified through suitable defuzification method.  $(D_i + R_i)^{def}$  shows how the important the strategic objectives are, and  $(D_i - R_i)^{def}$  shows which strategic objectives is cause and which one is effect. Generally, if  $(D_i - R_i)^{def}$  is positive, the objectives belong to the cause group, otherwise; the objectives belong to the effect group.

### 3. NUMERICAL EXAMPLE

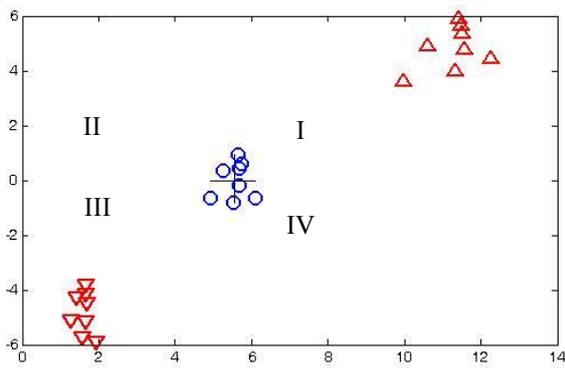
The green supply chain management practices (GSCM) (Lin, 2013) was examined by fuzzy DEMATEL. Lin (2013) shows the strategic objectives of GSCM, which include eight influence factors. By adopting a fuzzy triangular number, a fuzzy DEMATEL exertion will be put in place by expressing different degrees of influence or causality in crisp DEMATEL, with five linguistic terms as {NO, VL, L, H, VH} and their corresponding positive triangular fuzzy numbers. In this study, the fuzzy DEMATEL with Yager's t-norms is examined the same case, and display the results of parameterized fuzzy DEMATEL.

Figure 1 shows the cause and effect diagram with parameterized fuzzy DEMATEL ( $q=1, 10, 100, \text{ and } \infty$ ). In order to analyze the causes and effects, the four quadrants can be divided by an average of  $(D_i + R_i)^{def}$  on the x-axis, and  $(D_i - R_i)^{def}$  on the y-axis. Quadrant I contains the core factors while the factor  $(D_i + R_i)^{def}$  is larger than the average of  $(D_i + R_i)^{def}$ , and  $(D_i - R_i)^{def}$  is positive. The core factors ( $C_4, C_7, C_8$ ) should be considered or improved first in GSCM practices. Quadrant II contains driving factors while the factor  $(D_i + R_i)^{def}$  is smaller than the

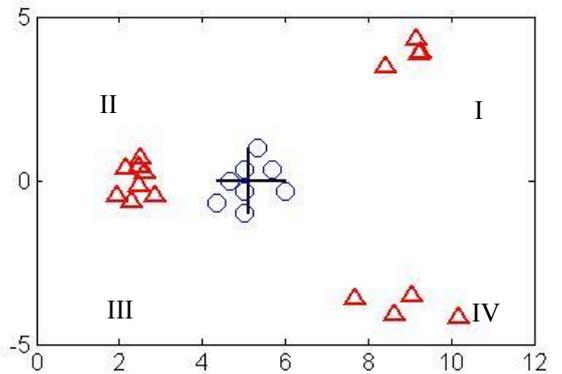
average of  $(D_i + R_i)^{def}$ , and  $(D_i - R_i)^{def}$  is positive. The driving factor ( $C_3$ ) only affects a few other factors. Quadrant III contains independent factors while the factor  $(D_i + R_i)^{def}$  is smaller than the average of  $(D_i + R_i)^{def}$ , and  $(D_i - R_i)^{def}$  is negative. The independent factors ( $C_2, C_6$ ) can be individually handled because they may not affect other factors. Quadrant IV contains affected factors while the factor  $(D_i + R_i)^{def}$  is larger than the average of

$(D_i + R_i)^{def}$ , and  $(D_i - R_i)^{def}$  is negative. The affected factors ( $C_1, C_5$ ) should be indirectly considered or improved. If a manager can effectively control the factors in quadrants I and II, the affected factors also can be well controlled. Furthermore, the fuzzy intervals with increasing  $q$  will be smaller which can be easily observed in Figure 1.

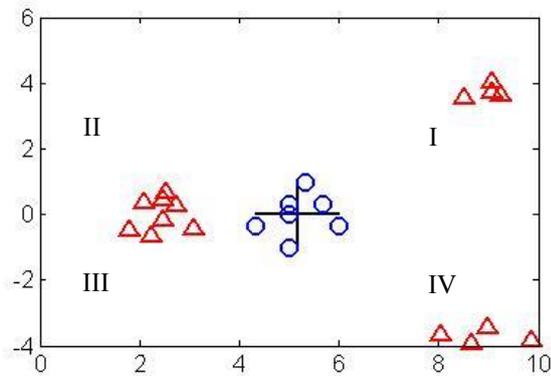
△ Fuzzy right bound   ▽ Fuzzy left bound   ○ Defuzzified value



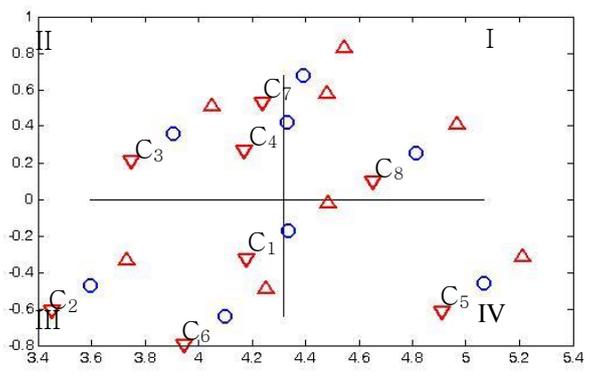
(a) Fuzzy DEMATEL with Yager's t-norms ( $q=1$ )



(b) Fuzzy DEMATEL with Yager's t-norms ( $q=10$ )



(c) Fuzzy DEMATEL with Yager's t-norms ( $q=100$ )



(d) Fuzzy DEMATEL with Yager's t-norms ( $q=\infty$ )

Figure 1: Comparison of cause and effect diagram.

## 5. CONCLUSION

Fuzzy DEMATEL has been proven to be an effective tool in evaluating cause and effect relationships. Thus, this paper proposed a fuzzy DEMATEL with Yager's t-norms operation to improve on fuzzy DEMATEL. The proposed

$T_P$  fuzzy DEMATEL was successfully applied to GSCM, with useful and more credible results obtained. The fuzzy DEMATEL with Yager's t-norms operation may change the decision-making process in that more accurate evaluations are possible.  $T_P$  fuzzy DEMATEL may provide more conservative information. Further research may investigate

a number of remaining issues, which may extend the realizable application of  $T_P$  fuzzy DEMATEL.  $T_P$  fuzzy DEMATEL can also integrate other evaluation methods such as Balance Scorecard (BSC) and analytic hierarchy process (AHP). Moreover, other industries may also consider using  $T_P$  fuzzy DEMATEL as a decision support to investigate cause and effect relations.

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## REFERENCES

- Chang, P.-T., Pai, P.-F., Lin, K.-P., & Wu, M.-S. (2006) Applying fuzzy arithmetic to system dynamics for the customer-producer-employment model. *International Journal of Systems Science*, **37**, 673–698.
- Fontela, E., and Gabus, A. (1976) The DEMATEL Observer. DEMATEL 1976 Report, Switzerland, Geneva, Battelle Geneva Research Center.
- Gabus, A., and Fontela, E. (1973) Perceptions of the World Problematique: Communication Procedure, Communicating with Those Bearing Collective Responsibility, DEMATEL Report No. 1, Battelle Geneva Research Center, Geneva, Switzerland.
- Hong, J.-S., Liu, C.-H., Chou, S.-F., and Tsai, C.-Y. (2013) Creativity as a critical criterion for future restaurant space design: Developing a novel model with DEMATEL application. *International Journal of Hospitality Management*, **33**, 96–105.
- Keresztfalvi, T. (1993) *Operations on fuzzy numbers extended by Yager's family of t-norms*. In H. Bandemer (ed.) *Modelling Uncertain Data, Mathematical Research*, vol. 68, Akademie Verlag, 163–167.
- Lin, R.-J. (2013) Using fuzzy DEMATEL to evaluate the green supply chain management practices. *Journal of Cleaner production*, **40**, 32–39.
- Yeh, T.-M., and Huang, Y.-L. (2014). Factors in determining wind farm location: Integrating GQM, fuzzy DEMATEL, and ANP. *Renewable Energy*, **66**, 159–169.