The Selection Scheme on Replacement Policies for Repairable Two-Component Systems

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Abstract. This paper investigates the selection scheme on replacement policies for repairable two-component systems. Two components can be connected in series or parallel. Due to the inevitable deterioration of the components, each component may fail more frequently as its age or usage increases. Therefore, an appropriate preventive replacement may be suitable for reducing the number of failures and maintains the operation of the component normally. When the age of component reaches a pre-specified time, it is replaced with a replacement cost, a downtime cost, and a setup cost. Any failure before replacement is rectified by minimal repair and it incurs a minimal repair cost and a downtime cost. In particular, when the downtime cost or setup cost is high, it might be worthwhile replacing both components at the same time (called group replacement policy; GRP) instead of replacing them separately (called individual replacement policy; IRP). In this paper, the selection schemes for IRP and GRP of series and parallel systems are proposed in order to minimize the long-run expected cost per unit time. Finally, the IRP and GRP of series and parallel systems are compared through numerical examples.

Keywords: minimal repair, replacement policy, series system, parallel system

1. INTRODUCTION

With the advance of science and technology, manufacturing processes of products become more and more complex. In general, any manufacturing system may involve multiple subsystems (or components) to complete a manufacturing process. To ensure the manufacturing process runs smoothly, a suitable maintenance strategy for subsystems (or components) should be planned and performed.

Over the past few decades, there are many research papers on the minimal repair and replacement policies for a single component. Barlow and Hunter (1960) first introduce the minimal repair concept into the reliability field and further, it is widely adopted. Phelps (1983) considered the replacement problem under minimal repair and showed that a particular form of control limit policy is optimal in the space of all possible policies. In 1983, Nakagawa and Kowada defined a minimal repair in the term of the failure rate and devices some probability quantities and reliability properties. Sheu (1991) proposed a generalization of the block replacement policy and analyzed for a multi-unit system which has the specific multivariate distribution. Chien and Sheu (2006) developed an extended optimal age-replacement policy. They considered an operating system suffers a shock and fails at a certain age, it is either replaced by a new system or it undergoes minimal repair. Li and Peng (2014) investigated a type of dynamic behavior in the multi-state series-parallel system and used the Markov reward model to calculate the system availability and the operation cost.

For replacement policies, Osaki and Nakagawa (1975) established the age-replacement cost model and further derived an optimal replacement policy such that the total maintenance cost is minimized. Berg and Epstein (1978) gave a rule for choosing the least costly of the above three policies under conditions specified. The implementation of this rule is illustrated for two special cases, where the distribution of item life-time is uniform, or 2-stage Erlang. Beichelt (1981) introduced a generalized block replacement policy and gave the long-run cost rate. Furthermore, integral equations of the renewal type for the basic reliability expressions are derived and a numerical example is presented. Sheu *et al.* (2010) considered a periodic replacement model with minimal repair based on cumulative repair-cost limit and then the minimum-cost replacement policy is studied by showing its existence, uniqueness, and structural properties.

In repair-replacement policy aspect, Elwany *et al.* (2011) considers a replacement problem for components whose degradation process can be monitored using dedicated sensors. Hsu *et al.* (2015) proposed the impact of the downtime cost on the replacement policies for a two-component series system. They considered the case when minimal repairs are carried out at failures and the component should be replaced at a certain age. However, most of the work did not consider the case when the components are cost-dependent in performing the maintenance actions.

In this paper, the individual and group replacement cost models of a two-component series or parallel systems are constructed and the optimal individual replacement policy (IRP) and group replacement policy (GRP) are derived. In addition, the selection scheme on IRP and GRP for series and parallel systems is proposed and the comparisons of IRP and GRP for series and parallel systems are illustrated through numerical examples.

This paper is organized as follows. The mathematical models are constructed in Section 2. In Section 3, the optimal IRP and GRP are derived and the select scheme on IRP and GRP for series and parallel systems is proposed. The comparisons of IRP and GRP for series and parallel systems are illustrated and the impacts of downtime cost and setup cost are analyzed through numerical examples in Section 4. Finally, some conclusions are drawn in the last section.

2. MATHEMATICAL MODELS

Consider a system consists of two components (P_i, i=1, 2) and the lifetime distributions of two components follow the Weibull distribution $f_i(t)=\alpha_i\beta_i(\alpha_i t)^{\beta_i-1}e^{-(\alpha_i t)^{\beta_i}}, t\geq 0$. By definition of a failure rate function, the failure rate function and the cumulative failure rate function of a Weibull distribution are $h_i(t)=\alpha_i\beta_i(\alpha_i t)^{\beta_i-1}$ and $H_i(t)=(\alpha_i t)^{\beta_i}$, respectively. The connection method of two components can be in series or parallel as shown in Figures 1 and 2.

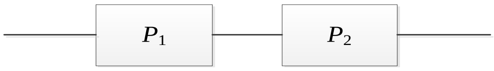

Figure 1. Two-component series system.

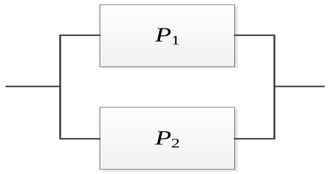

Figure 2. Two-component parallel system.

For a two-component series system, any component fails will lead to the system breakdown. However, for a two-component parallel system, either one component fails will not cause the system breakdown unless both fails. Due to the inevitable deterioration of the component, the component may fail more frequently as its age or usage increases. Therefore, an appropriate preventive replacement may be suitable to reduce the number of failures and maintains the operation of the component normally. To reduce the number of failures, a preventive replacement action is usually adopted to replace each component at a certain age. Under this replacement policy, two replacement models are considered: (1) individual replacement which each component is replaced separately; and (2) group replacement which two components are replaced simultaneously. For series and parallel systems, individual and group replacement cost models are constructed and compared as follows.

2.1 IRP - Series System

For the component of series system, suppose that the age of the component P_i reaches a certain time T_i, a replacement action is performed and incurs a replacement cost C_{ri}, i=1, 2, a system downtime cost C_{dr1}+C_{dr2} and a setup cost C_s. Therefore, the expected total replacement cost per unit time is $(C_{ri}+C_{dr1}+C_{dr2}+C_s)/T_i$, i=1, 2. When component P_i fails before replacement time T_i, the failed component is rectified by a minimal repair. Since the failed component is rectified by minimal repair, the failure process of the component P_i is a nonhomogeneous Poisson process. Each minimal repair of component P_i will incur a fixed repair cost C_{mi}, i=1, 2 and a system downtime cost C_{dm1}+C_{dm2}. Before replacing the component P_i, the expected total minimal repair cost per unit time becomes $(C_{mi}+C_{dm1}+C_{dm2})H_i(T_i)/T_i$=

$(C_{mi}+C_{dm1}+C_{dm2})(\alpha_i T_i)^{\beta_i}/T_i$, for i=1, 2. Therefore, the expected total cost per unit time is

$$E[TC_{SI}(T_1,T_2)] = \sum_{i=1}^{2} \frac{(C_{mi}+C_{dm1}+C_{dm2})(\alpha_i T_i)^{\beta_i}+C_{ri}+C_{dr1}+C_{dr2}+C_s}{T_i} \quad (1)$$

2.2 GRP- Series System

Suppose that the components P_1 and P_2 of series system are replaced at a common time T_g. Similar to the IRP, when components P_1 and P_2 are replaced at time T_g, the expected total replacement cost per unit time becomes $(C_{r1}+C_{r2}+C_{dr1}+C_{dr2}+C_s)/T_g$. Within group replacement time T_g, the expected total minimal repair cost per unit time is $(C_{mi}+C_{dm1}+C_{dm2})H_i(T_g)/T_g=(C_{mi}+C_{dm1}+C_{dm2})(\alpha_i T_g)^{\beta_i}/T_g$, i=1, 2. Therefore, within group replacement time T_g, the expected total cost per unit time of system is

$$E[TC_{SG}(T_g)] = \frac{\sum_{i=1}^{2}[(C_{mi}+C_{dm1}+C_{dm2})(\alpha_i T_g)^{\beta_i}+C_{ri}]+C_{dr1}+C_{dr2}+C_s}{T_g} \quad (2)$$

2.3 IRP- Parallel System

For parallel system, any component fails will not cause the system breakdown. The repair and replacement policy of components is the same as section 2.1. When the component P_i is replaced at time T_i, the expected total replacement cost per unit time is $(C_{ri}+C_{dri}+C_s)/T_i$, i=1, 2. Within the replacement time T_i, the expected total repair cost per unit time is $(C_{mi}+C_{dmi})H_i(T_i)/T_i=(C_{mi}+C_{dmi})(\alpha_i T_i)^{\beta_i}/T_i$, for i=1, 2. Therefore, the expected total cost per unit time of system is

$$E[TC_{PI}(T_1,T_2)] = \sum_{i=1}^{2} \frac{(C_{mi}+C_{dmi})(\alpha_i T_i)^{\beta_i}+C_{ri}+C_{dri}+C_s}{T_i} \quad (3)$$

2.4 GRP- Parallel System

Similar to sections 2.2 and 2.3, under the GRP, two components P_1 and P_2 of parallel system are replaced at a common time T_g. When two components P_1 and P_2 is replaced at time T_g, the expected total replacement cost per unit time is $(C_{r1}+C_{r2}+C_{dr1}+C_{dr2}+C_s)/T_g$. Within group replacement time T_g, the expected total minimal repair cost per unit time is $(C_{mi}+C_{dmi})H_i(T_g)/T_g=(C_{mi}+C_{dmi})(\alpha_i T_g)^{\beta_i}/T_g$, for i=1, 2. Therefore, within group replacement time T_g, the expected total cost per unit time of system is

$$E[TC_{PG}(T_g)] = \frac{\sum_{i=1}^{2}[(C_{mi}+C_{dmi})(\alpha_i T_g)^{\beta_i}+C_{ri}+C_{dri}]+C_s}{T_g} \quad (4)$$

The objective of this paper is to find the optimal individual replacement time (T_1^*, T_2^*) and group replacement time T_g^* for two components such that the expected total cost per unit time in Eqs. (1)-(4) is minimized.

3. OPTIMAL REPLACEMENT POLICY

For series and parallel systems, the optimal individual and group replacement times of two components are derived as follows.

3.1 Optimal IRP- Series System

From Eq. (1), the optimal individual replacement time T_i, i=1, 2 can be obtained by taking the first partial derivatives of Eq. (1) with respect to T_i and setting it equal to 0 as follows.

$$(\beta_i-1)(C_{mi}+C_{dm1}+C_{dm2})(\alpha_i T_i)^{\beta_i}=(C_{ri}+C_{dr1}+C_{dr2}+C_s) \quad (5)$$

From Eq. (5), the optimal individual replacement time can be obtained as follows.

$$T_i^* = \frac{1}{\alpha_i}\left[\frac{C_{ri}+C_{dr1}+C_{dr2}+C_s}{(\beta_i-1)(C_{mi}+C_{dm1}+C_{dm2})}\right]^{\frac{1}{\beta_i}}, i=1,2 \quad (6)$$

3.2 Optimal GRP- Series System

Similar to section 3.1, the optimal replacement times T_g can be obtained by differentiating Eq. (2) with respect to T_g and setting it equal to 0. Then, the result is obtained as follows.

$$\sum_{i=1}^{2}(\beta-1)(C_{mi}+C_{dm1}+C_{dm2})(\alpha_i T_g)^{\beta_i} = \sum_{i=1}^{2}(C_{ri}+C_{dri})+C_s \quad (7)$$

From Eq. (7), there is no close-form solution for solving T_g unless β_1=β_2. When $\beta_1 \neq \beta_2$, we can search a T_g to satisfy the Eq. (7) using any search method. When β_1=β_2=β, the optimal group replacement time T_g^* can be obtained by Eq. (7) as follows.

$$T_g^* = \left[\frac{C_{r1}+C_{r2}+C_{dr1}+C_{dr2}+C_s}{(\beta-1)[\alpha_1^{\beta}(C_{m1}+C_{dm1}+C_{dm2})+\alpha_2^{\beta}(C_{m2}+C_{dm1}+C_{dm2})]}\right]^{\frac{1}{\beta}} \quad (8)$$

3.3 Optimal IRP- Parallel System

In order to obtain the optimal individual replacement times T_i, i=1, 2, we can differentiate Eq. (3) with respect to T_i and set it equal to 0. Then, the result is obtained as follows.

$$(\beta_i - 1)(C_{mi} + C_{dmi})(\alpha_i T_i)^{\beta_i} - (C_{ri} + C_{dri} + C_s) = 0 \qquad (9)$$

From Eq. (9), the optimal individual replacement time can be obtained as follows.

$$T_i^* = \frac{1}{\alpha_i}\left[\frac{C_{ri} + C_{dri} + C_s}{(\beta_i - 1)(C_{mi} + C_{dmi})}\right]^{\frac{1}{\beta_i}} \qquad (10)$$

3.4 Optimal GRP - Parallel System

Similar to section 3.3, we can differentiate Eq. (4) with respect to T_g and setting it equal to 0. Then, the result is obtained as follows.

$$\sum_{i=1}^{2}(\beta_i - 1)(C_{mi} + C_{dmi})(\alpha_i T_g)^{\beta_i} = C_{r1} + C_{r2} + C_{dr1} + C_{dr2} + C_s \qquad (11)$$

Similar to section 3.2, in Eq. (11), the close-form solution does not exist unless β_1=β_2. When $\beta_1 \neq \beta_2$, we can search a T_g satisfying Eq. (11). When β_1=β_2=β, the optimal group replacement time T_g^* can be obtained by Eq. (11) as follows.

$$T_g^* = \left[\frac{C_{r1} + C_{r2} + C_{dr1} + C_{dr2} + C_s}{(\beta - 1)[\alpha_1^{\beta}(C_{m1} + C_{dm1}) + \alpha_2^{\beta}(C_{m2} + C_{dm2})]}\right]^{\frac{1}{\beta}} \qquad (12)$$

3.5 Comparisons of IRP and GRP – Series System

When β_1=β_2=β, we can find a condition to choose IRP or GRP for the components of series system. Let D_1 denote that $E[TC_{SG}(T_g^*)] - E[TC_{SI}(T_1^*, T_2^*)]$ and substituting the optimal individual replacement time (T_1^*, T_2^*) and group replacement time T_g^* into D_1. Then, the result can be obtained as follows.

$$\begin{aligned} D_1 &= (C_{m1} + C_{dm1} + C_{dm2})\alpha_1^{\beta}\beta[A_1 - B_1] \\ &\quad - (C_{m2} + C_{dm1} + C_{dm2})\alpha_2^{\beta}\beta[B_2 - A_1] \end{aligned} \qquad (13)$$

where $$A_1 = \left[\frac{(C_{r1} + C_{r2} + C_{dr1} + C_{dr2} + C_s)}{[\alpha_1^{\beta}(C_{m1} + C_{dm1} + C_{dm2}) + \alpha_2^{\beta}(C_{m2} + C_{dm1} + C_{dm2})]}\right]^{\frac{\beta-1}{\beta}},$$

$$B_1 = \left[\frac{(C_{r1} + C_{dr1} + C_{dr2} + C_s)}{\alpha_1^{\beta}(C_{m1} + C_{dm1} + C_{dm2})}\right]^{\frac{\beta-1}{\beta}},$$

$$B_2 = \left[\frac{(C_{r2} + C_{dr1} + C_{dr2} + C_s)}{\alpha_2^{\beta}(C_{m2} + C_{dm1} + C_{dm2})}\right]^{\frac{\beta-1}{\beta}}.$$

When D_1=0, the following equation can be obtained.

$$\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta}\frac{C_{m2} + C_{dm1} + C_{dm2}}{C_{m1} + C_{dm1} + C_{dm2}} = \frac{A_1 - B_1}{B_2 - A_1} \qquad (14)$$

In this case (D_1=0), performing IRP and GRP will result in the same expected total cost per unit time.

(a) If $\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta}\frac{C_{m2} + C_{dm1} + C_{dm2}}{C_{m1} + C_{dm1} + C_{dm2}} < \frac{A_1 - B_1}{B_2 - A_1}$ (i.e., D_1>0), then the IRP should be adopted.

(b) If $\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta}\frac{C_{m2} + C_{dm1} + C_{dm2}}{C_{m1} + C_{dm1} + C_{dm2}} > \frac{A_1 - B_1}{B_2 - A_1}$ (i.e., D_1<0), then the GRP should be adopted.

From (a) and (b), we can easily select the IRP or GRP for series system.

3.6 Comparisons of IRP and GRP – Parallel System

Similar to section 3.5, Let D_2 denote that $E[TC_{PG}(T_g^*)] - E[TC_{PI}(T_1^*, T_2^*)]$ and substituting the optimal individual replacement time (T_1^*, T_2^*) and group replacement time T_g^* into D_2. Then, the result can be obtained as follows.

$$\begin{aligned} D_2 &= (C_{m1} + C_{dm1})\alpha_1^{\beta}\beta[A_2 - B_3] \\ &\quad - (C_{m2} + C_{dm2})\alpha_1^{\beta}\beta[B_4 - A_2] \end{aligned} \qquad (15)$$

where $$A_2 = \left[\frac{(C_{r1} + C_{r2} + C_{dr1} + C_{dr2} + C_s)}{[\alpha_1^{\beta}(C_{m1} + C_{dm1}) + \alpha_2^{\beta}(C_{m2} + C_{dm2})]}\right]^{\frac{\beta-1}{\beta}},$$

$$B_3 = \left[\frac{(C_{r1} + C_{dr1} + C_s)}{\alpha_1^{\beta}(C_{m1} + C_{dm1})}\right]^{\frac{\beta-1}{\beta}},$$

$$B_4 = \left[\frac{(C_{r2} + C_{dr2} + C_s)}{\alpha_2^{\beta}(C_{m2} + C_{dm2})}\right]^{\frac{\beta-1}{\beta}}.$$

When D_2=0, the following equation can be obtained.

$$\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta}\frac{C_{m2} + C_{dm2}}{C_{m1} + C_{dm1}} = \frac{A_2 - B_3}{B_4 - A_2} \qquad (16)$$

In this case (D_2=0), performing IRP and GRP will result in the same expected total cost per unit time.

(a) If $\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta}\frac{C_{m2} + C_{dm2}}{C_{m1} + C_{dm1}} < \frac{A_2 - B_3}{B_4 - A_2}$ (i.e., D_2>0), then the IRP should be adopted.

(b) If $\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta}\frac{C_{m2} + C_{dm2}}{C_{m1} + C_{dm1}} > \frac{A_2 - B_3}{B_4 - A_2}$ (i.e., D_2<0), then the GRP should be adopted.

From (a) and (b), the IRP or GRP of the components can be easily chosen for parallel system.

4. NUMERICAL ANALYSIS

In this section, the performances of the optimal individual and group replacement policies for series and parallel systems are evaluated and the impacts of minimal repair downtime cost (C_{dmi}, i=1, 2), setup cost (C_s), and replacement downtime cost (C_{dri}, i=1, 2) for series and parallel systems are demonstrated through numerical examples. The following values of parameters are considered for two components in series and parallel systems: α_1=0.15, α_2=0.35, β_1=β_2=2, C_{m1}=200, C_{m2}=100, C_{r1}=600, C_{r2}=300.

4.1 Numerical Examples for Series System

Suppose that setup cost C_s=50 and replacement downtime cost C_{dr1}=C_{dr2}=C_{dr}=1000, Figure 3 shows that the choice of the optimal replacement policy does not change under various minimal repair downtime costs (C_{dm1}=C_{dm2}=C_{dm}). That is, the GRP should be adopted and the lower expected total cost per unit time of the system can be obtained when the replacement downtime cost is relatively high.

Figure 4 shows that when C_{dr}=132, performing IRP and GRP will result in the same expected total cost per unit time under C_s=50 and C_{dm}=1000. When C_{dr}<132, the IRP should be adopted. Otherwise, the GRP should be adopted. Similarly, Figure 5 shows that when setup cost C_s=214, performing IRP and GRP will result in the same expected total cost per unit time under C_{dr}=50 and C_{dm}=1000. When C_s<214, the IRP should be adopted. Otherwise, the GRP should be adopted.

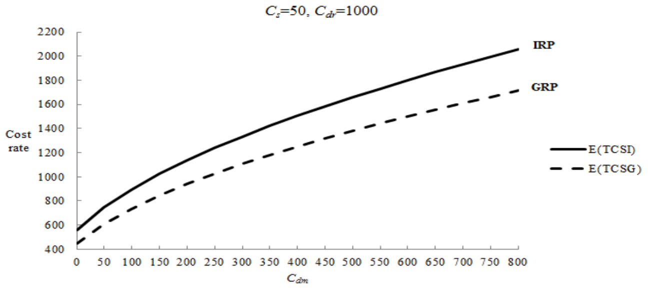

Figure 3: The impact of C_{dm} for optimal replacement policy of series system.

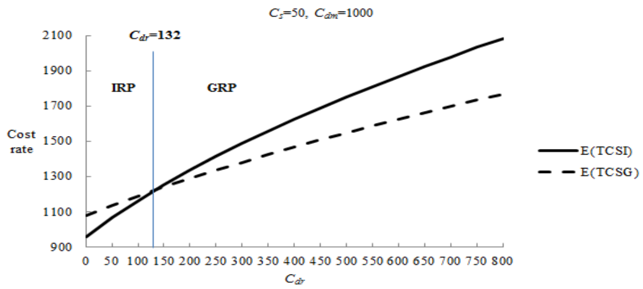

Figure 4: The impact of C_{dr} for optimal replacement policy of series system.

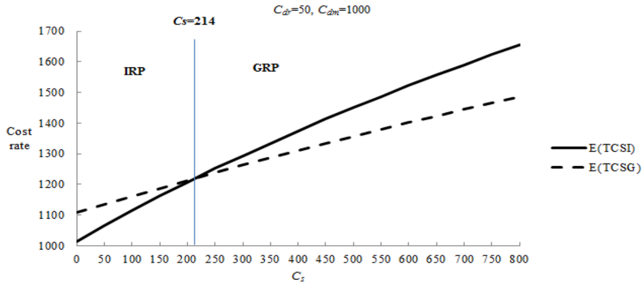

Figure 5: The impact of C_s for optimal replacement policy of series system.

4.2 Numerical Examples for Parallel System

Similar to section 4.1, Figures 6-8 show that the optimal replacement policy and the expected total cost per unit time of system under various minimal repair downtime costs (C_{dm1}=C_{dm2}=C_{dm}), setup cost (C_s) and replacement downtime cost (C_{dr1}=C_{dr2}=C_{dr}). Figures 6 and 7 show that C_{dm} and C_{dr} do not affect the choice of the optimal replacement policy. That is, the IRP should be adopted and the lowest expected total cost per unit time of system can be obtained.

Figure 8 shows that when setup cost C_s=318, performing IRP and GRP will result in the same expected total cost per unit time under C_{dr}=50 and C_{dm}=1000. When C_s<318, the IRP should be adopted. Otherwise, the GRP should be adopted.

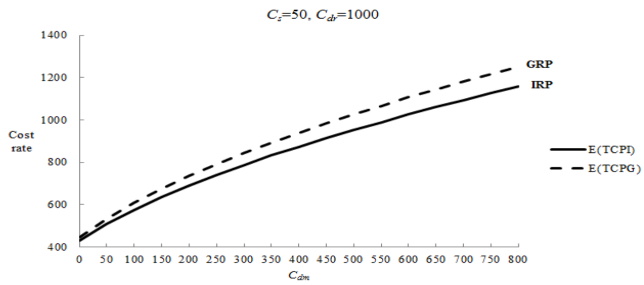

Figure 6: The impact of C_{dm} for optimal replacement policy of parallel system.

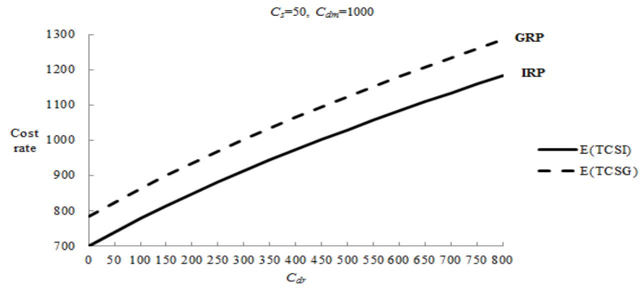

Figure 7: The impact of C_{dr} for optimal replacement policy of parallel system.

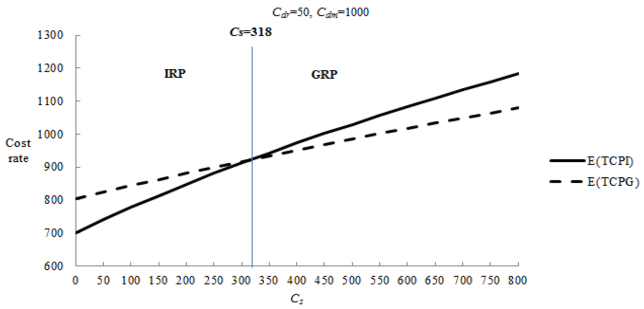

Figure 8: The impact of C_s for optimal replacement policy of parallel system.

4.3 Comparisons between Series and Parallel Systems

In this section, the comparisons of series and parallel systems on IRP. From Figure 9 and 10, there are some results can be obtained. Under various C_s, performing IRP or GRP on parallel system is better than series system and the reduction percentage (Δ) of expected total cost per unit time is significant.

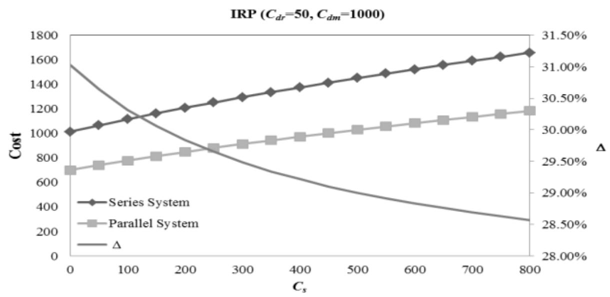

Figure 9: The comparison of series and parallel on IRP under various C_s.

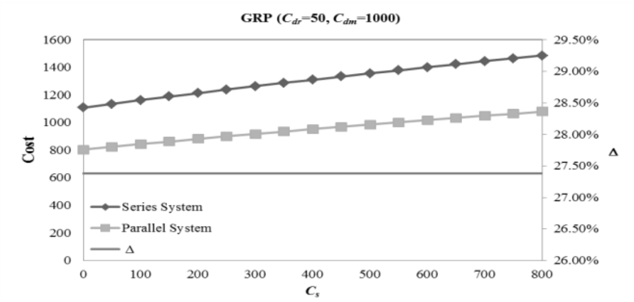

Figure 10: The comparison of series and parallel on GRP under various C_s.

5. CONCLUSIONS

This paper investigates optimal preventive replacement policies for both two-component series and parallel systems when minimal repairs are carried out at failures. The mathematical cost models of series and parallel systems are constructed under IRP and GRP. Furthermore, the optimal individual and group replacement times of two components are obtained such that the expected total cost per unit time is minimized. Moreover, the impacts of the downtime cost (C_{dm}, C_{dr}) and setup cost (C_s) on the optimal replacement policies are analyzed through numerical examples.

In general, we found that the expected total cost per unit time of series systems is higher than parallel systems. For series or parallel systems, when the setup cost is relatively high, we may choose the GRP instead of the IRP. In addition, when the downtime cost (C_{dr}) in series system is relatively high, the GRP should be adopted.

For further study, we may look for an appropriate preventive maintenance policy instead of replacing the components if such actions are available.

ACKNOWLEDGMENT

This research is supported in part by a grant (MOST-103-2221-E-011-061-MY3) from the Ministry of Science and Technology, Taiwan.

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