Project Selection via

Stochastic Multi-criteria Acceptability Analysis

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Abstract. Project selection problem has been largely discovered in extant literature, and attracted considerable attentions of academics and purchasing managers. Practical project selection problem is typically evolved of multi-criteria and a committee of experts. Given the exact values of the input data, certain expert may generate uncertain evaluation results on a project, because the exact weights associated with multi-criteria are difficult to reach consensus. SMAA-2 method is an effective tool to deal with stochastic multi-criteria decision making problem. In this paper, we first formulate the interval data describing all experts' judgment on all project s, and then apply SMAA-2 method to provide a fully rank of all candidate project s. The rank acceptability indices and holistic rank indices are obtained to support the project selection. A numerical example drawn from the previous paper is recalculated to show the effectiveness of our approach.

Keywords: Project selection; stochastic multi-criteria acceptability analysis; Rank

1. INTRODUCTION

The Project selection problem has received considerable attention in both decision analysis and supply chain management literature, and is becoming a fertile research topic for operations research and management science disciplines. Ho et al. (2010) exhaustively review the individual and integrated decision making approaches from 2000 to 2008 to aid the project selection problem. Chai et al. (2013) complementarily provide a systematic literature review of the decision making techniques assisting project selection from 2008 to 2012, which classifies the mentioned techniques into three categories: multiple criteria decision making (MCDM) techniques, mathematical programming techniques and artificial intelligence techniques.

The contemporary project management requires decision maker to maintain strategic partnership with few but reliable project s (Ho et al., 2010), which effectively reduce the project costs and improve the competitive advantages (Ghodsypour and O'Brien, 2001). Therefore, besides conventional price factor, promising project selection policy should also depend on a broad spectrum of qualitative and quantitative criteria such as quality, delivery, flexibility and lead time et al. (Chen et al., 2006). Dickson (1966) has identified 23 criteria to be considered during the process of the project manager determines project selection.

The project selection problem examined in this paper is described as follows. A set of candidate project s are evaluated in terms of criteria, with the involvement of a group of experts. Each expert has specific preference on the ordering of criteria importance. In the presence of deterministic values for each project associated with each criterion, each expert knows the lower and upper bounds about the evaluation results for each project. Therefore, individual expert may produce interval evaluation values to measure the performance of each project, such that an interval project selection matrix (ISSM) is formulated to support project evaluation and selection. Different experts may generate different intervals for certain projects. The interval formulation is motivated from the observation that in the domain of MCDM, different weight elicitation methods may generate different weights even for the same problem, and it is difficult to reach consensus about exact weights (Lahdelma and Salminen, 2001). Evaluating a set of project s using interval values is an important issue in decision analysis. The purpose of this paper is to develop a sophisticated technique for solving the aforementioned ISSM, and provide a comprehensive rank of the candidate project s. Although the large body of research on multicriteria project selection in literature is helpful to effectively guide project manager to choose appropriate projects, it is crucial to understand the impact of interval values on project evaluation and selection. To the best of our knowledge, the extant literature has left this interesting and important topic largely unexplored. The present study fills this gap by first formulating the ISSM and then applying stochastic multicriteria acceptability analysis (SMAA-2) to provide a holistic rank of candidate projects. Such investigation sheds much-needed light on potential incentives and directions for academic, managerial and policy-related implications.

Pioneered by Lahdelma et al. (1998), SMAA is a method intended to aid MCDM with multiple experts in cases where little or no weight information is available, and the criteria values are uncertain. It does not need the experts to describe their input data precisely or implicitly, and provides several meaningful and useful indices including acceptability index for each alternative measuring the variety of input data that give each alternative the best ranking position, central weight describing the preferences of an expert supporting an alternative, and confidence factor representing the reliability of the analysis. Lahdelma and Salminen (2001) extend SMAA by considering all ranks, and provide holistic SMAA-2 analysis to identify good compromise alternatives. For the problems with ordinal criteria information, Lahdelma et al. (2003) develop a new SMAA-O method. Durbach (2006) propose a SMAA using achievement functions (SMAA-A) for discrete-choice decision that investigating what combinations of aspirations are necessary to make each alternative the preferred one. Lahdelma and Salminen (2006a) develop cross confidence factors based upon calculating confidence factors for alternatives using other's central weights. Lahdelma and Salminen (2006b) combine DEA and SMAA-2 to evaluate multicriteria alternatives. Lahdelma and Salminen (2009) develop the SMAA-P method that combines the piecewise linear difference functions of prospect theory with SMAA. Lahdelma et al. (2006, 2009) present and compare simulation and multivariate Gaussian distribution methods to treat the uncertainty and dependency information about the SMAA-2 MCDM. Tervonen and Lahdelma (2007) present efficient methods for performing the computations through Monte Carlo simulation, analyze the complexity and evaluate the accuracy of the developed algorithms. Corrente et al. (2014) integrate SMAA and PROMETHEE methods to explore the parameters compatible with preference information provided by the decision maker. Angilella et al. (2015) and Angilella et al. (2015) combine the Choquet integral preference model with SMAA to obtain robust recommendations and robust ordinal regression, respectively. Durbach and Calder (2015) investigate the context where decision makers are unable or unwilling to assess trade-off information precisely in SMAA.

Besides the method development on SMAA, there exist substantial application papers in literature: facility location (Lahdelma et al., 2002), forest planning (Kangas et al., 2006), elevator planning (Tervonen et al., 2008), descriptive multiattribute choice model (Durbach, 2009a), estimation of a satisficing model of choice (Durbach, 2009b), DEA cross efficiency aggregation (Yang et al., 2012), mutual funds' performance assessment (Babalos et al., 2012) and project portfolio optimization (Yang et al., 2015).

The main contribution of this paper is summarized as follows. First, we formulate an ISSM to describe the project selection problem, in which each expert has specific but uncertain evaluation results on a set of candidate project s. Therefore, the project selection problem with interval values is deemed as a stochastic optimization problem. Second, SMAA-2 is introduced, along with the concepts of rank acceptability index, central weight vector and confidence factor. Third, we apply SMAA-2 to the project selection problem with interval data, and propose a holistic rank of the candidate project s. Even though the classical project selection problem has been largely explored in literature, such investigation in this study is completely new and of both academic and practical significances and values.
The reminder of this paper is organized as below. Section 2 presents the problem description. Section 3 introduces SMAA-2 and some related important indices. Section 4 applies SMAA-2 to solve the project selection with interval inputs. Section 5 concludes this study and proposes some future directions.

2. PROBLEM FORMULATION

The project selection problem studied in this paper is modeled as follows. A set of I candidate projects are evaluated in terms of J criteria, with the involvement of a committee of K experts. All criteria are assumed to be benefit. With regard to the cost-type criteria, we may take the transformation of negativity or reciprocal. Therefore, the basic framework of the multi-criteria project selection problem is depicted by a decision matrix

$G_{IJ}=\left[x_{ij}\right]_{IJ}$:

$$G_{IJ}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ x_{21} & x_{22} & \cdots & x_{2J} \\ \vdots & \vdots & \vdots & \vdots \\ x_{I1} & x_{I2} & \cdots & x_{IJ} \end{bmatrix} \quad (1)$$

Where $x_{ij}, x_{ij}\in[0,1], i=1,2,...,I, j=1,2,...,J$ are exact values for all experts and have been normalized to eliminate the effect of magnitude of data. The evaluation score of a project is calculated by the weighted sum of criteria measures with respect to the mentioned project, that is,

$$S_i=\sum_{j=1}^{J}x_{ij}w_{ij}, i=1,2,...,I \quad (2)$$

Where w_{ij} are the weights of criterion j associated with project i, and $\sum_{j=1}^{J}w_{ij}=1, w_{ij}\geq 0$.

Each expert $k, k=1,2,...,K$ is identified by a specific preference on the sequence of criteria. Without loss of generality, we assume that for typical expert $k, k=1,2,...,K$, the criteria are arranged in a descending order of importance, that is $w_{i1}^k\geq w_{i2}^k\geq\cdots\geq w_{iJ}^k$. This sequence definitely changes across different experts. Therefore, certain expert $k, k=1,2,...,K$ may formulate the following mathematical model to aggregate the most favorable performance for each project i:

$$US_i^k=\max\sum_{j=1}^{J}x_{ij}w_{ij}^k$$

$$\text{s.t. } w_{i1}^k\geq w_{i2}^k\geq\cdots\geq w_{iJ}^k, i=1,2,...,I, \quad (3)$$

$$\sum_{j=1}^{J}w_{ij}^k=1, w_{ij}^k\geq 0, i=1,2,...,I, k=1,2,...,K.$$

Theorem 1 (Ng, 2008). The optimal score of project i derived from the mathematical model (3) is

$$\max_{j=1,2,...,J}\left\{\frac{1}{j}\sum_{t=1}^{j}x_{it}\right\}.$$

Proof. After denoting

$$v_{ij}^k=w_{ij}^k-w_{i(j+1)}^k\geq 0, j=1,2,...,J-1, v_{iJ}^k=w_{iJ}^k\geq 0,$$

We obtain

$$\begin{aligned}\sum_{j=1}^{J}w_{ij}^k&=\left(w_{i1}^k-w_{i2}^k\right)+2\left(w_{i2}^k-w_{i3}^k\right)+\cdots+(J-1)\left(w_{i(J-1)}^k-w_{iJ}^k\right)+J\left(w_{iJ}^k\right)\\&=v_{i1}^k+2v_{i2}^k+\cdots+Jv_{iJ}^k\\&=\sum_{j=1}^{J}jv_{ij}^k\\&=1.\end{aligned} \quad (4)$$

We also incorporate $\varphi_{ij}^k=\sum_{t=1}^{j}x_{it}$, and then have

$$\begin{aligned}\sum_{j=1}^{J}x_{ij}w_{ij}^k&=x_{i1}w_{i1}^k+x_{i2}w_{i2}^k+\cdots+x_{iJ}w_{iJ}^k\\&=\left[\left(w_{i1}^k-w_{i2}^k\right)x_{i1}\right]+\left[\left(w_{i2}^k-w_{i3}^k\right)\left(x_{i1}+x_{i2}\right)\right]+\cdots+\\&\quad\left[\left(w_{i(J-1)}^k-w_{iJ}^k\right)\left(x_{i1}+x_{i2}+\cdots+x_{i(J-1)}\right)\right]+\left[w_{iJ}^k\left(x_{i1}+x_{i2}+\cdots+x_{iJ}\right)\right]\\&=v_{i1}^k\varphi_{i1}^k+v_{i2}^k\varphi_{i2}^k+\cdots+v_{iJ}^k\varphi_{iJ}^k\\&=\sum_{j=1}^{J}v_{ij}^k\varphi_{ij}^k.\end{aligned} \quad (5)$$

Therefore, the mathematical model (3) is equivalent to the following formulation:

$$US_i^k=\max\sum_{j=1}^{J}v_{ij}^k\varphi_{ij}^k$$

$$\text{s.t.} \sum_{j=1}^{J}jv_{ij}^k=1 \quad (6)$$

$$v_{ij}^k\geq 0, j=1,2,...,J.$$

The dual of (6) is

$$\min z_i^k$$

s.t. $z_i^k \geq \frac{1}{j}\varphi_{ij}^k$. (7)

The optimal objective value of (7) is obtained at the point that $z_i^k = \max_{j=1,2,\ldots,J}\left\{\frac{1}{j}\varphi_{ij}^k\right\}$, which is the optimal objective value of (3) in terms of $US_i^k = \max_{j=1,2,\ldots,J}\left\{\frac{1}{j}\sum_{t=1}^{j} x_{it}\right\}$.

This is the most favorable evaluation values determined by expert k for project i, with the given input of decision matrix (1). Given the determined sequence of criteria provided by typical expert, model (3) is easy-to-understand and simple-to-apply, and can be effectively solved without the elicitation of the exact values of weights.

Similarly, it is also necessary to consider the least favorable evaluation scores by expert k for project i. Therefore, an analogous mathematical model is presented below:

$$LS_i^k = \min \sum_{j=1}^{J} x_{ij} w_{ij}^k$$

s.t. $w_{i1}^k \geq w_{i2}^k \geq \cdots \geq w_{iJ}^k, i = 1, 2, \ldots, I$ (8)

$$\sum_{j=1}^{J} w_{ij}^k = 1, w_{ij}^k \geq 0, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K \cdot$$

Theorem 2. The optimal score of project i derived from the mathematical model (8) is $\min_{j=1,2,\ldots,J}\left\{\frac{1}{j}\sum_{t=1}^{j} x_{it}\right\}$.

On the strength of the obtained least and most favorable evaluation scores for project i by expert k, we formulate an ISSM $\Omega_{IK} = \left(\left[LS_i^k, US_i^k\right]\right)_{IK}$ that describes the uncertain judgment of each expert on each project. Reasonable evaluation of project i by expert k should lie in $\left[LS_i^k, US_i^k\right]$:

$$\Omega_{IK} = \begin{bmatrix} \left[LS_1^1, US_1^1\right] & \left[LS_1^2, US_1^2\right] & \cdots & \left[LS_1^K, US_1^K\right] \\ \left[LS_2^1, US_2^1\right] & \left[LS_2^2, US_2^2\right] & \cdots & \left[LS_2^K, US_2^K\right] \\ \vdots & \vdots & \vdots & \vdots \\ \left[LS_I^1, US_I^1\right] & \left[LS_I^2, US_I^2\right] & \cdots & \left[LS_I^K, US_I^K\right] \end{bmatrix}. \quad (9)$$

Be consistent with Yang et al. (2012), the derived ISSM can be viewed as a stochastic MCDM problem. In the following section, we briefly introduce the SMAA-2 method proposed by Lahdelma and Salminen (2001), which effectively solves these series of stochastic MCDM problems by providing a holistic rank of all alternatives.

3. STOCHASTIC MULTICRITERIA ACCEPTABILITY AALYSIS

SMAA represents a family of methods for assisting MCDM with uncertain, imprecise or partially missing input data. The rationale behind SMAA is exploring the weight space to describe the preferences that make each alternative the most preferred one, or grant a certain ranking position for a specific alternative. Lahdelma et al. (1998) initiate the adventure on this topic, and propose rank acceptability index, central weight vector and confidence factor for all alternatives. Lahdelma and Salminen (2001) extend the original SMAA method by considering all ranks in the analysis, and provide more holistic SMAA-2 analysis to graphically identify good compromise alternatives.

3.1 Preliminaries

In line with the ISSM introduced in Section 2, we consider that a committee of K experts has a set of I projects to be evaluated and selected. Neither expert-specific evaluation values nor weights are precisely known. We assume that the decision maker's preferences across all experts' evaluations can be represented by a real-value utility function $g(i, w), i = \{1, 2, \ldots, I\}$, where the weight vector w is to quantify decision maker's subjective preferences across experts' judgments. Moreover, the uncertain evaluation values from experts on projects are represented by stochastic variables ξ_{ik} with assumed or estimated density function $f(\xi)$ in the space $X \subseteq \Re^{I \times K}$. In addition, the unknown weight vector is represented by a weight distribution with density function $f(w)$ in the set of feasible weights defined as

$$W=\left\{w\subseteq\Re^{K}:\sum_{k=1}^{K}w_k=1,w_k\geq 0\right\}. \quad (10)$$

Total absence of weight vector information is represented in “Bayesian” spirit by a uniform weight distribution in W, i.e., $f(w)=\frac{1}{Vol(\mathrm{W})}=\frac{(K-1)!}{\sqrt{K}}$.

Based upon the above descriptions, the utility function is then used to map the stochastic experts' evaluation values and weight distributions into utility distributions $g(\xi_i,w)$.

We define a ranking function denoting the rank of each project as an integer from the best rank (=1) to the worst rank (= I) as follows:

$$rank\ (\xi_i,w)=1+\sum_{l}\rho(g(\xi_l,w)>g(\xi_i,w)), \quad (11)$$

Where $\rho(\text{true})=1$ and $\rho(\text{false})=0$.

The SMAA-2 method is totally relied on analyzing the sets of favorable rank weights $W_i^r(\xi)$ defined as

$$W_i^r(\xi)=\left\{w\in W:rank\left(\xi_i,w\right)=r\right\}, \quad (12)$$

in which a weight $w\in W_i^r(\xi)$ guarantees that alternative ξ_i obtains rank r.

3.2 Indices

This subsection introduces several useful indices proposed by SMAA-2 method. The first one is the rank acceptability index b_i^r, which is described as the expected volume of the set of favorable rank weights. More specifically, b_i^r measures the variety of different valuations that grant alternative ξ_i rank r, which is computed by

$$b_i^r=\int_X f(\xi)\int_{W_i^r(\xi)} f(w)dwd\xi. \quad (13)$$

Obviously, the rank acceptability index b_i^r belongs to the interval $[0,1]$, while $b_i^r=0$ shows that the alternative ξ_i never reaches rank r, and $b_i^r=1$ represents that the alternative ξ_i always obtains rank r, neglecting the impact of the choice about weights. Furthermore, the rank acceptability index can be employed directly in the multi-criteria evaluation of the alternatives. For large-scale problems, we develop an iterative process as below, in which the n best ranks (nbr) acceptability are analyzed at each interaction n:

$$a_i^n=\sum_{r=1}^{n}b_i^r. \quad (14)$$

The nbr-acceptability a_i^n is a measure of the variety different preferences that grant alternative ξ_i any of the n best rank. This analysis proceeds until one or more alternatives obtain a sufficient majority of the weights.

The weight space with respect to the n best rank associated with an alternative can be depicted by the concept of central nbr weight vector w_i^n as below:

$$w_i^n=\int_X f(\xi)\sum_{r=1}^{n}\int_{W_i^r(\xi)} f(w)wdwd\xi/a_i^n. \quad (15)$$

Considering the given weight distribution, the central nbr weight vector is the best single vector representation for the preferences of a decision maker who assigns an alternative any rank from 1 to n.

The third proposed index is the nbr confidence factor p_i^n, which is defined as the probability that the alternative reaches any rank from 1 to n if the central nbr weight vector is determined and computed by

$$p_i^n = \int_{\xi:rank(\xi_i, w_i^n)} f(\xi)d\xi \quad (16)$$

More detailed knowledge about these indices can be found in Lahdelma and Salminen (2001). The manual on implementing SMAA in practice is provided by Tervonen and Lahdelma (2007).

3.3 Holistic evaluation of rank acceptability

On the strength of the aforementioned rank acceptability, the following step is to develop a complementary approach that combines the rank acceptability into holistic acceptability indices associated with all alternatives as below

$$a_i^h = \sum_{r=1}^{I} \alpha^r b_i^r \quad (17)$$

Where α^r are described as metaweights for constructing holistic acceptability indices and satisfy $1=\alpha^1 \geq \alpha^2 \geq \cdots \geq \alpha^I \geq 0$.

The elicitation of so-called metaweights is essential a weight determination process about a lexicographic decision problem, which reasonably assign the largest value to α^1, and the least value to α^I. As for assigning weights to ranks, Barron and Barrett (1996) introduce three mechanisms, namely, rank-sum approach, i.e., $\alpha^r(RS)=\frac{2(I+1-r)}{I(I+1)}, r=1,2,...,I$, reciprocal of the ranks approach, i.e.,

$$\alpha^r(RR)=\frac{1/r}{\sum_{r=1}^{I} 1/r}, r=1,2,...,I,$$

and rank-order centroid approach, i.e.,

$$\alpha^r(ROC)=\frac{1}{I}\sum_{r=1}^{I}\frac{1}{r}, r=1,2,...,I.$$

We use ROC to determine $\alpha^r, r=1,2,...,I$ because they are more accurate, straightforward and efficacious, and provide an appropriate implementation tool (Barron and Barrett, 1996).

The holistic evaluation of rank acceptability indices generates an overall measure of the acceptability of all alternatives. This is helpful to effectively rank and sort alternatives.

4. NUMERICAL EXAMPLE

For the purpose of applying SMAA-2 to solve project selection problem, we draw the data from the multiple criteria project selection problem studied by Xia and Wu (2007). Three criteria, namely, price, quality and service are rated using the three-point scale, i.e., 1, 2 and 3, which indicate “low”, “middle” and “high” for price criterion, and “good”, “middle” and “poor” for quality and service criteria. The problem is to select 5 out of 14 candidate projects, with the involvement of a committee of 6 experts. Each expert has specific preference on the criteria importance, i.e., price $\succ$ quality $\succ$ service, price $\succ$ service $\succ$ quality, quality $\succ$ price $\succ$ service, quality $\succ$ service $\succ$ price, service $\succ$ price $\succ$ quality and service $\succ$ quality $\succ$ price, which are denoted by notations “1”, “2”, “3”, “4”, “5” and “6”, respectively.

Table 1: Data for project selection

Supplier	Price	Quality	Service	Price(Norm)	Quality(Norm)	Service(Norm)
1	2	1	1	0.0600	0.0370	0.0400
2	3	1	1	0.0400	0.0370	0.0400
3	1	2	2	0.1200	0.0741	0.0800
4	2	2	2	0.0600	0.0741	0.0800
5	3	2	1	0.0400	0.0741	0.0400
6	1	2	3	0.1200	0.0741	0.1200
7	1	3	1	0.1200	0.1111	0.0400
8	1	1	3	0.1200	0.0370	0.1200
9	2	2	1	0.0600	0.0741	0.0400
10	2	2	3	0.0600	0.0741	0.1200
11	3	3	1	0.0400	0.1111	0.0400
12	3	2	2	0.0400	0.0741	0.0800
13	2	3	1	0.0600	0.1111	0.0400
14	2	1	3	0.0600	0.0370	0.1200

The ISSM $\Omega_{IK}=\left(\left[LS_i^k, US_i^k\right]\right)_{IK}$ is obtained by formulations (3) and (8), in which the interval evaluations on all projects by all experts are reported in the following Table 2.

Table 2: Interval project selection matrix

Supplier	Expert					
	1	2	3	4	5	6
1	[0.0457,0.0600]	[0.0457,0.0600]	[0.0370,0.0485]	[0.0370,0.0457]	[0.0400,0.0500]	[0.0385,0.0457]
2	[0.0385,0.0400]	[0.0390,0.0400]	[0.0370,0.0390]	[0.0370,0.0390]	[0.0390,0.0400]	[0.0385,0.0400]
3	[0.0914,0.1200]	[0.0914,0.1200]	[0.0741,0.0970]	[0.0741,0.0914]	[0.0800,0.1000]	[0.0770,0.0914]
4	[0.0600,0.0714]	[0.0600,0.0714]	[0.0670,0.0741]	[0.0714,0.0770]	[0.0700,0.0800]	[0.0714,0.0800]
5	[0.0400,0.0570]	[0.0400,0.0514]	[0.0514,0.0741]	[0.0514,0.0741]	[0.0400,0.0514]	[0.0400,0.0570]
6	[0.0970,0.1200]	[0.1047,0.1200]	[0.0741,0.1047]	[0.0741,0.1047]	[0.1047,0.1200]	[0.0970,0.1200]
7	[0.0904,0.1200]	[0.0800,0.1200]	[0.0904,0.1156]	[0.0756,0.1111]	[0.0400,0.0904]	[0.0400,0.0904]
8	[0.0785,0.1200]	[0.0923,0.1200]	[0.0370,0.0923]	[0.0370,0.0923]	[0.0923,0.1200]	[0.0785,0.1200]
9	[0.0580,0.0670]	[0.0500,0.0600]	[0.0580,0.0741]	[0.0570,0.0741]	[0.0400,0.0580]	[0.0400,0.0580]
10	[0.0600,0.0847]	[0.0600,0.0900]	[0.0637,0.0847]	[0.0741,0.0970]	[0.0847,0.1200]	[0.0847,0.1200]
11	[0.0400,0.0756]	[0.0400,0.0637]	[0.0637,0.1111]	[0.0637,0.1111]	[0.0400,0.0637]	[0.0400,0.0756]
12	[0.0400,0.0647]	[0.0400,0.0647]	[0.0570,0.0741]	[0.0647,0.0770]	[0.0600,0.0800]	[0.0647,0.0800]
13	[0.0600,0.0856]	[0.0500,0.0704]	[0.0704,0.1111]	[0.0704,0.1111]	[0.0400,0.0704]	[0.0400,0.0756]
14	[0.0485,0.0723]	[0.0600,0.0900]	[0.0370,0.0723]	[0.0370,0.0785]	[0.0723,0.1200]	[0.0723,0.1200]

Furthermore, the metaweights to formulate the holistic acceptability indices are

$$\alpha^{12} = (1.00, 0.69, 0.54, 0.44, 0.36, 0.30, 0.25, 0.20, 0.16, 0.13, 0.10, 0.07, 0.05, 0.02). \quad (18)$$

The SMAA-2 model can be effectively solved by the open source software developed by Tervonen (2014).

4.1 Normal Distribution

We assume that the interval data $\left[LS_i^k, US_i^k\right]$ are normally distributed, the mean and variance of which are represented by $\mu_i^k = \frac{LS_i^k + US_i^k}{2}$ and $\left(\sigma^2\right)_i^k = \frac{US_i^k - LS_i^k}{6}$, respectively. The results about the rank acceptability indices and the holistic acceptability indices derived according to SMAA-2 are shown in Table 3 and graphically reported in Figure 1.

Table 3: Holistic acceptability indices and rank acceptability indices (Normal distribution)

Supplier	b^1	b^2	b^3	b^4	b^5	b^6	b^7	b^8	b^9	b^{10}	b^{11}	b^{12}	b^{13}	b^{14}	a^k
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.35	0.64	0.0307
2	0.01	0.01	0.00	0.01	0.02	0.03	0.03	0.02	0.04	0.04	0.08	0.11	0.27	0.33	0.0960
3	0.00	0.25	0.46	0.23	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5392
4	0.00	0.00	0.00	0.00	0.00	0.09	0.47	0.38	0.07	0.00	0.00	0.00	0.00	0.00	0.2311
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.55	0.37	0.03	0.0607
6	0.91	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.9708
7	0.07	0.24	0.16	0.16	0.21	0.12	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.5128
8	0.03	0.24	0.18	0.26	0.17	0.06	0.05	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.5035
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.10	0.59	0.27	0.01	0.00	0.0962
10	0.02	0.17	0.17	0.26	0.31	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.4833
11	0.00	0.00	0.00	0.01	0.03	0.08	0.13	0.12	0.22	0.29	0.10	0.01	0.00	0.00	0.1798
12	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.13	0.31	0.39	0.14	0.03	0.00	0.00	0.1464
13	0.01	0.00	0.01	0.05	0.09	0.28	0.14	0.20	0.16	0.06	0.01	0.00	0.00	0.00	0.2630
14	0.00	0.01	0.01	0.02	0.12	0.25	0.13	0.12	0.16	0.11	0.05	0.03	0.00	0.00	0.2426

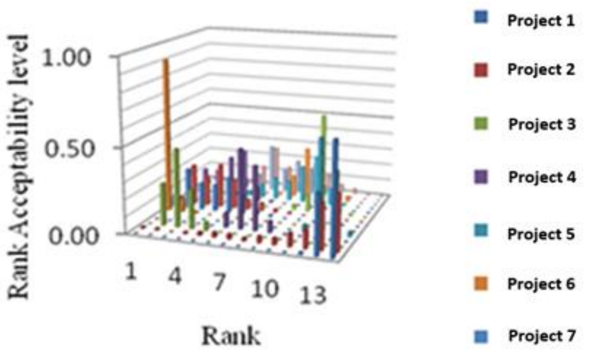

Figure 1: Rank acceptability indices (Normal distribution).

Based upon the holistic acceptability indices in Table 3, we obtain a full and comprehensive rank of all projects: $6 \succ 3 \succ 7 \succ 8 \succ 10 \succ 13 \succ 14 \succ 4 \succ 11 \succ 12 \succ 9 \succ 2 \succ 5 \succ 1$. The selected projects are projects 6, 3, 7, 8 and 10. More specifically, the most favorable project is project 6, whose holistic rank index is 97.08% and first rank support is 91% of the possibility, while the least favorable project is project 1, the holistic rank index and the last rank support of which are 3.07% and 64% of the possibility, respectively.

4.2 Uniform distribution

We alternatively assume that the interval data are uniformly distributed. With such assumptions, we report the holistic acceptability indices and the rank acceptability indices in the following Table 4 and Figure 2.

Table 4: Holistic acceptability indices and rank acceptability indices (Uniform distribution)

Supplier	b^1	b^2	b^3	b^4	b^5	b^6	b^7	b^8	b^9	b^{10}	b^{11}	b^{12}	b^{13}	b^{14}	a^h
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.55	0.41	0.0362
2	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.04	0.07	0.23	0.56	0.0587
3	0.02	0.23	0.36	0.26	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.5321
4	0.00	0.00	0.00	0.00	0.01	0.16	0.43	0.32	0.08	0.01	0.00	0.00	0.00	0.00	0.2365
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.14	0.61	0.20	0.02	0.0708
6	0.82	0.13	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.9359
7	0.10	0.19	0.17	0.16	0.17	0.12	0.04	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.5103
8	0.03	0.23	0.19	0.20	0.15	0.08	0.05	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.4813
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.06	0.17	0.55	0.19	0.01	0.00	0.1025
10	0.02	0.17	0.18	0.24	0.28	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.4717
11	0.00	0.01	0.01	0.02	0.05	0.10	0.12	0.14	0.19	0.24	0.09	0.02	0.00	0.00	0.1995
12	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.18	0.34	0.31	0.10	0.02	0.00	0.00	0.1592
13	0.01	0.01	0.03	0.05	0.11	0.22	0.16	0.16	0.14	0.10	0.01	0.00	0.00	0.00	0.2687
14	0.00	0.01	0.03	0.05	0.12	0.20	0.13	0.13	0.13	0.10	0.06	0.04	0.01	0.00	0.2495

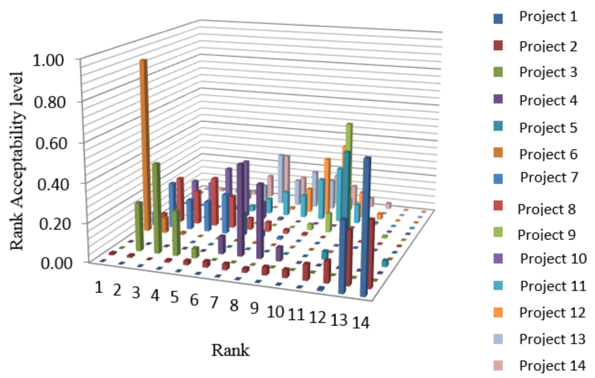

Figure 2: Rank acceptability indices (Uniform distribution).

It is observed that the sequence of candidate projects using SMAA-2 under uniform distribution are:

$$6 \succ 3 \succ 7 \succ 8 \succ 10 \succ 13 \succ 14 \succ 4 \succ 11 \succ 12 \succ 9 \succ 5 \succ 2 \succ 1,$$

and the selected projects are projects 6, 3, 7, 8 and 10 as well. This sequence is mildly different from that derived from norm distribution case. The only difference lies in the rank positions of projects 2 and 5. In details, the most favorable project 6's holistic rank index is 93.59% and first rank support is 82% of the possibility, both of which are lower than that of normal distribution case. Meanwhile, the holistic rank index and last rank support possibility of the least favorable suppler 1 are 3.62% and 41%, respectively.

In summary, SMAA-2 under both the normal distribution and uniform distribution assumptions may produce complete ranks with sufficient discrimination power among all alternatives, in the case of that each expert has uncertain evaluations across all projects.

4. CONCLUSION

Multi-criteria project selection problem with the involvement of a group of experts has been widely explored in decision science and supply chain management literature. Given the exact input data, different experts may generate uncertain evaluation results for all projects. However, the extant literature has left this topic largely undiscovered. This paper is initially engaged in this tremendous surge by first formulating the interval values to optimize, and then innovatively applying the SMAA-2 method to obtain an overall rank over all candidate projects. The interval data are assumed to be either normally or uniformly distributed in this study, and a metaweight scheme to derive holistic rank indices is elicited from the previous literature. A numerical example from the existing work is reexamined to show the effectiveness of our approach.

This paper not only provides the decision maker with more methodological options, but also enriches the theory and method of project selection problem. Future research should consider the determination of the uncertain sets for decision making, and investigate more practical distributions over the uncertainties.

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