Forecasting the Daily Shipping Data of Sanitary Materials

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Abstract: Correct sales forecasting is indispensable to industries. In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are many researches made on this. In this paper, we propose a new method to improve forecasting accuracy and confirm them by the numerical example. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. First, we make estimation of ARMA model parameter and then estimate smoothing constants, which is the theoretical solution. Combining the trend removing method with this method, we aim to improve forecasting accuracy. Furthermore, "a day of the week index" is newly introduced for the daily data and the forecasting is executed to the manufacturer's data of sanitary materials. We have obtained good result. The effectiveness of this method should be examined in various cases.

Key Words: Minimum Variance, Exponential Smoothing Method, Forecasting, Trend, Sanitary Materials

1. INTRODUCTION

Correct sales forecasting is indispensable in industries. Poor sales forecasting accuracy leads to increased inventory and prolonged dwell time of product. In order to improve forecasting accuracy, we have devised trend removal methods as well as searching optimal parameters and obtained good results. We created a new method and applied it to various time series and examined the effectiveness of the method. Applied data are sales data, production data, shipping data, stock market price data, flight passenger data etc.

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM) (Box Jenkins [1]), (R.G.Brown[2]), (Tokumaru et al.[3]), (Kobayashi[4]). Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend (Peter [5]), utilizing Kalman Filter (Maeda [6]), Bayes Forecasting (M.West et al. [7]), adaptive ESM (Steinar [8]), exponentially weighted Moving Averages with irregular updating periods

(F.R.Johnston [9]), making averages of forecasts using plural method (Spyros [10]) are presented. For example, Maeda[6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he couldn't grasp observation noise. It can be said that it doesn't pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii[11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before (Takeyasu et al. [12]). Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. "a day of the week index (DWI)" is newly introduced for the daily data and a day of the week trend is removed. Theoretical solution of smoothing constant of ESM is calculated for both of the DWI trend removing data and the non DWI trend removing data. Then forecasting is executed to the manufacturer's data of sanitary materials. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of the previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. "a day of the week index (DWI)" is newly introduced in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2.DESCRIPTION OF ESM USING ARMA MODEL[1]

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\begin{aligned}\hat{x}_{t+1} &= \hat{x}_t + \alpha(x_t - \hat{x}_t) \\ &= \alpha x_t + (1-\alpha)\hat{x}_t\end{aligned} \tag{1}$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$

x_t : realized value at t

α : smoothing constant $(0 < \alpha < 1)$

(1) is re-stated as:

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \tag{2}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \tag{3}$$

Generally, (p,q) order ARMA model is stated as:

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{4}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (4) is supposed to satisfy convertibility condition.

Utilizing the relation that:

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

we get the following equation from (3).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{5}$$

Operating this scheme on $t+1$, we finally get:

$$\begin{aligned}\hat{x}_{t+1} &= \hat{x}_t + (1-\beta)e_t \\ &= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t)\end{aligned} \tag{6}$$

If we set $1-\beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1 (Box Jenkins [1]), (Tokumaru et al.[3]).

Comparing with (3) and (4), we obtain:

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases}$$

From (1), (6),

$$\alpha = 1 - \beta$$

Therefore, we get:

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \tag{7}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (4) be:

$$\begin{cases} \tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} & (8) \\ \tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} & (9) \end{cases}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (8), (9), we get the following non-linear equations which are well known[3].

$$\left.\begin{aligned} \tilde{r}_k &= \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ & \quad 0 & (k \ge q+1) \\ \tilde{r}_0 &= \sigma_e^2 \sum_{j=0}^{q} b_j^2 & \end{aligned}\right\} \quad (10)$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (3) (4) (7) (10), we get:

$$\left.\begin{aligned} & q = 1 \\ & a_1 = -1 \\ & b_1 = -\beta = \alpha - 1 \\ & \tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ & \tilde{r}_1 = b_1 \sigma_e^2 \end{aligned}\right\} \quad (11)$$

If we set:

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \quad (12)$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \quad (13)$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \quad (14)$$

In order to have real roots, ρ_1 must satisfy:

$$\left|\rho_1\right| \le \frac{1}{2} \quad (15)$$

From invertibility condition, b_1 must satisfy:

$$\left|b_1\right| < 1$$

From (13), using the next relation,

$$\left(1 - b_1\right)^2 \ge 0$$
$$\left(1 + b_1\right)^2 \ge 0$$

(15) always holds.

As

$$\alpha = b_1 + 1$$

b_1 is within the range of:

$$-1 < b_1 < 0$$

Finally we get:

$$\left.\begin{aligned} b_1 &= \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \quad (16)$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

Here ρ_1 must satisfy:

$$-\frac{1}{2} < \rho_1 < 0 \quad (17)$$

in order to satisfy $0 < \alpha < 1$.

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1 st order autocorrelation function.

3.TREND REMOVAL METHOD

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set:

$$y = a_1 x + b_1 \tag{18}$$

as a linear function.

[2] Non-linear function

We set:

$$y = a_2 x^2 + b_2 x + c_2 \tag{19}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{20}$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function

We set:

$$\begin{aligned} y &= \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2) \\ &\quad + \alpha_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \end{aligned} \tag{21}$$

$$\begin{aligned} &0 \le \alpha_1 \le 1 \quad , 0 \le \alpha_2 \le 1 \quad , 0 \le \alpha_3 \le 1 \\ &\alpha_1 + \alpha_2 + \alpha_3 = 1 \end{aligned} \tag{22}$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function.

Trend is removed by dividing the data by (21).

4. A DAY OF THE WEEK INDEX

"a day of the week index (DWI)" is newly introduced for the daily data of sanitary materials.. The forecasting accuracy would be improved after we identify the "a day of the week index" and utilize them. This time in this paper, the data we handle consist by Monday through Sunday, we calculate $DWI_j \ (j = 1, \cdots, 7)$ for Monday through Sunday.

For example, if there is the daily data of L weeks as stated bellow:

$$\{x_{ij}\} \ (i = 1, \cdots, L) \ (j = 1, \cdots, 7)$$

where $x_{ij} \in R$ in which L means the number of weeks (Here $L = 10$), i means the order of weeks (i-th week), j means the order in a week (j-th order in a week; for example $j = 1$: Monday, $j = 7$: Sunday) and x_{ij} is the daily sales data of sanitary materials. Then, DWI_j is calculated as follows.

$$DWI_j = \frac{\frac{1}{L} \sum_{i=1}^{L} x_{ij}}{\frac{1}{L} \cdot \frac{1}{7} \sum_{i=1}^{L} \sum_{j=1}^{7} x_{ij}} \tag{23}$$

DWI trend removal is executed by dividing the data by (23). Numerical examples both of DWI removal case and non-removal case are discussed in section 5.

5. FORECASTING THE SANITARY MATERIALS DATA

5.1 Analysis Procedure

The shipping data of 2 cases from January 31, 2012 to April 2, 2012 are analyzed. First of all, graphical charts of these time series data are exhibited in Figure 5-1 to 5-2.

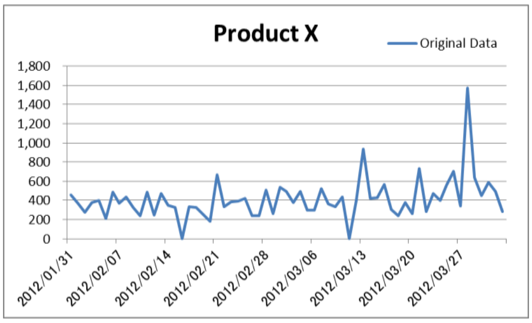

Fig. 5-1. Daily Shipping Data of Product X

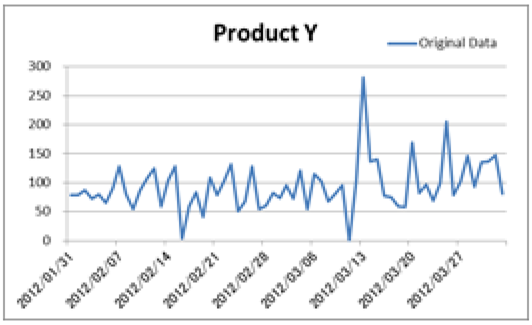

Fig. 5-2. Daily Shipping Data of Product Y

Analysis procedure is as follows. There are 68 daily data for each case. We use 49 data(1 to 49) and remove trend by the method stated in section 3. Then we calculate a day of the week index (DWI) by the method stated in section 4. After removing DWI trend, the method stated in section 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 50th and the forecast for 51st data is executed consecutively, which finally reaches forecast of 63rd data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 50th to 63rd data. Final forecasting data is obtained by multiplying DWI and trend.

Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{24}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{25}$$

Variance of forecasting error is calculated by:

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{26}$$

In this paper, we examine the two cases stated in Table 5-1.

Table 5-1. The combination of the case of trend removal and DWI trend removal

Case	Trend	DWI trend
Case1	Removal	Removal
Case2	Removal	Non removal

5.2. Trend Removing

Trend is removed by dividing original data by (21). Here, the weight of α_1 and α_2 are shifted by 0.01 increment in (21) which satisfy the equation (22). The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (18), (19) and (20) are exhibited in Table 5-2. Data are fitted to (18), (19) and (20), and using the least square method, parameters of (18), (19) and (20) are estimated. Estimation results of weights of (21) are exhibited in Table 5-3. The weighting parameters are selected so as to minimize the variance of forecasting error.

Table 5-2. Coefficient of (18),(19) and (20)

	1st		2nd			3rd			
	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
X	1.51	333.97	0.07	−1.92	363.15	−0.01	0.81	−16.81	428.29
Y	0.37	78.32	0.01	−0.26	83.69	−0.00	0.08	−1.71	90.03

Table 5-3. Weights of (21)

	Case	α_1	α_2	α_3
Product X	Case1	1.00	0.00	0.00
	Case2	0.75	0.25	0.00
Product Y	Case1	1.00	0.00	0.00
	Case2	1.00	0.00	0.00

As a result, we can observe the following two patterns.

① Selected liner model:
Product X Case1, Product Y Case1, Product Y Case2

② Selected 2nd order model:
Product X Case2

Graphical charts of trend are exhibited in Figure 5-3 to 5-4.

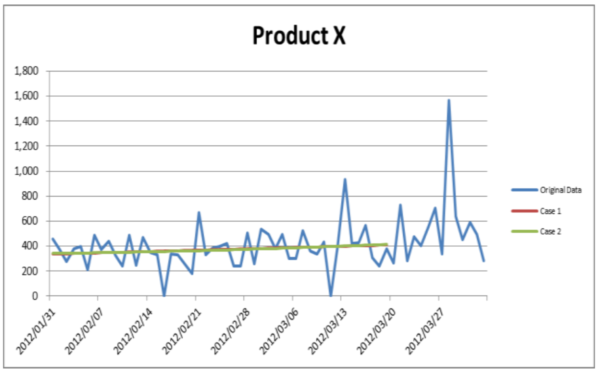

Fig. 5-3. **Daily Shipping Data of Product X**

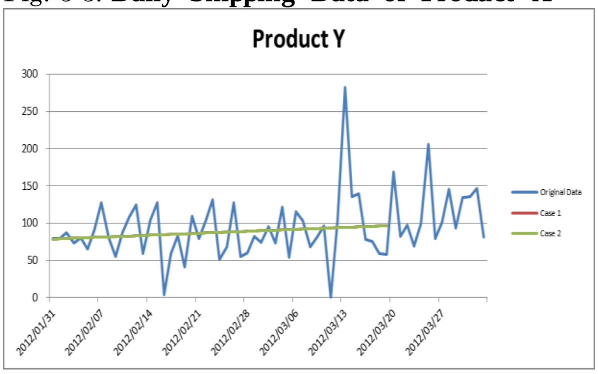

Fig. 5-4. **Daily Shipping Data of Product Y**

5.3. Removing Trend by DWI

After removing trend, a day of the week index is calculated by the method stated in 4. Calculation result for 1st to 49th data is exhibited in Table 5-4.

Table 5-4. a day of the week index

	Case	a day of the week index						
		Thu.	Fri.	Sat.	Sun.	Mon.	Tue.	Wed.
Product X	Case 1	1.384	1.038	0.883	1.050	1.061	0.645	0.940
Product Y	Case 1	1.378	1.165	0.908	0.855	0.952	0.883	0.859

5.4. Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing DWI trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (16). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (17). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 49th data is exhibited in Table 5-5.

Table 5-5. Estimated Smoothing Constant with Minimum Variance

	Case	ρ_1	α
Product X	Case1	-0.1264	0.87
	Case2	-0.1261	0.87
Product Y	Case1	-0.0277	0.97
	Case2	-0.3695	0.56

5.5. Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 50th to 63rd data. Final forecasting data is obtained by multiplying DWI and trend. Variance of forecasting error is calculated by (26).

Forecasting results are exhibited in Figure 5-5 and 5-6

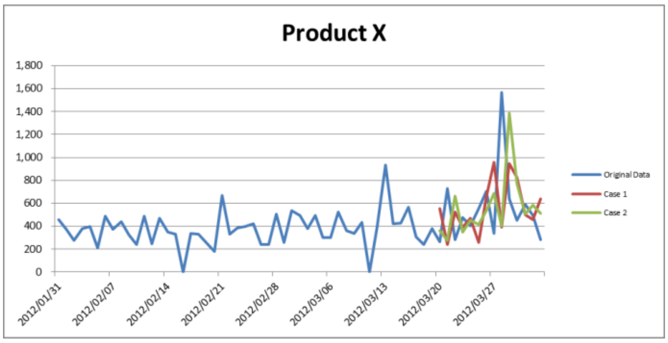

Fig. 5-5. Forecasting Results of **Product X**

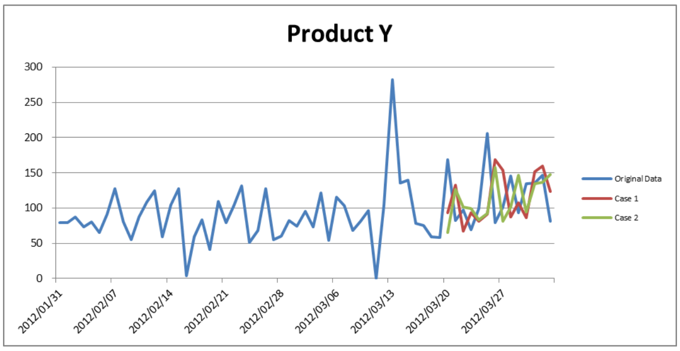

Fig. 5-6 Forecasting Results of **Product Y**

Variance of forecasting error is exhibited in Table 5-6.

Table 5-6. Variance of Forecasting Error

	Case	Variance of Forecasting Error	
Product X	Case1	92,304	*
	Case2	96,008	
Product Y	Case1	946	*
	Case2	1,267	

6. CONCLUSIONS

Correct sales forecasting is indispensable to industries. Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. First, we made estimation of ARMA model parameter and then estimated smoothing constants, which was the theoretical solution.

Furthermore, combining the trend removal method with this method, we aimed to increase forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the daily shipping data of sanitary materials. The combination of linear and non-linear function was also introduced in trend removing. “a day of the week index (DWI)” is newly introduced for the daily data and a day of the week trend is removed. Theoretical solution of smoothing constant of ESM was calculated for both of the DWI trend removing data and the non DWI trend removing data. Then forecasting was executed on these data.

Regarding both data, the forecasting accuracy of case1 (DWI is imbedded) was better than those of case 2 (DWI is not imbedded). It can be said that the introduction of DWI has worked well. It is our future works to ascertain our newly proposed method in many other cases. The effectiveness of this method should be examined in various cases.

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