Analysis of Evolutionary Stability for Behavior Strategies on Recycling Promotion Activity in Green Supply Chain

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Abstract. For solving problems about environment protection and resource saving, establishment of a green supply chain(GSC), which collects used products, reuses the recycled parts in production of products and sells the products, has been promoted. It is necessary to analyze behavior of GSC members to determine the optimal operation. This paper discusses a GSC with one retailer and one manufacturer and verifies the behavior strategies of GSC members which may change over time as to changes of parameters regarding recycling promotion activity(RPA) with collection of used products and sales promotion of the products reusing the recycled parts. A retailer takes three behavior strategies: maximum cooperation, minimum cooperation and non-cooperation in RPA. A manufacturer takes two behavior strategies: monitoring and non-monitoring of the retailer's behavior. Evolutionary game theory combining evolutionary theory by Darwin with game theory is adopted to clarify analytically evolutionary outcomes by a change in each behavior of GSC members over time. The evolutionary stable strategies(ESSs) for GSC members' behaviors are derived by using the replicator dynamics. The analysis numerically illustrates how (i) compensation cost, (ii) collection promotion cost, (iii) sales promotion cost, (iv) monitoring cost, (v) penalty cost affect the judgment of ESSs of behaviors of GSC members.

Keywords: green supply chain, recycling promotion activity, evolutionary game theory, evolutionarily stable strategy, replicator dynamics

1. INTRODUCTION

In recent years, for the purpose of solving the problem regarding environment protection and resource saving, certain measures and policies have been promoted to establish a green supply chains (GSCs) with material flows from collection of used products to reuse of recycled parts in production of products. It is necessary to analyze behaviors of members in a GSC to determine the optimal operation. In general, the game theory is used to analyze members' behaviors in a GSC. The orthodox game theory usually assumes that members in a GSC can share correctly the following information: (i) each other's defined rule of game; (ii) common and full knowledge of rationality; (iii) the preference of each other's behavior; (iv) each other's optimal action under rational situation. However, in reality, all assumptions mentioned above may not be held between members in a GSC. The behavior strategy of the individual member in a GSC may change over time due to changes of members' situations such as cost parameters and the quantity of transaction. Under above situations, the evolutionary game theory, which combines the evolutionary theory by Darwin with the game theory, has become a powerful tool to analyze evolutionary process and the outcome which are driven by a change in behavior of the individual member in a GSC (Zhou and Deng, 2006; Zhu and Dou, 2007; Yu et al., 2009; Barari, et al., 2012).

Regarding GSC, some previous papers have dealt with applications of the evolutionary game theory into the analysis of behaviors of members in a GSC. Zhu and Dou (2007) discussed green supply chains (GSCs) consisting of governments and core enterprises, and studied the game relationships between governments and core enterprises in GSCs. The only respective costs and benefits between them without and with GSC activity were analyzed, and established the evolutionary game model in GSCs. Different equilibrium results for the evolutionary game model were analyzed, and explored win-win ways between governments and core enterprises in GSCs. However, the recycling cooperation activity between governments and core enterprises were not discussed. The effects of GSC activity on both the collection quantity of used products and the product demand were not discussed. Barari et al. (2012) discussed a case study regarding a GSC with one producer and one retailer, and verified synergetic alliance between the environmental and commercial benefits by establishing coordination in the GSC by using evolutionary dynamics. Green financial burden sharing contract was considered between the producer and consumers in the GSC. The degree of sales effort of products were considered for the retailer. The actual costs related to GSC activity was combined with the objective of profit maximization of members in the GSC by using evolutionary dynamics. The equilibrium point which not only balanced the price with the green benefits of the product, but also optimized members' revenue in the GSC was found by using the replicator dynamics equation. However, formulations of costs and profits related to a GSC was specialized as the conceptual framework for a case study which the GSC was dealt with.

Differently from the previous papers mentioned above, this paper focuses on the followings: (1) Analysis of a GSC using the evolutionary dynamics and presentation of the evolutionary stable strategies (EESs) for behavior strategy of members in a GSC, (2) Analysis of recycling promotion activity (RPA) between members in a GSC by using two concepts: (i) compensation and penalty regarding collection of used products and (ii) maximum sales promotion and minimum sales promotion of the products reusing the recycled parts, (3) Analysis of effect of RPA on both the collection quantity of used products and the product demand, (4) formulations and analysis of the evolutionary dynamics by using costs and profits of members related to the operation in a GSC with mathematical expressions and numerical calculation. Concretely, this paper discusses a GSC with one retailer and one manufacturer and verifies the behavior strategies of GSC members which may change over time as to changes of parameters regarding RPA with collection of used products and sales promotion of the products reusing the recycled parts. A retailer takes three behavior strategies: maximum cooperation, minimum cooperation and non-cooperation in RPA. A manufacturer takes two behavior strategies: monitoring and non-monitoring of the retailer's behavior. Evolutionary game theory combining evolutionary theory by Darwin with game theory is adopted to clarify analytically evolutionary outcomes by a change in each behavior of GSC members over time. The evolutionary stable strategies(ESSs) for GSC members' behaviors are derived by using the replicator dynamics. The analysis numerically illustrates how (i) compensation cost, (ii) collection promotion cost, (iii) sales promotion cost, (iv) monitoring cost, (v) penalty cost affect the judgment of ESSs of behaviors of GSC members. The different ESSs for behavior strategies of members in a GSC are analyzed through numerical calculation. The contribution of this paper is to provide the optimal setting of system parameters and its practices to construct and operate a GSC and the informative motivations for researchers and policymakers to manage a GSC.

2. NOTATIONS

- Behavior strategies of a retailer and a manufacturer

R_1 : maximum cooperation for the recycling promotion activity (RPA) which a retailer collects of used product aggressively from customers and invests the maximum sales promotion cost to sell a single type of products using recycled parts

R_2 : minimum cooperation for RPA which a retailer collects of used product passively from customers and invests the minimum sales promotion cost to sell a single type of products using recycled parts

R_3 : non-cooperation for RPA

M_1 : monitoring of the retailer's RPA

M_2 : non-monitoring of the retailer's RPA

(R_i, M_j) $(i=1,2,3,\ j=1,2)$: behavior strategy of GSC members

- System parameters in GSC

A : potential quantity of used products collected from customers

t_1 : the unit collection promotion cost of used product from customers when a retailer takes behavior strategy R_1

t_2 : the unit collection promotion cost of used product from customers when a retailer takes behavior strategy R_2

t_0 : the unit collection promotion cost of used product from customers when a retailer takes behavior strategy R_3

β_1 : increase rate of collection quantity of used products from customers when GSC members take (R_1, M_1)

β_2 : increase rate of collection quantity of used products from customers when GSC members take (R_2, M_1)

β_3 : increase rate of collection quantity of used products from customers when a manufacturer takes behavior strategy M_2 $(\beta_3 < \beta_2 < \beta_1)$

D : potential product demand in a market

α_1 : increase rate of product demand when GSC members take (R_1, M_1)

α_2 : increase rate of product demand when GSC members take (R_2, M_1)

α_3 : increase rate of collection quantity of used products when a manufacturer takes behavior strategy M_2 $(\alpha_3 < \alpha_2 < \alpha_1)$

$c_{sp\max}$: the maximum sales promotion cost

$c_{sp\min}$: the minimum sales promotion cost

c_s : monitoring cost of a manufacturer

re_1 : compensation cost per used product of a manufacturer to pay to a retailer for the remanufacturing quantity of used products when a retailer takes behavior strategy R_1

re_2 : compensation cost per used product of a manufacturer to pay to a retailer for the remanufacturing quantity of used products when a retailer takes behavior strategy R_2

re_0 : compensation cost per used product of a manufacturer to pay to a retailer for the remanufacturing quantity of used products when a retailer takes behavior strategy R_3

c_p : penalty cost of a retailer to pay to a manufacturer when a retailer takes behavior strategy R_3

p : the unit sales price of product

c_t : the unit delivery cost of used products

w : the unit wholesale price of product

c_m : the unit production cost of product

c_a : disassembly and inspection cost per used product

r: recycling rate to recycle a single type of parts from used products

c_r : the unit remanufacturing cost of recyclable parts

c_d : the unit disposal cost of un-recycled part

c_n : the unit production cost of new parts

3. MODEL DESCRIPTIONS

3.1 Operational Flows of a GSC

A green supply chain (GSC) with one retailer and one manufacturer is considered. Also, it is assumed that a single type of products such as consumer electronics (mobile phone, personal computer) are produced and are sold in a market. The operational flows of a GSC are shown as follows:

(1) A retailer decides one behavior strategy among R_1, R_2, R_3 (See section 2) for the recycling promotion activity (RPA). The retailer collects used products from customers as to the own behavior strategy, and delivers all the collection quantity of the used products at the unit cost c_t to a manufacturer. Also, the retailer places an order D with the manufacturer for a single type of products. Here, it is assumed that the collection quantity of used products for the unit collection inventive is smaller than the retailer's order quantity D.

(2) The manufacturer disassembles the used products, and inspects their quality at the unit cost c_a. After that, the manufacturer remanufactures some of recyclable parts at unit cost c_r by a recycling rate r. All the un-recycled parts are disposed at unit cost c_d.

(3) The manufacturer produces the required quantity of new parts at the unit cost c_n if the quantity of the recycled parts is unsatisfied with the required quantity D of parts for the product order quantity D.

(4) The manufacturer produces the product order quantity D at the unit cost c_m, and sells them to the retailer at the unit wholesale price w.

(5) According to the behavior strategy decided in (1) on sales promotion of the products reusing the recycled parts, the retailer sells products in a market with the unit sales price p during a single period.

(6) A manufacturer decides one behavior strategy among M_1 and M_2 (See section 2) for monitoring the retailer's RPA. The manufacturer pays the compensation cost to the retailer for the retailer's collection promotion cost as to the retailer's action on collection of used product.

(7) The retailer pays the penalty cost to the manufacturer when the retailer does not cooperate RPA and the manufacturer monitors the retailer's action without cooperation of RPA.

3. 2 Behavior Strategies of Members in GSC

This paper defines that the recycling promotion activity (RPA) in a GSC includes the collection of used products and the sales promotion of products reusing parts recycled from the used products. Regarding RPA, a retailer takes three behavior strategies: R_1, R_2, R_3 regarding RPA (See section 2), meanwhile a manufacturer takes two behavior strategies: M_1 and M_2 (See section 2). In behavior strategies between a retailer and a manufacturer in a GSC, the retailer does not know which behavior strategy the manufacturer will take before the retailer takes the own behavior strategy and vice versa.

Table 1 shows the effects of the behaviors strategies of GSC members on the collection quantity of used products, product demand and the related costs of each member, based on 3.1 the operational flows of a GSC.

4. PROFITS OF A RETAILER AND A MANUFACTURER AS TO BEHAVIOR STRATEGIES OF GSC MEMBERS

Table 2 shows the payoff matrix between a retailer and a manufacturer in a GSC as to their behavior strategies. Profits of both member in a GSC are formulated as to their behavior strategies, (R_i, M_j) $(i=1,2,3, j=1,2)$ from section 3 and Table 1.

- Profits of members in behavior strategy (R_1, M_1)

The profit of a retailer taking (R_1, M_1), $\pi_{R\,(R_1,M_1)}$ consists of the collection promotion cost of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of the products, the sales of products, the maximum sales promotion cost and the compensation income from a manufacturer.

$$\pi_{R\,(R_1,M_1)} = -t_1\beta_1 A - c_t\beta_1 A - w\alpha_1 D + p\alpha_1 D - c_{sp\max} + re_1\beta_1 Ar \ . \qquad (1)$$

The profit of a manufacturer taking (R_1, M_1), $\pi_{M\,(R_1,M_1)}$ consists of the disassembly and the inspection costs of used products, the remanufacturing cost of reusable parts, the disposal cost of un-recycled parts, the production cost of new

Table 1: Effects of behaviors strategies of GSC members on collection quantity of used products, product demand and the related costs of each member

Effects	Behavior strategies of members in GSC					
	(R_1, M_1)	(R_2, M_1)	(R_3, M_1)	(R_1, M_2)	(R_2, M_2)	(R_3, M_2)
Product demand	$\alpha_1 D$	$\alpha_2 D$	D	$\alpha_3 D$	$\alpha_3 D$	D
Collection quantity	$\beta_1 A$	$\beta_2 A$	A	$\beta_3 A$	$\beta_3 A$	A
Collection promotion cost	t_1	t_2	t_0	t_1	t_2	t_0
Sales promotion cost	c_{spmax}	c_{spmin}	-	c_{spmax}	c_{spmin}	-
Monitoring cost	c_s	c_s	c_s	-	-	-
Compensation cost	re_1	re_2	re_0	re_1	re_2	re_0
Penalty cost	-	-	c_p	-	-	-

Table 2: Payoff matrix between a retailer and a manufacturer in GSC as to the behavior strategy

Manufacturer / Retailer	Monitoring M_1	Non- monitoring M_2
Maximum Cooperation : R_1	$\left(\pi_{R\,(R_1.M_1)},\ \pi_{M\,(R_1.M_1)}\right)$	$\left(\pi_{R\,(R_1.M_2)},\ \pi_{M\,(R_1.M_2)}\right)$
Minimum Cooperation : R_2	$\left(\pi_{R\,(R_2.M_1)},\ \pi_{M\,(R_2.M_1)}\right)$	$\left(\pi_{R\,(R_2.M_2)},\ \pi_{M\,(R_2.M_2)}\right)$
Non-cooperation: R_3	$\left(\pi_{R\,(R_3.M_1)},\ \pi_{M\,(R_3.M_1)}\right)$	$\left(\pi_{R\,(R_3.M_2)},\ \pi_{M\,(R_3.M_2)}\right)$

parts, the production cost of products, the wholesale of products, the compensation cost to a retailer and the monitoring cost.

$$\pi_{M\,(R_1,M_1)} = -c_a\beta_1 A - c_r\beta_1 Ar - c_d\beta_1 A(1-r) - c_n(\alpha_1 D - \beta_1 Ar)$$
$$-c_m\alpha_1 D + w\alpha_1 D - re_1\beta_1 Ar - c_s\ . \quad (2)$$

- Profits of members in behavior strategy (R_1, M_2)

The profit of a retailer taking (R_1, M_2), $\pi_{R\,(R_1,M_2)}$ consists of the collection promotion cost of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of the products, the sales of products, the maximum sales promotion cost and the compensation income from a manufacturer.

$$\pi_{R\,(R_1,M_2)} = -t_1\beta_3 A - c_t\beta_3 A - w\alpha_3 D + p\alpha_3 D - c_{sp\max} + re_1\beta_3 Ar\ . \quad (3)$$

The profit of a manufacturer taking (R_1, M_2), $\pi_{M\,(R_1,M_2)}$ consists of the disassembly and the inspection costs of used products, the remanufacturing cost of reusable parts, the disposal cost of un-recycled parts, the production cost of new parts, the production cost of products, the wholesale of products and the compensation cost to a retailer.

$$\pi_{M\,(R_1.M_2)} = -c_a\beta_3 A - c_r\beta_3 Ar - c_d\beta_3 A(1-r) - c_n(\alpha_3 D - \beta_3 Ar)$$
$$-c_m\alpha_3 D + w\alpha_3 D - re_1\beta_3 Ar\ . \quad (4)$$

- Profits of members in behavior strategy (R_2, M_1)

The profit of a retailer taking (R_2, M_1), $\pi_{R\,(R_2,M_1)}$ consists of the collection promotion cost of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of the products, the sales of products, the minimum sales promotion cost and the compensation income from a manufacturer.

$$\pi_{R\,(R_2,M_1)} = -t_2\beta_2 A - c_t\beta_2 A - w\alpha_2 D + p\alpha_2 D - c_{sp\min} + re_2\beta_2 Ar\ . \quad (5)$$

The profit of a manufacturer taking (R_2, M_1), $\pi_{M\,(R_2,M_1)}$ consists of the disassembly and the inspection costs of used products, the remanufacturing cost of reusable parts, the disposal cost of un-recycled parts, the production cost of new parts, the production cost of products, the wholesale of products, the compensation cost to a retailer and the monitoring cost.

$$\pi_{M\,(R_2,M_1)} = -c_a\beta_2 A - c_r\beta_2 Ar - c_d\beta_2 A(1-r) - c_n(\alpha_2 D - \beta_2 Ar)$$
$$-c_m\alpha_2 D + w\alpha_2 D - re_2\beta_2 Ar - c_s\ . \quad (6)$$

- Profits of members in behavior strategy (R_2, M_2)

The profit of a retailer taking (R_2, M_2), $\pi_{R\,(R_2,M_2)}$ consists of the collection promotion cost of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of the products, the sales of products, the minimum sales promotion cost and the compensation income from a manufacturer.

$$\pi_{R\,(R_2,M_2)} = -t_2\beta_3 A - c_t\beta_3 A - w\alpha_3 D + p\alpha_3 D - c_{sp\min} + re_2\beta_3 Ar\ . \quad (7)$$

The profit of a manufacturer taking (R_2, M_2), $\pi_{M\,(R_2,M_2)}$ consists of the disassembly and the inspection costs of used products, the remanufacturing cost of reusable parts, the disposal cost of un-recycled parts, the production cost of new parts, the production cost of products, the wholesale of products and the compensation cost to a retailer.

$$\pi_{M\,(R_2.M_2)} = -c_a\beta_3 A - c_r\beta_3 Ar - c_d\beta_3 A(1-r) - c_n(\alpha_3 D - \beta_3 Ar)$$
$$-c_m\alpha_3 D + w\alpha_3 D - re_2\beta_3 Ar\ . \quad (8)$$

- Profits of members in behavior strategy (R_3, M_1)

The profit of a retailer taking (R_3, M_1) $\pi_{R\,(R_3,M_1)}$ consists of the collection promotion cost of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of products, the sales of products, the compensation income from a manufacturer and the penalty cost to a manufacturer.

$$\pi_{R\,(R_3,M_1)} = -t_0 A - c_t A - wD + pD + re_0 Ar - c_p \,. \tag{9}$$

The profit of a manufacturer taking (R_3, M_1) $\pi_{M\,(R_3,M_1)}$ consists of the disassembly and the inspection costs of used products, the remanufacturing cost of reusable parts, the disposal cost of un-recycled parts, the production cost of new parts, the production cost of products, the wholesale of products, the compensation cost to a retailer, the monitoring cost and the penalty income from a retailer.

$$\pi_{M\,(R_3,M_1)} = -c_a A - c_r Ar - c_d A(1-r) - c_n(D-Ar)$$
$$-c_m D + wD - re_0 Ar - c_s + c_p \,. \tag{10}$$

- Profits of members in behavior strategy (R_3, M_2)

The profit of a retailer taking (R_3, M_2) $\pi_{R\,(R_3,M_2)}$ consists of the collection promotion cost of used products from customers, the delivery cost of used products to a manufacturer, the procurement cost of products, the sales of products and the compensation income from a manufacturer.

$$\pi_{R\,(R_3,M_2)} = -t_0 A - c_t A - wD + pD + re_0 Ar \,. \tag{11}$$

The profit of a manufacturer taking (R_3, M_2) $\pi_{M\,(R_3,M_2)}$ consists of the disassembly and the inspection costs of used products, the remanufacturing cost of reusable parts, the disposal cost of un-recycled parts, the production cost of new parts, the production cost of products, the wholesale of products and the compensation to a retailer.

$$\pi_{M\,(R_3,M_2)} = -c_a A - c_r Ar - c_d A(1-r) - c_n(D-Ar)$$
$$-c_m D + wD - re_0 Ar \,. \tag{12}$$

5. ANALYZING BEHAVIOR STRATEGIES WITH THE APPLICATION OF AN EVOLUTIONARY GAME THEORY

In behavior strategies between a retailer and a manufacturer in a GSC, the retailer does not know which behavior strategy the manufacturer will take before the retailer takes the own behavior strategy and vice versa. Under the situation, the evolutionary game theory can shed more insights on the evolutionary mechanism of a GSC in this paper, and examine the trend of the evolutionary stability for the behavior strategy of GSC members from a time-based perspective.

This paper uses the evolutionary game theory to analyze behavior strategies of a retailer and a manufacturer in a GSC, and verify the evolutionary stabilities of a retailer and a manufacturer. The evolutionary game theory combines the static feature of an evolutionary stable strategy (ESS) with the dynamic nature of the replicator dynamics (Maynard-Smith, 1974; Taylor and Jonker, 1978; Friedman, D., 1999; Zhou and Deng, 2006; Zhu and Dou, 2007; Yu et al., 2009; Barari, et al., 2012). Concretely, using the replicator dynamics, behavior strategies of one member of the retailer' population and one member of the manufacturer' population are analyzed by time unit (Zhou and Deng, 2006; Zhu and Dou, 2007; Yu et al., 2009; Barari, et al., 2012)).

Suppose x_1 as the rate of the retailer's population taking R_1, x_2 as that taking behavior strategy R_2, and $x_3 = (1\text{-}x_1\text{-}x_2)$ as that taking behavior strategy R_3. Meanwhile, suppose y_1 as the rate of the manufacturer's population taking behavior strategy M_1, $y_2 = (1\text{-}y_1)$ as that taking behavior strategy M_2. Under the situation, the expected profits of one retailer and one manufacturer are formulated using Table 2.

The expected profit of one retailer taking behavior strategy R_1 under behavior strategies M_1 and M_2 is obtained as

$$E_{R1} = \pi_{R\,(R_1,M_1)} y_1 + \pi_{R\,(R_1,M_2)} y_2 = \pi_{R\,(R_1,M_1)} y_1 + \pi_{R\,(R_1,M_2)} (1-y_1) \,. \tag{13}$$

The expected profit of one retailer taking behavior strategy R_2 under behavior strategies M_1 and M_2 is obtained as

$$E_{R2} = \pi_{R\,(R_2,M_1)} y_1 + \pi_{R\,(R_2,M_2)} y_2 = \pi_{R\,(R_2,M_1)} y_1 + \pi_{R\,(R_2,M_2)} (1-y_1) \,. \tag{14}$$

The expected profit of one retailer taking behavior strategy R_3 under behavior strategies M_1 and M_2 is obtained as

$$E_{R3} = \pi_{R\,(R_3,M_1)} y_1 + \pi_{R\,(R_3,M_2)} y_2 = \pi_{R\,(R_3,M_1)} y_1 + \pi_{R\,(R_3,M_2)} (1-y_1) \,. \tag{15}$$

Using Eqs. (13)-(15) and Table 2, the expected profit of retailer's population considering proportions of the retailer's population, x_1, x_2 and $x_3= (1\text{-}x_1\text{-}x_2)$, using behavior strategy for three behavior strategies, R_1, R_2 and R_3 is obtained as

$$\overline{E_R} = x_1 E_{R1} + x_2 E_{R2} + (1-x_1-x_2) E_{R3} \,. \tag{16}$$

The expected profit of one manufacturer taking behavior strategy M_1 under behavior strategies R_1 and R_2 is obtained as

$$E_{M1} = x_1 \pi_{M\,(R_1,M_1)} + x_2 \pi_{M\,(R_2,M_1)} + (1-x_1-x_2) \pi_{M\,(R_3,M_1)} \,. \tag{17}$$

The expected profit of one manufacturer taking behavior strategy M_2 under behavior strategies R_1 and R_2 is obtained as

$$E_{M2} = x_1 \pi_{M\,(R_1,M_2)} + x_2 \pi_{M\,(R_2,M_2)} + (1-x_1-x_2) \pi_{M\,(R_3,M_2)} \,. \tag{18}$$

Using Eqs. (17) and (18) and Table 2, the expected profit of manufacturer's population considering proportions of the manufacturer's population, y and $(1\text{-}y)$, using behavior strategy for two behavior strategies, M_1 and M_2 is obtained as

$$\overline{E_M} = y_1 E_{M1} + (1-y_1) E_{M2} \,. \tag{19}$$

Using the values of E_{R1} and $\overline{E_R}$, the replicator dynamics equation of one retailer can be obtained as

$$\dot{x}_1 = \frac{dx_1}{dt} = x_1(E_{R1} - \overline{E_R}) . \tag{20}$$

Eq. (20) indicates the time variation of the behavior strategy R_1 in retailer's population.

Using the values of E_{R2} and $\overline{E_R}$, the replicator dynamics equation of one retailer can be obtained as

$$\dot{x}_2 = \frac{dx_2}{dt} = x_2(E_{R2} - \overline{E_R}) . \tag{21}$$

Eq. (21) indicates the time variation of the behavior strategy R_2 in retailer's population.

Using the values of E_{M1} and $\overline{E_M}$, the replicator dynamics equation of one manufacturer can be obtained as

$$\dot{y}_1 = \frac{dy_1}{dt} = y_1(E_{M_1} - \overline{E_M}) . \tag{22}$$

Eq. (22) indicates the time variation of the behavior strategy M_1 in retailer's population.

The stable state of the replicator dynamics equation is the equilibrium of the non-linear system (Zhou and Deng, 2006; Zhu and Dou, 2007; Yu et al., 2009; Barari, et al., 2012). When $\dot{x}_1 = 0$ in Eq. (20), $\dot{x}_2 = 0$ in Eq. (21) and $\dot{y}_1 = 0$ in Eq. (22), the following six equilibriums (strategy combinations) except $(x_1^{*1}, x_2^{*1}, y_1^{*1}) = (1,1,0)$ and $(x_1^{*1}, x_2^{*1}, y_1^{*1}) = (1,1,1)$ are obtained as

$$\begin{cases} (x_1^{*1}, x_2^{*1}, y_1^{*1}) = (1,0,0),\ (x_1^{*2}, x_2^{*2}, y_1^{*2}) = (1,0,1), \\ (x_1^{*3}, x_2^{*3}, y_1^{*3}) = (0,1,0),\ (x_1^{*4}, x_2^{*4}, y_1^{*4}) = (0,1,1), \\ (x_1^{*5}, x_2^{*5}, y_1^{*5}) = (0,0,0),\ (x_1^{*6}, x_2^{*6}, y_1^{*6}) = (0,0,1). \end{cases} \tag{23}$$

When an equilibrium of the replicator dynamics equation is an evolutionary equilibrium, which equals to the locally asymptotically, it is judged as the evolutionary stable strategy (ESS). We use the standard Jacobian Matrix (J) for differential equations in Eqs. (20)-(22) to evaluate the asymptotic stability of an equilibrium strategy combination $(x_1^{*k}, x_2^{*k}, y_1^{*k})$ $(k = 1,..,6)$ and obtain ESS (Friedman, 1999; Zhou and Deng, 2006; Zhu and Dou, 2007; Yu et al., 2009; Barari, et al., 2012). Six equilibriums in Eq. (23) are judged if they are ESS, using the following procedures.

【Step 1】 Using differential equations of Eqs. (20)-(22) in terms of time t, obtain the Jacobian Matrix J as

$$J = \begin{pmatrix} \partial\dot{x}_1/\partial x_1 & \partial\dot{x}_1/\partial x_2 & \partial\dot{x}_1/\partial y_1 \\ \partial\dot{x}_2/\partial x_1 & \partial\dot{x}_2/\partial x_2 & \partial\dot{x}_2/\partial y_1 \\ \partial\dot{y}_1/\partial x_1 & \partial\dot{y}_1/\partial x_2 & \partial\dot{y}_1/\partial y_1 \end{pmatrix}. \tag{24}$$

$$\frac{\partial\dot{x}_1}{\partial x_1} = (1-2x_1)\left\{ y_1\left(\pi_{R\,(R_1,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_1,M_2)} + \pi_{R\,(R_3,M_2)} \right) + \pi_{R\,(R_1,M_2)} - \pi_{R\,(R_3,M_2)} \right\}$$
$$- x_2\left\{ y_1\left(\pi_{R\,(R_2,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_2,M_2)} + \pi_{R\,(R_3,M_2)} \right) + \pi_{R\,(R_2,M_2)} - \pi_{R\,(R_3,M_2)} \right\} \tag{25}$$

$$\frac{\partial\dot{x}_1}{\partial x_2} = -x_1\left\{ y_1\left(\pi_{R\,(R_2,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_2,M_2)} + \pi_{R\,(R_3,M_2)} \right) + \pi_{R\,(R_2,M_2)} - \pi_{R\,(R_3,M_2)} \right\} \tag{26}$$

$$\frac{\partial\dot{x}_1}{\partial y_1} = x_1\left\{ (1-x_1)\left(\pi_{R\,(R_1,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_1,M_2)} + \pi_{R\,(R_3,M_2)} \right) \right.$$
$$\left. - x_2\left(\pi_{R\,(R_2,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_2,M_2)} + \pi_{R\,(R_3,M_2)} \right) \right\} \tag{27}$$

$$\frac{\partial\dot{x}_2}{\partial x_1} = -x_2\left\{ y_1\left(\pi_{R\,(R_1,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_1,M_2)} + \pi_{R\,(R_3,M_2)} \right) + \pi_{R\,(R_1,M_2)} - \pi_{R\,(R_3,M_2)} \right\} \tag{28}$$

$$\frac{\partial\dot{x}_2}{\partial x_2} = (1-2x_2)\left\{ y_1\left(\pi_{R\,(R_2,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_2,M_2)} + \pi_{R\,(R_3,M_2)} \right) + \pi_{R\,(R_2,M_2)} - \pi_{R\,(R_3,M_2)} \right\}$$
$$- x_1\left\{ y_1\left(\pi_{R\,(R_1,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_1,M_2)} + \pi_{R\,(R_3,M_2)} \right) + \pi_{R\,(R_1,M_2)} - \pi_{R\,(R_3,M_2)} \right\} \tag{29}$$

$$\frac{\partial\dot{x}_2}{\partial y_1} = x_2(1-x_1-x_2)\left\{ \pi_{R\,(R_1,M_1)} - \pi_{R\,(R_3,M_1)} - \pi_{R\,(R_1,M_2)} + \pi_{R\,(R_3,M_2)} \right\} \tag{30}$$

$$\frac{\partial\dot{y}_1}{\partial x_1} = y_1(1-y_1)\left\{ \pi_{M\,(R_1,M_1)} - \pi_{M\,(R_3,M_1)} - \pi_{M\,(R_1,M_2)} + \pi_{M\,(R_3,M_2)} \right\} \tag{31}$$

$$\frac{\partial\dot{y}_1}{\partial x_2} = y_1(1-y_1)\left\{ \pi_{M\,(R_2,M_1)} - \pi_{M\,(R_3,M_1)} - \pi_{M\,(R_2,M_2)} + \pi_{M\,(R_3,M_2)} \right\} \tag{32}$$

$$\frac{\partial\dot{y}_1}{\partial y_1} = (1-2y_1)\left\{ x_1\left(\pi_{M\,(R_1,M_1)} - \pi_{M\,(R_3,M_1)} - \pi_{M\,(R_1,M_2)} + \pi_{M\,(R_3,M_2)} \right) \right.$$
$$\left. + x_2\left(\pi_{M\,(R_2,M_1)} - \pi_{M\,(R_3,M_1)} - \pi_{M\,(R_2,M_2)} + \pi_{M\,(R_3,M_2)} \right) + \pi_{M\,(R_3,M_1)} - \pi_{M\,(R_3,M_2)} \right\} . \tag{33}$$

【Step 2】 Substitute equilibriums $(x_1^{*k}, x_2^{*k}, y_1^{*k})$ $(k = 1,..,6)$ in Eq. (23) into the Jacobian Matrix J in Eqs. (24)-(33).

【Step 3】 Find eigen values, λ_1^k, λ_2^k, λ_3^k $(k = 1,..,6)$ of the Jacobian Matrix J obtained in **【Step 2】**, using numerical calculation methods.

【Step 4】 By investigating that eigen values, λ_1^k, λ_2^k, λ_3^k $(k = 1,..,6)$ obtained in **【Step 3】**, are either positive or negative, it is judged whethere equilibrium (strategy combination) $(x_1^{*k}, x_2^{*k}, y_1^{*k})$ $(k = 1,..,6)$ of a retailer and a manufacturer are ESS or not.

If the following conitions regarding eigen values

$$\lambda_1^k < 0 \ \ \& \ \ \lambda_2^k < 0 \ \ \& \ \ \lambda_3^k < 0 \quad (k = 1,..,6) . \tag{34}$$

are satisfied, equilibrium (strategy combination) $(x_1^{*k}, x_2^{*k}, y_1^{*k})$ $(k = 1,..,6)$ is the asymptotically stable and judged as ESS. If Eq. (34) is unsatisfied, $(x_1^{*k}, x_2^{*k}, y_1^{*k})$ $(k = 1,..,6)$ is not the asymptotically stable and judged as ESS. If the following conitions regarding eigen values

$$\lambda_1^k = 0 \ \ \& \ \ \lambda_2^k = 0 \ \ \& \ \ \lambda_3^k = 0 \quad (k = 1,..,6) . \tag{35}$$

are satisfied, it is impossible to eveluate whether $(x_1^{*k}, x_2^{*k}, y_1^{*k})$ $(k = 1,..,6)$ is ESS or not.

6. NUMERICAL ANALYSIS

The analysis numerically investigates how parameters, related to the recycling promotion activity (RPA) in a GSC, affect the judgment of the evolutionary stable strategy (ESS) of behavior strategies of a retailer and a manufacturer in a GSC by using analysis of both the evolutionary game theory and the replicator dynamic equation in sections 4 and 5. This paper focuses on (i) compensation costs re_1, re_2 , (ii) collection promotion costs t_1, t_2 , (iii) sale promotion costs c_{spmax}, c_{spmin}, (iv) monitoring cost c_s, (v) penalty cost c_p as parameters related to RPA. The initial data sources (System parameters) of the numerical examples in a GSC are provided as follows:

D=100, $\alpha_1 > \alpha_2 > \alpha_3$, $\beta_1 > \beta_2 > \beta_3$,α_1=1.3, β_1=1.2, α_2=β_2=1.05, α_3=β_3=1.03, p=150, w=50, A=100, t_1=3, t_2=1,t_0=0, c_f=1, r=0.3, c_r=35, c_m=7, c_a=10, c_d=10, re_1=10, re_2=2, re_0=1, c_n=40, c_s=100, c_p=800, c_{spmax}=150, c_{spmin}=100.

This paper clarifies how each of parameters (i)-(v) related to RPA affects the evaluation of the ESS for the behavior strategies of a retailer and a manufacturer in a GSC. So, this paper conducts some sensibility analyses by changing each of parameters (i)-(v) one by one in the following ranges for the operation of the GSC where (i) $8 \le re_1 \le 16$, $1 \le re_2 \le 8$, (ii) $1.5 \le t_1 \le 4$, $0.3 \le t_2 \le 1.3$, (iii) $50 \le c_{sp\max} \le 180$, $5 \le c_{sp\min} \le 100$, (iv) $1 \le c_s \le 200$, (v) $10 \le c_p \le 1500$.

All data sources set used here are modifiable if needed.

- Effect of compensation costs re_1 and re_2 on ESS

Table 3 shows the effects of changes in compensation costs re_1 and re_2 on the judgment of ESS for behavior strategies of a retailer and a manufacturer in a GSC. As re_1 is higher, the behavior strategy (R_1, M_2) is judged as EES between both members. This result means that the retailer's population always tends to take R_1 and the manufacturer's population always tend to take M_2. From section 3 and Table 1, the more a retailer taking R_1 pays the maximum sales promotion cost c_{spmax}, the more compensation income from a manufacturer based on re_1 is. As re_2 is higher, the behavior strategy (R_2, M_2) is judged as EES between both members. The reason of this result is similar as that when re_1 is higher. The manufacturer can guess that the retailer's population tends to take either R_1 when re_1 is high or R_2 when re_2 is high regardless of the manufacturer's act. Therefore, the manufacturer's population tends to take M_2 since the monitoring of the retailer's act is unneeded. As either re_1 or re_2 is lower, neither (R_1, M_2) nor (R_2, M_2) is judged as ESS.

- Effect of collection promotion costs t_1 and t_2 on ESS

Table 4 shows the effects of changes in collection promotion costs t_1 and t_2 on the judgment of ESS for behavior strategies in a GSC. As t_1 is lower, the behavior strategy (R_1, M_2) is judged as EES between both members. From section 3 and Table 1, the higher rate of retailer paying collection promotion cost t_1 by taking R_1 is, the more used products can be collected and the more compensation income from a manufacturer based on re_1 is. As t_2 is lower, the behavior strategy (R_2, M_2) is judged as EES between both members. The reason of this result is similar as that when t_1 becomes lower.

Table 3: Effect of changes in compensation costs re_1 and re_2 on judgment of ESS for behavior strategies in a GSC

Behavior strategy	Compensation cost re_1				
	8	10	12	14	16
(R_1, M_2)	-	-	ESS	ESS	ESS
Behavior strategy	Compensation cost re_2				
	1	2	4	6	8
(R_2, M_2)	-	ESS	ESS	ESS	ESS

Table 4: Effect of changes in collection promotion costs t_1 and t_2 on judgment of ESS for behavior strategies in a GSC

Behavior strategy	Collection promotion cost t_1					
	1.5	2	2.5	3	3.5	4
(R_1, M_2)	ESS	ESS	ESS	-	-	-
Behavior strategy	Collection promotion cost t_2					
	0.3	0.5	0.7	0.9	1.1	1.3
(R_2, M_2)	ESS	ESS	ESS	ESS	-	-

Table 5: Effect of changes in maximum and minimum sales promotion costs c_{spmax} and c_{spmin} on judgment of ESS for behavior strategies in a GSC

Behavior strategy	Sales promotion cost c_{spmax}					
	50	80	100	120	150	180
(R_1, M_2)	ESS	ESS	ESS	ESS	-	-
Behavior strategy	Sales promotion cost c_{spmin}					
	5	10	30	50	80	100
(R_2, M_2)	ESS	ESS	ESS	ESS	ESS	ESS

The manufacturer can guess that the retailer's population tends to take either R_1 when t_1 is low or R_2 when t_2 is low regardless of the manufacturer's act. Therefore, the manufacturer's population tends to take M_2 since the monitoring of the retailer's act is unneeded. As either t_1 or t_2 is higher, neither (R_1, M_1) nor (R_2, M_2) is judged as ESS.

- Effects of maximum sales promotional costs c_{spmax} and minimum sales promotional cost c_{spmin} on ESS

Table 5 shows the effects of changes in the maximum sales promotion cost c_{spmax} and minimum sales promotional cost c_{spmin} on the judgment of ESS for behavior strategies in a GSC. As c_{spmax} is lower, the behavior strategy (R_1, M_2) is judged as EES between both members. From section 3 and Table 1, the higher rate of retailer paying c_{spmax} by taking R_1 is, the more product demand is based on a constant rate. As c_{spmax} is lower, R_1 results in the increase of the retailer's product sales. The manufacturer can guess that the retailer's population tends to take R_1 as c_{spmax} is lower regardless of the manufacturer's act. Therefore, the manufacturer's population tends to take M_2

since the monitoring of the retailer's act is unneeded.

Table 6: Effect of change in monitoring cost c_s on judgment of ESS for behavior strategies in a GSC

Behavior strategy	Monitoring cost c_s					
	1	30	50	100	150	200
(R_2, M_2)	ESS	ESS	ESS	ESS	ESS	ESS

Table 7: Effect of change in penalty cost c_p on judgment of ESS for behavior strategies in a GSC

Behavior strategy	Penalty cost c_p					
	10	300	400	800	1000	1500
(R_2, M_2)	ESS	ESS	ESS	ESS	ESS	ESS

As c_{spmax} is higher, (R_1, M_2) is not judged as ESS. When $5 \le c_{sp\min} \le 100$, the behavior strategy (R_2, M_2) is always judged as EES between both members. The reason of this result is similar as that when c_{spmax} is lower.

- Effect of monitoring cost c_s on ESS

Table 6 shows the effect of change in monitoring cost c_s on the judgment of ESS for behavior strategies in a GSC. When $1 \le c_s \le 200$, the behavior strategy (R_2, M_2) is judged as EES between both members. From section 3 and Table 1, R_2 can bring the increase of product sales to a retailer and the compensation income to a retailer from a manufacturer based on t_2. Therefore, the manufacturer's population tends to take M_2 since the monitoring of the retailer's act is unneeded.

- Effect of penalty cost c_p on ESS

Table 7 shows the effect of change in penalty cost c_p on the judgment of ESS for behavior strategies in a GSC. When $10 \le c_p \le 1500$, the behavior strategy (R_2, M_2) is judged as EES between both members. From section 3 and Table 1, R_2 has no penalty cost from a retailer to a manufacturer. Also, R_2 can bring the increase of product sales to a retailer and the compensation income to a retailer from a manufacturer based on t_2. Therefore, the manufacturer's population tends to take M_2 since the monitoring of the retailer's act is unneeded.

7. CONCLUSIONS

This paper discussed a green supply chain (GSC) with one retailer and one manufacturer and verified the behavior strategies of GSC members which might change over time as to changes of parameters regarding recycling promotion activity (RPA) with collection of used products and sales promotion of the products reusing the recycled parts. A retailer took three behavior strategies: maximum cooperation, minimum cooperation and non-cooperation in RPA. A manufacturer took two behavior strategies: monitoring and non-monitoring of the retailer's behavior. Evolutionary game theory was adopted to clarify analytically evolutionary outcomes by a change in each behavior of GSC members over time. The evolutionary stable strategies (ESSs) for GSC members' behaviors were derived by using the replicator dynamics. The analysis numerically illustrated how (i) compensation cost, (ii) collection promotion cost, (iii) sales promotion cost, (iv) monitoring cost, (v) penalty cost affected the judgment of ESSs of behaviors of GSC members. From the research outcomes in this paper, the contribution of this paper was to provide the optimal setting of system parameters and its practices to construct and operate a GSC and the informative motivations for researchers and policymakers to manage a GSC.

As future researches, it will be necessary to extend the following topics into the evolutional stable analysis of GSC in this paper: (i) Index of RPA of CO_2 emission and so on, (ii) Effect of customers' green image from RPA, (iii) the optimal operations for a GSC with the evolutionary dynamics.

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REFERENCES

Barari, S., Agarwal, G., Zhang, W.J.(Chiris), Mahanty, B. and Tiwari, M. K. (2012) A Decision Framework for the Analysis of Green Supply Chain Contracts: An Evolutionary Game Approach, *Expert Systems with Applications*, **39**, 2965-2976.

Friedman, D. (1999) Evolutionary Games in Economics, *Econometrica*, **59**, 637-666.

Maynard-Smith, J. (1974) The Theory of Games and the Evolution of Animal Conflicts, *Journal of Theoretical Biology*, **47**, 209-221.

Taylor, P. J. and Jonker, L. (1978) Evolutionary Stable Strategies and Game Dynamics, *Mathematical Bioscience*, **40**, 145-156.

Yu, H., Zeng, A. Z. and Zhao, L. (2009) Analyzing the Evolutionary Stability of the Vendor-Managed Inventory Supply Chains, *Computers & Industrial Engineering*, **56**, 274-282.

Zhou, M. and Deng, F. Q. (2006) *Evolutionary Dynamics of an Asymmetric Game Between a Supplier and a Retailer. In: Jiao, L., Wang, L., Gao, X., Liu, J. and Wu, F.(eds.), Advances in Natural Computation: Lecture Notes in Computer Science,* ***4222****, Springer-Verlag, Heidelberg, Berlin,* 466-469.

Zhu, Q. H. and DOU, Y. J. (2007) Evolutionary Game Model between Governments and Core Enterprises in Greening Supply Chains, *Systems Engineering-Theory & Practice*, **27**, 85-89.