Modeling Location Diffusion, Resource Allocation and Rebalance in Car Sharing Industry

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Abstract. Economic, environmental and social impacts have increased popularity of car sharing program. More firms consider entering this market to satisfy rising demand from public. In general, a car sharing company faces three practical problems: 1). Station Location Selection; 2). Station Size/Capacity; 3). Strategies for imbalance of vehicles distribution for each station. Although literature presents that such questions have been studied in the past, almost all of them use optimization models to address these questions and most of time those optimization models are difficult to be implemented in practice. In this study, we build two novel models to tackle above questions.

Keywords: Location Diffusion, Resource Allocation, Rental Car Redistribution, Car Sharing

1. INTRODUCTION

Car-sharing programs start from 1994 in United States. Programs offer car rental for consumers in usual shorter rental time period compared with traditional rental car companies. Most of programs charge users by hours/half hour or actual driven miles. They are attractive to consumers who make occasional use or short time period use of vehicles. The programs usually select stations in particular regions and deploy vehicles in specified stations for consumers' use. Consumers can book any available vehicle in the station, pick up the vehicle at the scheduled time and return the vehicle either the original station or other companies' permitted stations. Since programs are perfectly designed for short time period users, a reserved parking space, prepaid fuel and insurance are all included in the hourly rental fee. It helps consumers financially not to pay unused time and any other additional fees.

In recent years, along with recognition of car-sharing on environment and society, car-sharing has emerged as an important alternative as public transportation choices, the market of car-sharing expands extremely fast. Barth et al. (2002) depict all scenarios for those who could be favor using car-sharing programs. In terms of the report from Shaheen et al.(2013), Membership of car-sharing has grown exponentially from 16,000 in 2002 to more than 1 million at the beginning of 2013. And based on the issue of 2014 Fall from Transportation Sustainability Research Center-University of California, Berkeley, the vehicles used in car-sharing programs in United States have grown from 696 in 2002 to 19,115 in 2014. And according to Navigant Consulting, the total global revenue for car-sharing industry reached $1 billion in 2013 and will continue growing to $6.2 billion by 2020.

Research has been active in studying these problems. However, almost all of these studies fall into using optimization models to solve the problems. In general, optimization models can perfectly answer problems; however these models are difficulty to be generalized and very question oriented, and validating models require collecting large data. In practice, these high standards may not be achieved by all car-sharing companies, therefore a relative simple and easy to be implemented model may serve a better role to provide managerial suggestions and while keeping less input. Our research takes a few steps in proposing a brand new integral mathematical framework to address location diffusion, resource allocation and redistribution problems. We study these tradition problems from a new perspective and our models settings are different traditional optimization models. And through our computational study, we showed our models require less information and achieve ideal forecasts.

The paper organizes the remaining structure as follows. In section 2, we will review the literature on all car-sharing business decisions from methodology perspective. Section 3 introduces the models on market diffusion, vehicle allocations and redistribution; after introducing each model, we solve the models and give solutions of each model. Section 4 summarizes final resultsand gives future research directions.

2.LITERATURE REVIEW

Car-sharing business models include the round-trip, one-way and free-float three types of rental contracts. Round-trip rental means uses have to return vehicles to original pick-up places; One-way allows users to return vehicles to any other permitted places; Free floats rental just emerge recently, which doesn't have any fixed spots for users to pick up vehicles, vehicles could be parked in a certain range area within walkable distance; users are not required to return the vehicles to a specific spot, it could return the vehicles in any space in an area. The research on round-trip type rental is no longer active due to less flexibility provided. The research on free-float is just emerging and presenting a promising research direction. The free float car sharing system has been studied recently. Weikl et al.(2012) developed two decision support systems(simulation) for user-based relocation strategy and operator-based relocation strategy separately. The most recent study on car-sharing system is to use data analytics. Wagner et al. (2015) present a decision support system to derive indicators for the attractiveness of certain areas based on points of interest in their vicinity. Authors employ data mining techniques to develop the whole system and use real city of Berlin data to validate proposed model.

Currently most of work concentrates on one-way rental, and our research is also falling into this category. Compared with traditional operational scheme, we think differently. We think car-sharing decision is driven by market demands, market demands should help companies determine where could be the next best interests to offer services. Conventional operational models usually opt-out marketing considerations, and put more focuses on reducing potential costs. In our view, cost is really secondary and market demands should be firstly emphasized. If and only if a company has ideas to open business in a certain area, operational models then can step in to play their roles.

3.MODEL

We analyze the location problem based on market diffusion to two phases: phase 1, determine the best location, phase 2, effective deployment and redistribution.We use equation theory to model the firm's decisions.

3.1 Location modeling

We use the following notations:

1: length of time (the length of time can be 1 year/month/week/day/hour, etc.).

t : discrete time point, $t = 0, 1, 2, \cdots$.

$F(t)$: The number of the full member at time t.

$P(t)$: The number of the potential member at time t.

N: The total population of a certain region, $N = F(t) + P(t)$, constant.

β: The number of potential members have been contacted by full member during aunit time interval.

γ : The dropout rate of the full member.

b_j : The growth rate of population.

d_j : The reduction rate of population.

p_j : The net growth rate of population.

$\frac{A}{m+1}$: length of each subinterval.

$N_j(t)$: total population in age interval $[\frac{jA}{m+1}, \frac{(j+1)A}{m+1})$ at time t.

$P_j(t)$: number of the potential member in age interval $[\frac{jA}{m+1}, \frac{(j+1)A}{m+1})$ at time t.

$F_j(t)$: number of the full member in age interval $[\frac{jA}{m+1}, \frac{(j+1)A}{m+1})$ at time t.

$V_j(t)$: number from potential member to full member in age interval $[\frac{jA}{m+1}, \frac{(j+1)A}{m+1})$ at time t.

v_{j0} : transfer rate from potential member to full member in each age interval.

v_{j1} : transfer rate from full member to potential member in each age interval.

θ_j : natural transfer rate between different age interval.

μ_j : turnover rate of each age group besides the conversion of membership.

λ_j : interval coefficient, $\beta_k \lambda_j$ means contact number from full member in age interval $[\frac{kA}{m+1}, \frac{(k+1)A}{m+1})$ to potential member in age interval $[\frac{jA}{m+1}, \frac{(j+1)A}{m+1})$ during a unit time under the circumstance of population mobility.

γ_j : drop out rate of the full member in age interval $[\frac{jA}{m+1}, \frac{(j+1)A}{m+1})$.

R_0 : reproductive number, indicate the average number of people affected by a fullmember over their membership period.

Suppose the influence brought by the population mobility and age structure can be ignored (*P-F-P* model is built based on a fixed demographic process). Consider the following discrete-time difference system:

$$\begin{cases} P(t+1) = P(t) - \frac{\beta}{N} P(t) \cdot F(t) + \gamma F(t) \\ F(t+1) = F(t) + \frac{\beta}{N} P(t) \cdot F(t) - \gamma F(t) \end{cases} \quad (1)$$

It is assumed that the parameters are positive, $0 < \beta < 1$, $0 < |\gamma| < 1$. It follows that $P(t) + F(t) = N$ for all time, the total population size remains constant.

From $P(0)+F(0)=N$, we can obtain

$$\begin{cases} P(t)\ge 0 \\ F(t)\ge 0 \\ P(t)+F(t)=N \end{cases} \qquad (2)$$

From system (2), we easily know the solution of system (1) exists, non-negative and unique.

Using the substitution $P(t)=N-F(t)$, the $F-$equation in system(1) becomes

$$F(t+1)=F(t)(1+\beta-\gamma-\frac{\beta}{N}F(t)) \qquad (3)$$

where $0\le F(t)\le N$.

If the total population assumes a positive steady state, then the ratio R_0 determines the asymptotic behavior of system (1), we will call this ratio as Determination Ratio. Here, we introduce R_0 and use it to predict the successful influence of the full member modeled in (1). In constant environments

$$R_0=\frac{\beta}{\gamma}$$

R_0 is the average number of people affected by a full member over their membership period. In case 1, we use the same R_0 to prove that $R_o\le 1$ implies the number of people from potential member to full member to zero and $R_o>1$ implies the number of people from potential member to full member persistence increase.

Theorem 1. (i) If $R_0\le 1$, then solution to system (1) approach the diffusion-free equilibrium $\lim_{t\to\infty}F(t)=0$, $\lim_{t\to\infty}P(t)=N$. (ii) If $R_0>1$, then solutions to system (1) persistence and approach unique positive diffusion equilibrium

$$\lim_{t\to\infty}F(t)=\tilde{F}>0, \qquad \lim_{t\to\infty}P(t)=\tilde{P}>0.$$

Proof. Denote therightside of $F(t+1)$ in (1) by $h(F)$.

$$h(F)=F+\frac{\beta}{N}P\cdot F-\gamma F$$

Note that

$$h'(F)=1+\frac{\beta}{N}P-\gamma$$

Since $0<\gamma<1$ for $F\in[0,N]$, it follows that $h'(F)>0$ for $F\in[0,N]$.

For case (i), where $R_o\le 1$, $h(0)=0$ and $0<h'(0)<1$. Since $h'(F)<1$ or $h(F)<F$ for $F\in[0,N]$. It follows that $\{F(t)\}$ is a strictly decreasing sequence bounded below by zero and must approach a fixed point of h on $[0,N]$. The only fixed point of h on $[0,N]$ is 0, hence, $\lim_{t\to\infty}F(t)=0$.

For case (ii), where $R_o>1$, it shown that there exists a unique $\tilde{F}>0$ such that $h(\tilde{F})=\tilde{F}$, $h(F)>F$ for $F\in(0,\tilde{F})$ and $h(F)<F$ for $F\in(\tilde{F},N)$. In this case, $h(0)=0$, $h(N)<N$ and $0<h'(0)<1$. Thus, there exists at least one fixed point $\tilde{F}>0$, $h(\tilde{F})=\tilde{F}$. Let $\tilde{F}$ be the smallest positive fixed point, then $h(F)>F$ for $F\in(0,\tilde{F})$. It follows that $h'(\tilde{F})<1$. Since $h'(F)<h'(\tilde{F})<1$ for $F\in(\tilde{F},N]$. Integration of the last inequality over the interval $[\tilde{F},F]$ shows that $h(F)<F$ for $F>\tilde{F}$. Thus, h has a unique positive fixed point $\tilde{F}$. The proof of $\tilde{P}$ is analogous.

Specially, when $\beta=N\gamma$, $h(F)=F+\beta$. It follows that $\{F(t)\}$ is a strictly increasing sequence associated with the increasing of β and $\sup_{\beta+\gamma N-F(t)} F(t)\to N$.

This completes the proof. By using the conclusion of theorem 1, we can easily solve the location problem.

3.2 Deployment and redistribution modeling

We use the following notations.

Z : time interval of two continuous replenishments. Z is a random variable which satisfies independent identically distributed, its value range is $[z_1,z_2]$.

$D(t)$: demand for a product in a certain business outlet at time t . $D(t)$ has the following form:

$$D(t)=\begin{cases} r+\alpha I(t), & I(t)\ge 0 \\ r, & I(t)<0 \end{cases}$$

$I(t)$: inventory level in a certain business outlet at time t .

r : basic demand in unit time, $r>0$.

α : coefficient of shortage influences rental rates, $0\le\alpha\le 1$.

$D'(t)$: deferred supply rate.

β : coefficient of supply shortage influences deferred supply rate, $0\le\beta\le 1$.

$S(t)$: product demand when shortage occurs at time t .

t_s : total time from the highest inventory level to zero.

Based on the notations ,the inventory level $I(t)$ with respect to time t can be described by the following differential equation:

$$\frac{dI(t)}{dt}=\begin{cases} -r-\alpha I(t), & 0\le t\le t_s \\ -r, & t_s\le t\le T \end{cases} \qquad (4)$$

Where T is the length of the replenishment cycle. From $I(t_s)=0$, we have

$$I(t)=\begin{cases}\frac{r}{\alpha}(e^{\alpha(t_s-t)}-1), & 0\le t\le t_s\\ r(t_s-t), & t_s\le t\le T\end{cases} \quad (5)$$

$$I(0)=\frac{r}{\alpha}(e^{\alpha t_s}-1) \quad (6)$$

(i). Expected inventory in per cycle

$$\begin{aligned}\bar{I}&=\int_{z_1}^{t_s}\int_0^z\frac{r}{\alpha}(e^{\alpha(t_s-t)}-1)f(z)dtdz\\&+\int_{t_s}^{z_2}\int_0^{t_s}\frac{r}{\alpha}(e^{\alpha(t_s-t)}-1)f(z)dtdz\\&=\int_{z_1}^{t_s}[\frac{r(e^{\alpha t_s}-e^{\alpha(t_s-z)})}{\alpha^2}-\frac{rz}{\alpha}]f(z)dz\\&+\int_{t_s}^{z_2}[\frac{r(e^{\alpha t_s}-1)}{\alpha^2}-\frac{rt_s}{\alpha}]f(z)dz\end{aligned} \quad (7)$$

Eq. (7) represents the total expected inventory in per cycle, where the first term represents the expected inventory when out of stock did not occur; the second term represents the expected inventory when out of stock occurred.

(ii). Expected deferred supply and lease loss in per cycle

When shortage occurs, there is a linear relationship between deferred supply speed and shortage, thus, $S(t)$ satisfies the following equation:

$$\begin{cases}\frac{dS(t)}{dt}=r-\beta S(t), & t_s\le t\le T\\ S(t_s)=0.\end{cases} \quad (8)$$

The solution of Eq. (8) is

$$S(t)=\frac{r}{\beta}(1-e^{\beta(t_s-t)}),\quad t_s\le t\le T \quad (9)$$

Since the total expected shortage in one cycle is the sum of expected deferred supply and expected lease capacity, while the expected deferred supply of each cycle is

$$\begin{aligned}\bar{B}&=\int_{t_s}^{z_2}S(z)f(z)dz\\&=\int_{t_s}^{z_2}\frac{r}{\beta}(1-e^{\beta(t_s-z)})f(z)dz\end{aligned} \quad (10)$$

Thus, the expected lease loss of each cycle is

$$\begin{aligned}\bar{L}&=\int_{t_s}^{z_2}r(z-t_s)f(z)dz-\int_{t_s}^{z_2}S(z)f(z)dz\\&=\int_{t_s}^{z_2}[r(z-t_s)-\frac{r}{\beta}(1-e^{\beta(t_s-z)})]f(z)dz\end{aligned} \quad (11)$$

In Eq. (11), the first term represents the total expected shortage when out of stock occurred; the second term represents the expected deferred supply when out of stock occurred.

(iii). Expected replenishment in per cycle

Since the replenishment policy of every cycle is to replenish to the expected inventory or initial inventory, thus, the expected replenishment in one cycle is the sum of expected lease capacity and expected deferred supply, that is

$$\begin{aligned}\bar{Q}&=\int_{z_1}^{t_s}[I(0)-I(z)]f(z)dz\\&+\int_{t_s}^{z_2}[I(0)+S(z)]f(z)dz\\&=\int_{z_1}^{t_s}\frac{r(e^{\alpha t_s}-e^{\alpha(t_s-z)})}{\alpha}f(z)dz\\&+\int_{t_s}^{z_2}[\frac{r(e^{\alpha t_s}-1)}{\alpha}+\frac{r}{\beta}(1-e^{\beta(t_s-z)})]f(z)dz\end{aligned} \quad （12）$$

CONCLUSIONS AND FUTURE WORK

Car-Sharing industry grows rapidly in the past decades, the popularity of car-sharing programs not only help consumers financially, but also create tremendous social benefits for public and environment. As a fast growing business, companies constantly seek business expand strategies and react to growing demands from public.

In this paper, we develop novel models to help car-sharing companies make decisions on new station locations, station capacity/size and vehicles relocation. The future work in car-sharing study could be extended inrelocation problems between regions under free float scenario. It would be very interesting to see how decisions would be changed if competitions have been taken into account.

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