Maintenance Service Contracts for Repairable Product Involving Cooperative Relationship between OEM and Agent

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Abstract. Common, maintenance service contracts typically have two parties an Original Equipment Manufacturer/OEM (or an agent) and a customer. In many situations where an external agent has capability to provide corrective maintenance and preventive maintenance such as Mercedes Benz, the OEM and the agent would be competitive or cooperative in offering the customer with service contract options. This paper deals with a maintenance service contract for a repairable product (such as dump-trucks, excavators) with the involvement of the cooperative relationship between OEM and agent as a service-provider (SP). Game theory approach is used to model the interaction among OEM, agent and customer as the owner of the product. Under semi-cooperative games, the optimal sale price for the SP and the optimal maintenance cost or repair cost for the agent are obtained by maximizing their profits. The satisfaction of the customer is also maximized by being able to choose one of the suggested options from the SP and the agent, based on the risk parameter.

Keywords: Maintenance, Warranty, Three-level service contract, Semi-cooperative game.

1. INTRODUCTION

As loading and transporting mining material are major activity in mining sites, we focus our study to the heavy equipments involving in those activity such as a dump truck. As the equipments deteriorate with age and usage, an economical way to carry out maintenance is to outsource the maintenance works to an external agent or to an Original Equipment Manufacturer/OEM due to its complexity and its expensive cost in maintaining the equipments. Preventive maintenance (PM) actions are done to prevent for excessive degradation (whether it is an age based or condition based maintenance), while corrective maintenance (CM) is performed to restore the failed equipment to the operational state.

In order to get the maximum profit the owner have to manage the equipments maintenance cost. On the other hand, the agent's decision problem or the OEM is to determine the price of each option offered that maximises its profit too.

Maintenance service contracts involving repairable items have received attention in the literature (see [1], [2], [3]). Further more, [4], [5], [6] and [7] has purposed an incentives to motivate the the agent to increase the equipment's performance beyond the target but they have not considered the PM level and number of PM in doing the maintenance activity during the contract.

In this paper, we assume that the OEM and the agent cooperate and act as an integrated service-provider and propose a two dimensional service contract where the service-provider proposed imperfect PM in order to reduce the number of failure during the contract coverage. The paper is organised as follows. In section 2 we give model formulation for the service contract studied. Sections 3 and 4 deal with model analysis to obtain the optimal price structure for the service-provider and the optimal service option for the customer as the owner. Finally, we conclude with topics for further research in Section 5.

2. MODEL FORMULATION

2.1 Warranty Policy

We consider that each dump truck purchased is covered by a two-dimensional warranty, and the warranty also covers PM to provide more protection to the buyer. All failures under warranty are rectified at no cost to the buyer. Hence, after the warranty ends, the responsibility to do maintenance (CM and PM actions) shifts to the buyer (or the owner). The warranty coverage is characterized by a rectangle region $\Omega_W = [0,W) \times [0,U)$ where *W* and *U* are the time, and the usage limits. For a given usage rate (*y*) of a dump truck, the warranty ceases at $W_y = W$ for $y \le U/W$, or $W_y = U/y$, for $y > U/W$ (See Fig.1). The decision problem for the OEM is to determine the optimal PM degree according to various usage pattern and the mining operational condition that minimizes the expected warranty cost.

2.2 Maintenance Service Contract

We consider a two dimensional maintenance contract where the contract has two limits (or parameters) representing age and usage limits (e.g. the maximum coverage for L (e.g. 1 year) or K (e.g. 100.000 km). Hence the service contract is characterised by a rectangle region Ω_S (see Fig.1). For $y \le \gamma (= U/W)$ the region is given by $[(W, W+L) \times (U_y, U_y + K)]$ and $[(W_y, W_y + L) \times (U, U+K)]$ for $y > \gamma$ where $W_y = U/y$ and $U_y = yW$.

The service contract offered just before the warranty ends are considered as follows. Here we assume the OEM and the agent cooperate together as a service-provider (SP) in offering service contract. For a fixed price of service contract P_G, the SP agrees to perform both PM and CM (full coverage) for a period of time, L or usage K, whichever occurs first. The contract starts at the end of warranty, W_y, here the SP assures a minimum down time (repair time and waiting time) for each failure as stated in the contract. As the maintenance service is full coverage (PM and CM), then a penalty cost incurs the SP if the actual down time falls above the target. But if it falls below the target, the SP will earn an incentive. If the down time for each failure over the contract be $D(t)$ is more than the down time target ζ, then the SP should pay a penalty cost. The amount of the penalty cost is proportional to $\Delta = D(t) - \zeta$. The penalty cost, $\mathcal{C}_{\mathcal{P}}$ is viewed as a penalty given by the SP. The SP earns some incentives C_I if $\Delta < 0$.

The decision problem for the SP is to determine the optimal price structure (i.e. the price of service contract and the optimal PM degree according to various usage pattern and the mining operational condition that maximizes the expected profit.

2.3 Failure modelling

2.3.1 Approaches to modelling failures

We use the one dimensional approach developed in [9] to model product failures. Let *Y* be the usage rate for a given truck. It is assumed that Y varies across the trucks but it is constant for a given truck. For $Y = y$, the conditional hazard function for the time to first failure is given by $r_y(t)$ which is a non-decreasing function of t (the age of the truck) and y. We consider that usage rate of the truck and a land contour of a mining area where the truck is operated may strongly affect the degradation of the truck. To incorporate the effect of usage rate and the operating condition on degradation of the truck, we use the accelerated failure time (AFT) model as in [8].

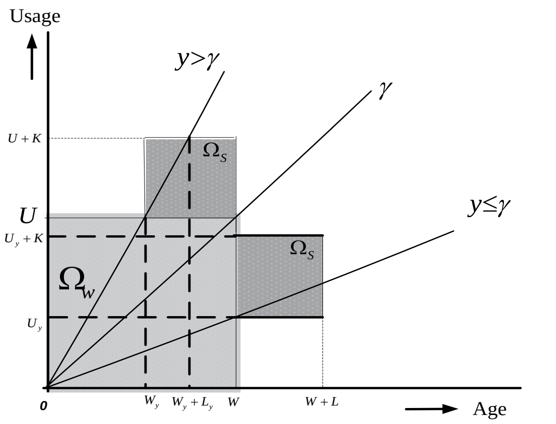


Fig 1. Warranty region Ω_W and service contract Region Ω_S for $y \le \gamma$ and $y > \gamma$.

If the distribution function for T_0 is given by $F_0(T, \alpha_0)$, where α_0 is the scale parameter, then the distribution function for T_y is the same as that for T_0 but with a scale parameter given by

$$\alpha_y = (y_0/y)^\rho \alpha_0 \tag{1}$$

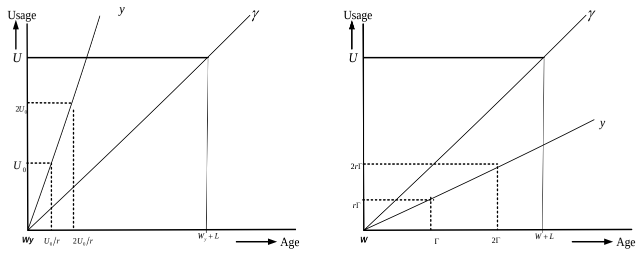
with $\rho \ge 1$ where ρ is a parameter representing the operating condition of a truck. Hence, we have $F(t,\alpha_y) = F_0((y_0/y)^\rho t, \alpha_y)$. The hazard and the cumulative hazard functions associated with $F(t, \alpha_y)$ are given by $r_y(t) = f(t,\alpha_y)/(1-F(t,\alpha_y))$ and $R_y(t) = \int_0^t r_y(x)dx$ respectively where $f(t,\alpha_y)$ is the associated density function.

Preventive Maintenance Policy:

We consider that for a given $Y = y$, PM done by the OEM during the warranty period and and the SP after warranty expired are an imperfect repair. The PM policy for a given y, is characterised by single parameter $\tau_y[\nu_y]$ during Ω_W [Ω_S]. The equipment is periodically maintained at $k.\tau_y$ [$l.\nu_y$]. Any failure occurring between pm is minimally repaired (See Fig. 2). Note $(k+1)\tau_y = W$ [$(\ell+1)\nu_y = L$] where $k[\ell]$ is an integer value.

2.3.2 Modelling of PM effect

For a given usage rate y, the effect of imperfect PM actions on the intensity function is given by $r(t_j) = r(t_{j-1}) - \delta_j$ with $0 \le \delta_j \le r(t_{j-1}) - \sum_{i=0}^{j} \delta_i$, δ_j denotes the reduction of the intensity function after $j^{th}, j \ge 1$, PM action. If the PM action is done at $j^{th}, j \ge 1$ the intensity function is reduced by δ_j , then for $t_j \le t < t_{j+1}$ the intensity function is given by $r_j(t) = r(t) - \sum_{i=0}^{j} \delta_i$.



(a). PM region for $y > \gamma$ (b). PM region for $y \le \gamma$

Fig 2. The two dimensional PM region

with $\delta_0 = 0$. For simplicity we assume that for each PM action $\delta_j = \delta_{j+1} = \delta$ then $r_j(t) = r(t) - j\delta$ (See Fig. 3). If any failure occurring between pm is minimally repaired, then the expected total number of minimal repairs in $([t_{j-1}, t_j), 1 \le j \le k+1)$ is given by

$$N = \sum_{j=1}^{k+1} \int_{t_{j-1}}^{t_j} r_{j-1}(t')dt' = R(W) - \sum_{j=1}^{k} (W - j\tau)\delta_j .$$

For $t_j - t_{j-1} = \tau_y$ then the expected number of minimal repairs in [0,W) is defined as

$$N(W) = N(k,\tau_y) = R(W) - \sum_{j=1}^{k} \left[(W - j\tau_y)\right]\left[r(j\tau_y) - r((j-1)\tau_y)\right] \tag{2}$$

where $R_y(W) = \int_0^W r_y(t)dt$.

And after the warranty ends, the expected number of minimal repairs in [W, W+L), with $1 \le m \le \ell + 1$ is given by

$$N(L) = N(\ell,\tau_y) = R_k(W, W+L) - \sum_{m=1}^{l} (L - jv)\delta_m,$$
$$= R_k(W, W+L) - \sum_{m=1}^{\ell}\left[\left(L - iv_y\right)\right]\left[r\left(iv_y\right) - r\left((i-1)v_y\right)\right] \tag{3}$$

where $R_k(W, W+L) = \int_W^L r_y(t)dt$.

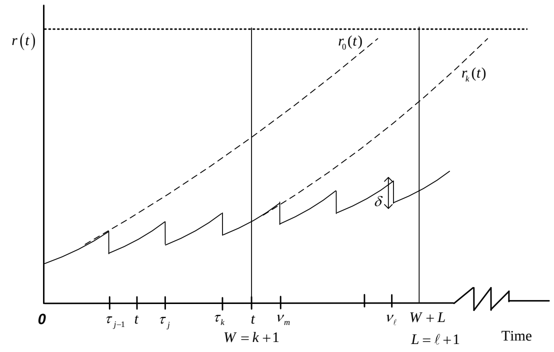


Fig 3. Failure rate function for $Y = y$

Notations:

W,U	:Warranty time, and usage limits
X_i	:Downtime caused by the *i*-th failure and waiting time
ζ	:Total repair time allowed
D(t)	:Total downtime in (*0,t*]
F(t)	:Distribution function of downtime
Υ, L	:Revenue , maintenance contract leght
Y	:Usage rate
C_r	:Repair cost done by OEM
C_m	:Repair cost done by SP
C_0	:Preventive maintenance cost per PM
C_v	:Additional cost for level PM created per PM
$\mathcal{C}_{\mathcal{P}}$	:Penalty cost per unit of time
$\phi_y(O.)$	:Profit owner
$\pi_y(O.)$	:Profit OEM
C_b	:The product cost over the contract period
P_0	:PM cost done in-house over the contract period
$F(t,\alpha_y)$	:Conditional failure distribution for a given usage rate *y*
$r_y(t)$, $R_y(t)$	:Hazard, and Cumulative hazard functions associated with $F(t,\alpha_y)$

3. MODEL ANALYSIS

We consider a situation where the SP is the only provider of maintenance services and hence the service provider has more bargaining power in the contract negotiation. As a result, we can view the SP as the leader and the owner as the follower.

Furthermore, we consider the owner condition i.e risk as given by:

$$U(\omega) = \begin{cases} \dfrac{1-e^{-r\pi}}{r}, r > 0 \text{ risk averse} \\ \pi, r = 0 \text{ neutral} \end{cases} \tag{4}$$

3.1 Owner's Decision Problem

We obtain the owner's expected profit for two options and each needs to consider two cases –i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

For case (i),

Option O_1 **:** the expected profit is given by

$$E\left[\phi_{1y}\right] = \Upsilon\left\{\tau - N(\ell, v_y)\int_0^\infty yg(y)dy\right\} - C_s N(\ell, v_y) - P_0 - C_b \tag{5}$$

With the expected utility function,

$$E\left[U_y(O_1)\right] = \frac{1}{r}\left(1 - e^{-r\left(\Upsilon\tau - P_0 - C_b\right)} e^{-N(\ell,v_y)\left(2 - \frac{\lambda}{\lambda - \Upsilon r} - e^{rC_s}\right)}\right) \tag{6}$$

where $N(\ell, v_y) = R_{1y}(\tau)$.

Option O_2 **:** the expected profit profit of the owner is given by

$$E\left[Profit\right] = E[uptime\ revenue] + E[Pinalty\ cost]$$
$$-E[Incentive\ cost] - E[Service\ Contract\ cost] - E[Purchase\ cost]$$

$$E\left[\phi_y\right] = \Upsilon\left\{\tau - N(\ell, v_y)\int_0^\infty yg(y)dy\right\} + EP(L) - E\left[\text{Incentive}\right] - P_G - C_b \tag{7}$$

Where $EP(L)$ is the expected penalty viewed as a compensation received by the owner (see 3.2).

For case (i), the expected utility function is

$$E\left[U_y(O_2)\right] = \frac{1}{r}\left(1 - e^{-r\left(\Upsilon\tau - P_G - C_b\right)} e^{-N(\ell,v_y)\left(2 - \frac{\lambda}{\lambda - \Upsilon r} + \int_v^\infty e^{-rC_P(x-v)} g(x)dx - \int_0^v e^{rC_I(v-x)} g(x)dx\right)}\right) \tag{8}$$

For case (ii),the expected profit of the owner is given by (5) and (7) replacing *W* with W_y and *L* with L_y.

3.2 Service-Provider's Decision Problem

Here, we consider on two cases–i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

For case (i),

During Ω_W, the OEM's expected cost is given by

$$E[Cost_y] = E[\text{PM cost}] + E[\text{CM cost}] \tag{9}$$

The expected PM and repair cost conditional on $Y=y$, is

$$E(\psi_y) = C_r R_0(0,W) + kC_0 - \sum_{j=1}^{k+1}\left[C_r\left(W - j\tau_y\right) - C_v\right]\left[r_y\left(j\tau_y\right) - r_y\left((j-1)\tau_y\right)\right]$$

Option O_1: the SP's expected profit is given by

$$E\left[\Psi_{1y}\right] = \left(C_s - C_m\right)N(\ell, \nu_y), C_s > C_m > C_r, \tag{10}$$

Option O_2: during Ω_S, the SP's expected profit is given by

$$E[\Psi_{2y}] = P_G + E[\text{Incentive}] - E[\text{ Penalty}] - E[\Theta_y] \tag{11}$$

where,

$$E[\Theta_y] = E[PM] + E[CM]$$

Expected of Penalty Cost:

Let $D(t)$ and ζ denote the sum of down time after a failure (including repair time), and down time target of the equipment in $(0,t)$. The expected penalty cost is given by $EP(L) = \mathcal{C}_{\mathcal{P}}\bar{G}(\zeta)N\left(\ell, \nu_y\right)$ where

$\bar{G}(\zeta) = \int_{\zeta}^{\infty}(z-\zeta)g(z)dz$, $\mathcal{C}_{\mathcal{P}}$ is the penalty cost and $N(\ell, \nu_y)$ denotes the expected number of failure in interval $(W, W+L]$.

Expected Incentive Cost:

The expected of incentive earned in $(W, W+L]$ is given by

$$EI(L) = C_I\int_0^{\zeta} G(z)dz$$

Expected of CM cost:

Let C_m is minimal repair cost then the expected repair is given by

$$EC(W,L) = C_m N(\ell, \upsilon_y)$$

where $N(\ell, \nu_y)$ is expected number of failures in $(W, W+L]$.

Expected PM cost

With cost of ℓ PM is given by $\ell C_0 + C_v\sum_{m=1}^{\ell}\delta_m$ then the expected PM cost is

$$E[PM\ \text{ cost}] = \ell C_0 - \sum_{m=1}^{l}\left[C_m\left(L - m\nu_y\right) - C_v\right]\delta$$

Where $\delta = [r(m\nu_y) - r((m-1)\nu_y)]$

For case (ii), the expected profit of the SP is given by (9) and (10) but it needs to replace W with W_y and L with L_y.

If the owner chooses Option O_0, the SP.s expected profit becomes

$$E\left[\Psi_{0y}\right] = 0 \tag{11}$$

4. OPTIMAL OPTION

This section presents the optimal option of the owner by maximizing the expected profit for each option, an d then we find the optimal price and repair cost for t he service-provider.

4.1 Owner's Optimal Option

We first obtain the optimal option for O_1 and then for O_2. Two cases need to be considered–i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

For $y \le \gamma$, Option O_1 is prefered to Option O_2 if $E[U(O_1;C_S)] > E[U(O_2;P_G)]$, and Option O_2 is preferred if $E[U(O_1;C_S)] < E[U(O_2;P_G)]$. The owner is indifferent between two option if $E[U(O_1;C_S)] = E[U(O_2;P_G)]$, which is equivalent to

$$P_G = P_0 - \frac{1}{r}\left[N(\ell, \nu_y)\left(4 - \frac{2\lambda}{\lambda - \Upsilon r} - e^{rC_s} + \int_v^{\infty} e^{-rC_p(x-v)}g(x)dx - \int_0^{v} e^{rC_I(v-x)}g(x)dx\right)\right] \tag{12}$$

Let $\Phi_1(C_s)$ express the right-hand side of equation (12). Then, Option O_1 is compared with Option O_0. Option O_1 is better than Option O_0 if $E[U(O_1;C_S)] > 0$, and if $E[U(O_1;C_S)] < 0$, Option O_0 is preferred. By solving

$E[U(O_1;C_S)]=0$ with respect to C_S, we have $\bar{C}_S$

$$\bar{C}_S=\frac{1}{r}\ln\left[\frac{r\left(\Upsilon\tau-P_0-C_b\right)}{R_{1y}(\tau)}+2-\frac{\lambda}{\lambda-\Upsilon r}\right] \quad (13)$$

Now, we compare Options O_2 and O_0 as follows. Define $\bar{P}_G$ as the value that satisfies $E[U(O_2;P_G)]=0$. We have,

$$\bar{P}_G=\Upsilon\tau-C_b-\frac{1}{r}\left[N(\ell,\nu_y)\left(2-\frac{\lambda}{\lambda-\Upsilon r}+\int_v^{\infty}e^{-rC_p(x-v)}g(x)dx-\int_0^{v}e^{rC_l(v-x)}g(x)dx\right)\right] \quad (14)$$

Let $\Omega_i\ (i=0,1,2)$ be defined by

$\Omega_0=\{(P_G,C_S);P_G\ge\bar{P}_G,C_S\ge\bar{C}_S\}$

$\Omega_1=\{(P_G,C_S);P_G<\Phi_1(C_S),P_G<\bar{P}_G\}$

$\Omega_2=\{(P_G,C_S);P_G\ge\Phi_1(C_S),C_S>\bar{C}_S\}$

Then, the optimal option for the owner becomes

$$O^*(P_G,C_S)=\begin{cases}O_0, & if\quad (P_G,C_S)\in\Omega_0\\ O_1, & if\quad (P_G,C_S)\in\Omega_1\\ O_2, & if\quad (P_G,C_S)\in\Omega_2\end{cases} \quad (15)$$

For $y>\gamma$, $\bar{C}_S$ and $\bar{P}_G$ are given by (13) and (14) replacing W with W_1. Using (15) we get the optimal option for the owner.

4.2 Service-Provider's Optimal Option

The SP's optimal option for P_G and C_S is obtained by maximizing the SP's expected profit based on the owner's optimal option $O^*(P_G,C_S)$. Again, two cases need to be considered–i.e. (i) $y\le\gamma$ and (ii) $y>\gamma$.

For $(P_G,C_S)\in\Omega_1$, the owner optimal option is O_1. Hence the SP's expected profit is given by eq. (10). Since $\frac{dE[\Psi_{1y}]}{dC_s}>0$, the SP's expected profit becomes maximum by a certain point on the curve $P_G=\Phi_1(C_s)$. By substitute $C_S=\bar{C}_S$ to (10), the expected profit SP is given by eq. (16).

Then, the maximum expected profit of the SP is obtained for $P_G^*>\Phi_1(C_S^*)+0$ and $C_S^*\to\bar{C}_S-0$.

For $(P_G,C_S)\in\Omega_2$, the owner optimal option is O_2. The SP's expected profit is given by eq. (11) and becomes maximum if $\Phi_1(C_S)=\bar{P}_G$. Substitute $P_G=\bar{P}_G$ to (11), we have eq. (17).

Then, the maximum expected profit of the OEM is obtained for $P_G^*\to\bar{P}_G-0$ and $C_S^*\to\bar{C}_S+0$.

Finally, for $(P_G,C_S)\in\Omega_0$, the owner optimal option is O_0. The SP's expected profit is given by eq. (12).

The optimal option for the SP is the one gives a larger positive expected profit between Option O_1 and Option O_2. If both are negative, then the best option is Option O_0 (or do not buy any option offered).

For case (ii), the optimal repair cost, P_G^* and the optimal expected profit are given in (14) and (17) by replacing W with W_y and L with L_y.

Here, by using Stackelberg game theoretic formulation the optimal decision is the one that maximizes the expected profit both the service-provider and the owner. Then, we have number of PM optimal, PM degree optimal (δ^{y*}) and the optimal expected profit of each party given by the optimal level maintenance for the owner, and the optimal price for the service-provider.

5. CONCLUSIONS

In this paper, we have studied maintenance contracts which taken into account risk attitude of the owner to the contract and developed models to determine the optimal option for the owner and the optimal price for the SP. In this paper, we consider only one service provider –i.e. OEM and agent. In many cases, the OEM offers more options –with warranty and without warranty and the agent gives service contract after the expired of warranty. These further research topics are currently under investigation.

$$E\left[\Psi_{1y}\right]=\left(\frac{1}{r}\ln\left[\frac{r\left(\Upsilon\tau-P_0-C_b\right)}{R_{1y}\left(\tau\right)}+2-\frac{\lambda}{\lambda-\Upsilon r}\right]-C_m\right)R_{1y}\left(\tau\right) \quad (16)$$

$$\begin{aligned}E\left[\Psi_{2y}\right]&=\Upsilon\tau-C_b-\frac{1}{r}\left[N(\ell,\nu_y)\left(2-\frac{\lambda}{\lambda-\Upsilon r}+\int_v^{\infty}e^{-rC_p(x-v)}g(x)dx-\int_0^{v}e^{rC_I(v-x)}g(x)dx\right)\right]\\&+\int_0^{v}C_I(v-x)g(x)dx-\int_v^{\infty}C_p(x-v)g(x)dx-\left\{C_mN(\ell,\upsilon_y)+\ell C_0-\sum_{m=1}^{l}\left[C_m\left(L-m\nu_y\right)-C_v\right]\delta_m\right\}\end{aligned} \quad (17)$$

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