# Factor Screening Method for Quantile-based Performance Measure

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**Abstract.** Screening experiments are often used to identify the important factors affecting intended systems' response significantly. In the literature, factor screening approaches mostly adopted expectation as performance measures for stochastic simulation experiments. Nevertheless, even though a few alternatives were known to offer insightful perspectives for a more complete view of the statistical landscape, they were rarely discussed due to technical difficulties of developing methodologies. Quantile is an important alternative to the expectation for spatial data and applications of risk control, however, unlike the expectation, quantile lacks nice distributional properties so developing quantile-based applications would be a challenge. In this study, we propose a novel approach of factor screening for quantile-based performance measure. The proposed method is able to address large-scale problems based on statistical inference. Both Type I error and Power are considered to handle the risk with given conditions. A numerical study was conducted to evaluate the performance of the proposed methodology, and an empirical problem based on real data was solved to validate its practical viability.

**Keywords:** Factor Screening, Quantile Estimation

## 1. INTRODUCTION

Screening experiments are designed to eliminate the unimportant factors so that important factors can be investigated in further detail. Because of the sparsity of effect principle, we intend that only a few factors will influence the output among all factors. Experiments can be conducted by physical experiments or simulation experiments. However, for many real world problems, conducting physical experiments may be difficultly or costly. The drawbacks may due to too much stochastic factors in the problems. As a result, simulation experiments are conducted when studying those problems.

In order to analyze the problems which cannot be investigated by physical experiments, simulation experiments are conducted. Simulation software packages allow us to build a model which can take most factors and properties in the problem into consideration. This advantage of simulation experiments can let us study the problem more realistically, but may result in a too complex simulation model. For simulation experiments, in pursuance of statistically significance, replications are run to eliminate the variance due to the stochastic factors in the problem. However, if the simulation model is too complex to conduct experiments, experimental cost will be high and experimental time will be long. As a result, it is critical to find out the important factors that will influence the system significantly so that the limiting computing resource can be used in the more influential factors.

In order to spend the computing resource efficiently, many screening methods have been developed with the purpose of identifying the important factors. The problem we study can then be analyzed by the important factors to avoid spending resource on the factors which may not affect the output significantly. Sequential Bifurcation (SB) (Bettonvil and Kleijnen, 1997) is one of the methods which has been widely used as the basis for other screening methods with simulation experiments. SB groups factors together and classifies the factors by its group effect. The process is efficient to screen out the important factors but the result is not theoretically guaranteed. In other words, the factors screened out by SB cannot be proved to be the true important factors. As a result, Wan et al. (2006) proposed a modified SB called Controlled Sequential Bifurcation (CSB). CSB integrates the hypothesis testing with SB to control the Type Ⅰ error $\delta$ and power $\gamma$ of the screening result. By controlling these two probabilities, the screened out factors can be mathematically proved to be the true important factors. Though CSB guarantees the screening result to be true with a certain probability, however, the condition for CSB is that the direction of the factor effect should be known since it can only deal with the factors with same direction. Another method extended from CSB which deals with factor interaction is called CSB-X (Wan et al., 2006). CSB-X utilizes fold-over approach to cope with two-way interactions which can eliminate the effect caused by interaction and give us the unbiased result of the main effect factors. Sanchez et al. (2009) proposed Fractional Factorial CSB (FFCSB) to cope with the limitary of CSB that the direction of the effect should be known prior to experimentation. FFCSB uses an efficient fractional factorial design to identify the direction of factor effect and then conducts CSB for the factors with positive effect and negative effect respectively.

CSB is proposed to guarantee the accuracy of the screening result for the true important factors. It is extended from SB which utilizes the concepts of group screening (Lewis and Dean, 2001) and sequential procedure. In group screening, if the group effect of a set of factors is considered to be insignificant, then all the factors in the group will be viewed as unimportant factors. In contrast, if the effect of a group is significant, then it is assumed that there is at least one factor in the group which is an important factor. In order to identify the important factors, the group with significant group effect will be divided into subgroups. The subgroups will be continued tested until there is only one factor in a group. By this testing structure, all factors are classified as important or unimportant. Additionally, in the sequential procedure, the insights of factors are accumulated as the experiments processing. The sequential nature makes the screening methods applicable for simulation experiments. CSB improves SB in providing assurance for the “correctness” of the screening results. It combines the concept of thresholds and hypothesis testing in the screening procedure to control the accuracy for selecting the true important factors.

Most factor screening approaches adopt expectation as performance measure since expectation has nice distributional properties. However, expectation is not applicable to every problem. For example, expectation is not a proper performance for the common risk control problem. To control the risk, probability for specific event in the problem would be a useful performance rather than expectation. Further, compared to expectation, other alternatives provide a more complete view of the statistical landscape, such as quantile. Quantile is an important alternative to the expectation for spatial data. Though estimating quantile is a huge challenge, building a factor screening method with quantile-based performance measure is critical since screening methods with expectation performance are not appropriate for all the problems. For instance, the delivery problem in service-driven industry, such as manufacturing, is a problem which requires quantile as the performance in order to attain accurate delivery time. The quantiles of cycle time distribution can produce accurate estimates of cycle time and thus provide decision makers in the industry various levels of delivery time. Consequently, in order to conduct factor screening method in this kind of problem or other problems with quantile performance, quantile-based screening method is needed.

The integration between quantile and screening method could be a challenge. There are many approaches discussing about quantile estimation. Eickhoff (2006) proposed a quantile estimator from mean value analysis and multiple independent replications to study the long-time behavior. Bekki et al. (2014) compared performances of two promising metamodeling tools, stochastic kriging and quantile regression, on steady-state quantile parameter estimation. Chen and Kelton (2001) implemented a sequential procedure to construct proportional half-width confidence intervals for a simulation estimator of the steady-state quantiles. Nakayama (2015) provided an asymptotically valid confidence interval for quantile by using a variance-reduction technique combined with stratified sampling and control variates. Jin et al. (2003) analyzed the probability that a simulation-based quantile estimator fails to lie in a prespecified neighborhood of the true quantile. Though there are many methods for quantile estimation, not all the methods are applicable. There are some limitations to adopt quantile methods in our research. Those methods with confidence interval as their results are not applicable since the estimated quantile will be calculated in our method. In addition to the result form, multiple simulation replications need to be conducted in the classifying process. In order to avoid spending too much computation resource, quantile needs to be estimated by smaller samples as possible. As a result, we adopt the method by Pandey (2000) in our methodology. He presented a distribution-free method for estimating the quantile function by using the principle of maximum entropy principle (MaxEnt). In the approach, probability-weighted moments (PWMs) are interpreted as the moment of quantile function. By integrating MaxEnt and PWMs, quantile can be estimated from small samples.

In this paper, we propose a modified CSB method with quantile-based performance measure. During the CSB procedure, the performance in the classifying process will be the quantile effect of the factors. In order to specify the important factor with quantile-based performance, for each group of factors, the quantile should be estimated with factors set at high level and low level respectively. Then we can obtain the effect of these factors on quantile. Thus, as to obtain one quantile estimation, replications are conducted. This will cost much more computation efforts than CSB. In order to avoid spending too much computation resource, we use Pandey (2000) to estimate the quantile since it can estimate with small samples but does not loss the accuracy too much. This paper will apply quantile estimation method by Pandey (2000) to estimate the quantile of the output data and utilize the estimated quantile in CSB procedure to screen out the factors which are important to quantile performance.

This paper is organized as follows: In Section 2, we define the model for experiment and the purpose of screening procedure. Section 3 describes the detail of the screening method. Conclusions are provided in Section 4.

## 2. MODEL DESCRIPTION

### 2.1 QUANTILE MAIN-EFFECTS MODEL

The quantile model of the $M$ factors at α-quantile is denoted by $Q^{\alpha}(\mathrm{Y}(\mathbf{x}))$.

$$Q^{\alpha}(Y(\boldsymbol{x})) = \beta_0^{\alpha} + \beta_1^{\alpha} x_1 + \cdots + \beta_M^{\alpha} x_M \tag{1}$$

Where $x_i$ is the level setting of factor $i$ for $i = 1 \dots M$, $\mathbf{x} = (x_1, x_2, \dots, x_M)$ denotes the group level of $M$ factors, and $\beta_i^{\alpha}$ is the effect coefficients of factor $i$ on $\alpha-$quantile for $i = 1 \dots M$. $\mathrm{Y}(\mathbf{x})$ is the response variable with group level $\mathbf{x}$. $Q^{\alpha}(\mathrm{Y}(\mathbf{x}))$ is the $\alpha-$quantile of $\mathrm{Y}(\mathbf{x})$. The meaning of $Q^{\alpha}(\mathrm{Y}(\mathbf{x}))$ is that the output of $\mathrm{Y}(\mathbf{x})$ will be less than $Q^{\alpha}(\mathrm{Y}(\mathbf{x}))$ with probability α. In this paper, the factor coefficient $\beta_i^{\alpha}$ is the core we would like to study. Given a probability α, we are longing to recognize the factor effect on quantile in order to find the important factors in the screening procedure. It should be noticed that the model only consider main effects and the direction of the factor effect should be same.

### 2.2 DETERMINATION OF FACTOR LEVELS

There are total $M$ factors. The group level is defined as $\mathbf{x} = (x_1, x_2, \dots, x_M)$. The $\mathbf{x}$ for an experiment at level $l$ is defined as $\mathbf{x}(l)$. Thus, the $x_i$ of $\mathbf{x}$ will be presented as $x_i(l)$ in $\mathbf{x}(l)$. $x_i(l)$ is $x_i$ at level $l$.

$$\boldsymbol{x}(l) = (x_1(l), x_2(l), \dots, x_M(l)) \tag{2}$$

$$x_i(l) = \begin{cases} 1, & i = 1, \cdots, l \\ 0, & i = (l+1), \cdots, M \end{cases} \tag{3}$$

For every factor, the high level setting is 1 and low level setting is 0. If an experiment is set at $\mathbf{x}(l)$, it means factors $1, \cdots, l$ in $\mathbf{x}$ are set at high level, which is 1, and other factors are set at low level, which is 0.

In order to test the factors' effect, experimentations will set the factors we have interests in, which we assume are factors $k_1, \cdots, k_2$, at high level and low level respectively. We denote the experiments as high level experiment and low level experiment. For factors $k_1, \cdots, k_2$, the difference between high level experiment and low level experiment is that they will be set at high level in high level experiment and set at low level in low level experiment. By setting factors $k_1, \cdots, k_2$ at high level and low level respectively and, at the same time, let other factors which we are not interested remain the same level during the experiments, we can specify the factor effect for factors $k_1, \cdots, k_2$. As a result, for factors $k_1, \cdots, k_2$, the high level experiment is set at $\mathbf{x}(k_2)$ and the low level experiment is set at $\mathbf{x}(k_1 - 1)$.

### 2.3 EXPERIMENTS AND QUANTILE EFFECT

Since the true quantile is unknown, $Q^{\alpha}(\mathrm{Y}(\mathbf{x}))$ can only be obtained by estimating the $\alpha-$quantile of $\mathrm{Y}(\mathbf{x})$. In order to get the data of $\mathrm{Y}(\mathbf{x})$, experiments are conducted several times. For one quantile value, $r$ replications will be run to estimate it. In one iteration, we will conduct $2r$ replications with $r$ replications for high level experiments and $r$ replications for low level experiments. The high level $\alpha-$quantile and low level $\alpha-$quantile can be estimated based on the experiment results. Thus, one high level $\alpha$ −quantile and one low level $\alpha$ −quantile can be obtained in one iteration. The difference between the two quantiles is defined as the quantile effect. As a result, $2r$ replications are needed for one quantile effect.

Denote $\mathrm{Y}(\mathbf{x}(l))$ as the output vector of $\mathrm{Y}(\mathbf{x})$ with level at $\mathbf{x}(l)$. The $i$th output of $\mathrm{Y}(\mathbf{x}(l))$ is $\mathrm{Y}_i(\mathbf{x}(l))$ for $i = 1,\cdots,r$.

$$Y(\boldsymbol{x}(l)) = \{Y_1(\boldsymbol{x}(l)),\cdots,Y_r(\boldsymbol{x}(l))\} \tag{4}$$

As a result, for factors $k_1,\cdots,k_2$, the low level experiment output is $\mathrm{Y}(\mathbf{x}(k_1-1))$ and the high level experiment output is $\mathrm{Y}(\mathbf{x}(k_2))$. Denote the $\alpha-$quantile of $\mathrm{Y}(\mathbf{x}(l))$ at $i$th iteration is $Q_i^\alpha\left(\mathrm{Y}(\mathbf{x}(l))\right)$. Thus, the high level $\alpha$ −quantile and low level $\alpha$ −quantile at $i$th iteration is $Q_i^\alpha\left(\mathrm{Y}(\mathbf{x}(k_2))\right)$ and $Q_i^\alpha\left(\mathrm{Y}(\mathbf{x}(k_1-1))\right)$.

By subtracting the high level $\alpha-$ quantile, $Q_i^\alpha\left(\mathrm{Y}(\mathbf{x}(k_2))\right)$, with the low level $\alpha-$ quantile, $Q_i^\alpha\left(\mathrm{Y}(\mathbf{x}(k_1-1))\right)$, we can get the quantile effect. Define the quantile effect at $i$th iteration is $QE_i^\alpha(k_1,k_2)$.

$$QE_i^\alpha(k_1,k_2) = Q_i^\alpha\left(Y(\boldsymbol{x}(k_2))\right) - Q_i^\alpha\left(Y(\boldsymbol{x}(k_1-1))\right) \tag{5}$$

With $QE_i^\alpha(k_1,k_2)$, we can conduct the screening method in Section 3.

## 2.4 OBJECTIVE OF SCREENING PROCEDURE

In the screening procedure, the objective is to find out the important factors by classifying each factor into the important group or the unimportant group. We can classify the factors by comparing the factor effect with a user-specified threshold $\Delta_0$. Important factors will have effect $\beta_i^\alpha > \Delta_0$ and unimportant factors will have effect $\beta_i^\alpha \le \Delta_0$. However, there must be error in classifying procedure when facing stochastic environment. In order to provide a correct result, the probability of incorrectly classifying must be controlled. We will utilize the idea in CSB of controlling the Type Ⅰ error $\delta$ and power $\gamma$ by hypothesis testing. There will be two user-specified thresholds given in CSB, $\Delta_0$ and $\Delta_1$. The first threshold $\Delta_0$ is defined as the degree of effect that one factor must achieve to avoid being labeled as an unimportant factor. The second threshold $\Delta_1$ is the effect that we don't want to miss in the process. If factor effect $\beta_i^\alpha \le \Delta_0$, the probability that the factor be classified as important factor should be less than $\alpha$. Or if factor effect $\beta_i^\alpha \ge \Delta_1$, the probability that the factor be classified as important factor should be more than $\gamma$. Otherwise, when the factor effect falls between the two thresholds, the group will be divided into two subgroups for further testing since there may be important factors in the group but we are not sure. With these two error control, the screened out factors can be guaranteed to be the true important factors. However, compared with CSB, the meaning of importance is changed in this paper.

The meaning of importance in quantile is different from that in expectation. As we want to specify the factor effect on quantile, experiments need to be conducted for the factors we have interest in. By setting these factors at high level and low level respectively, the difference between the outputs shows the effect of the factors on quantile. Thus, we can learn that if we consider the output with these factors set at high level, how much the quantile will increase compared to set at low level. Further, for those factors which we are not interested in during the experiments, they are set at the same level. By setting the indifferent factors at same level during the high level experiments and low level experiments, their quantile effects can be eliminated by subtracting the outputs of the high level experiments and low level experiments.

## 3. QUANTILE-BASED CONTROLLED SEQUENTIAL BIFURCATION

### 3.1 QUANTILE ESTIMATION

In many quantile estimation methods, the form of estimated results can be a value or a confidence interval. In order to transform the performance measure of the factor screening methods into quantile-based, the estimation methods with confidence interval as their results are not suitable in the screening procedure. In the screening procedure, quantile results will be calculated to analyze the quantile effect of the factors. As a result, it is preferable to choose a quantile estimation method with value rather than confidence interval as its result. Besides the type of the estimated result is restrict, it is desirable to estimate a quantile with less observations. The reason for less observations is because, when conducting screening method, we need several samples to identify the effect for specific factors. That is, several quantile results would be needed in the screening procedure. With the purpose of not spending too much computation resource, the observations needed for one estimation of quantile can't be too much. According to the above, we need a quantile estimation method which does not need to estimate a quantile with much observations and the type of its estimated result is value.

We adopt the method from Pandey (2000) which use the maximum entropy principle (MaxEnt) to estimate a quantile from small samples. The concept of Pandey (2000) is that he builds the quantile function in terms of probability-weighted moments (PWMs) and then transform the quantile function by maximum entropy principle to estimate the quantile. In the first part, quantile function is expressed in terms of PWMs. In the second part, the quantile function is obtained by combining PWMs and MaxEnt.

In the first part, the expected maximum $E[X_{n:n}]$ and the expected minimum $E[X_{1:n}]$ of the n observations are expressed as

$$E[X_{n:n}] = n\int_0^1 x(u)u^{n-1}du \tag{6}$$

$$E[X_{1:n}] = n\int_0^1 x(u)(1-u)^{n-1}du \tag{7}$$

Note that $u$ is the probability that the value of an experiment sample is less than $x$ and $x(u)$ denotes the quantile function of a random variable. $E[X_{n:n}]$ and $E[X_{1:n}]$ can be transformed into the two forms of PWMs, $\alpha_k$ and $\beta_k$.

$$\alpha_k = \int_0^1 x(u)(1-u)^k du\,, \qquad k = 0,1,\cdots,n \tag{8}$$

$$\beta_k = \int_0^1 x(u)u^k du\,, \qquad k = 0,1,\cdots,n \tag{9}$$

Thus, we can get

$$\alpha_k = \frac{1}{k}E[X_{1:n}] \tag{10}$$

$$\beta_k = \frac{1}{k}E[X_{n:n}] \tag{11}$$

Because PWMs are the expected maximum and expected minimum of observations with size $k$, the unbiased estimates of $\alpha_k$ and $\beta_k$, $a_k$ and $b_k$, can be obtained.

$$a_k = \frac{1}{n}\sum_{i=1}^{n}\binom{n-1}{k}X_i\Big/\binom{n-1}{k}, \quad k = 0,1,\cdots,(n-1) \tag{12}$$

$$b_k = \frac{1}{n}\sum_{i=1}^{n}\binom{i-1}{k}X_i\Big/\binom{n-1}{k}, \quad k = 0,1,\cdots,(n-1) \tag{13}$$

In the second part, the quantile function is transformed by combining PWMs and entropy. The entropy of the quantile function is as below

$$H(u) = -\int_0^1 [x(u)lnx(u)]du \tag{14}$$

The information is expressed in the form of PWMs

$$\int_0^1 u^k x(u)du = b_k \tag{15}$$

By ( 14 ) and ( 15 ), the quantile can be estimated as below

$$x(u) = exp\left[-\sum_{k=0}^{N}\lambda_k u^k\right] \tag{16}$$

The quantile can be obtained by calculating ( 16 ). $u$ is the probability we are interested in for the quantile. The Lagrangian multiplier $\lambda_k$ can be solved by ( 15 ) and ( 16 ).

## 3.2 QUANTILE-BASED CSB PROCEDURE

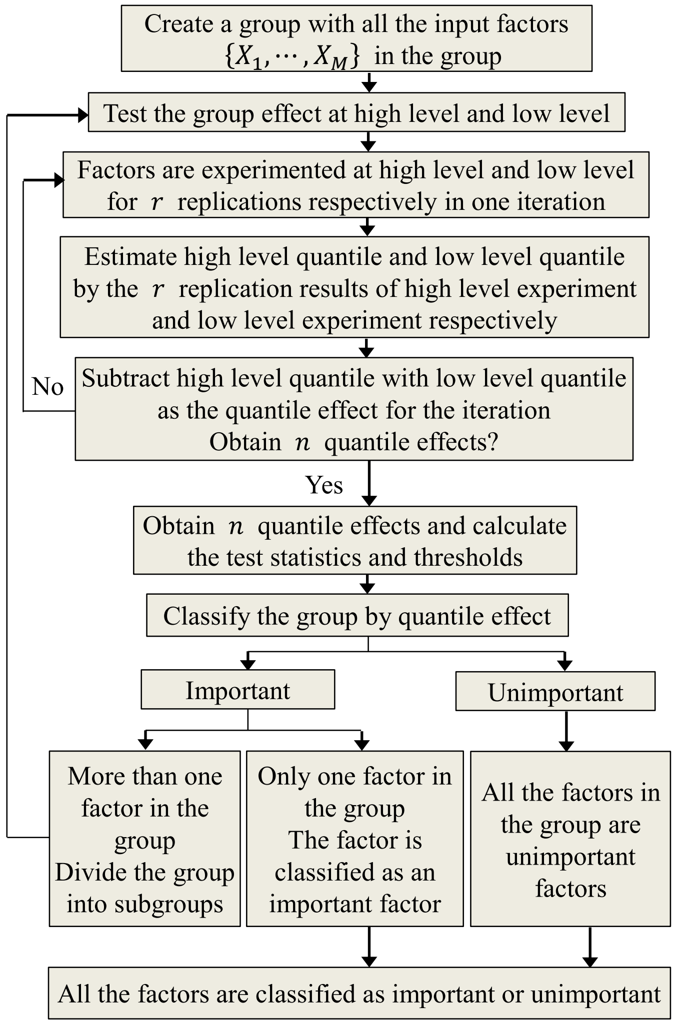
An overview procedure of quantile-based CSB is shown in Figure 1. There are three phases in the procedure. In the first phase, experiments are operated. In the second phase, some values for the third phase are calculated. And in the third phase, factors are classified. The input for the model is all the $M$ factors we would want to specify. The factors will be grouped together as an input.

In the first phase, factors in the group are experimented at high level and low level. The experiments will run $n$ iterations in total. In each iteration, high level experiments and low level experiments will be run $r$ times respectively. There are $2r$ replications in an iteration. After the experiments, one high level quantile and one low level quantile will be estimated by the $r$ observations with high level and the $r$ observations with low level. Thus, for $n$ iterations, $n$ quantile estimations for high level and $n$ quantile estimations for low level are obtained. Subtract one high level quantile with one low level quantile and get $n$ quantile effects at the end of the phase. The $n$ quantile effects will be passed to the second phase.

In the second phase, we will calculate the expected quantile effect, quantile effect variance and the thresholds in the screening process. The quantile effect of a testing group is the difference between one quantile estimation at high level and one at low level. Thus, by the results of the first phase, we can obtain $n$ quantile effects. In Section 0, the high level quantile estimation and low level quantile estimation are denoted as $Q_i^\alpha\left(\mathrm{Y}\left(\mathbf{x}(k_2)\right)\right)$ and $Q_i^\alpha\left(\mathrm{Y}\left(\mathbf{x}(k_1-1)\right)\right)$. Hence, the quantile effect is the difference between high level and low level and is denoted as $QE_i^\alpha(k_1,k_2)$. $QE_i^\alpha(k_1,k_2)$ is the α quantile effect for factors $k_1,\cdots,k_2$ at $i$th iteration.

Figure 1: Quantile-based CSB Procedure

After getting the quantile effect, we can calculate the expected quantile effect $\overline{QE}^\alpha(k_1,k_2)$ and the quantile effect




variance $S^2(k_1, k_2)$ for the group with factors $k_1, \cdots, k_2$ based on these $n$ quantile effects. It should be noticed that $\overline{QE}^\alpha(k_1, k_2)$ is the test statistics in the classifying process.

$$\overline{QE}^\alpha(k_1, k_2) = \frac{1}{n}\sum_{i=1}^{n} QE_i^\alpha(k_1, k_2) \tag{17}$$

$$S^2(k_1, k_2) = \frac{1}{n-1}\sum_{i=1}^{n}\left(QE_i^\alpha(k_1, k_2) - \overline{QE}^\alpha(k_1, k_2)\right)^2 \tag{18}$$

With $S^2(k_1, k_2)$, we can determine the thresholds for group with factors $k_1, \cdots, k_2$ by CSB method. Given user specified $\Delta_0$ and $\Delta_1$, there will be two stages in the third phase. In stage Ⅰ, there are two thresholds, $U_\mathrm{I}(k_1, k_2)$ and $L(k_1, k_2)$ used to determine the critical region. $U_\mathrm{I}(k_1, k_2)$ is a critical change in the quantile output that we do not want to ignore and $L(k_1, k_2)$ is the minimum change in the quantile output. If the group effect is more than $U_\mathrm{I}(k_1, k_2)$, the group will be considered as an important group. In another case that the group effect is less than $L(k_1, k_2)$, all the factors in the group will be classified as unimportant factors. Otherwise, the classifying process will go to the second stage. In stage II, there is only one threshold $U_\mathrm{II}(k_1, k_2)$. If the group effect is more than $U_\mathrm{II}(k_1, k_2)$, the group will be classified as an important group. If not, all the factors in the group will be classified as unimportant factors.

$$U_\mathrm{I}(k_1, k_2) = \Delta_0 + t_{\sqrt{1-\delta}, n-1} S(k_1, k_2)/\sqrt{n} \tag{19}$$

$$L(k_1, k_2) = \Delta_0 - t_{(1-\gamma)/2, n-1} S(k_1, k_2)/\sqrt{n} \tag{20}$$

$$U_\mathrm{I}(k_1, k_2) = \Delta_0 + t_{\sqrt{1-\delta}, n-1} S(k_1, k_2)/\sqrt{n'} \tag{21}$$

$t_{\theta,\nu}$ is the $\theta$ quantile of the $t$ distribution with $\nu$ degree of freedom. $n'$ is the sample number of quantile effect in the second stage.

In the third phase, the group of factors will be classified as important or unimportant. The test statistic is the expected quantile effect $\overline{QE}^\alpha(k_1, k_2)$. $\overline{QE}^\alpha(k_1, k_2)$ will be compared with the thresholds. There are two stages in the classifying process. $\overline{QE}^\alpha(k_1, k_2)$ will first be compared in stage Ⅰ. If the group cannot be classified in stage Ⅰ, then we go to stage Ⅱ. There are four kinds of results in stage Ⅰ.

1. $\overline{QE}^\alpha(k_1, k_2) \le L(k_1, k_2)$: Since the group effect is less than the minimum effect threshold, all the factors in the group, which are factors $k_1, \cdots, k_2$, are classified as unimportant factors.
2. $\overline{QE}^\alpha(k_1, k_2) \le U_\mathrm{I}(k_1, k_2)$ and $n \ge N(k_1, k_2)$ : $N(k_1, k_2)$ is the maximum number of experiments we would spend on the group with factors $k_1, \cdots, k_2$. Since we have done enough experiments on studying the group effect of factors $k_1, \cdots, k_2$, factors $k_1, \cdots, k_2$ will be directly classified as unimportant factors because their quantile effect is less than $U_\mathrm{I}(k_1, k_2)$.
3. $\overline{QE}^\alpha(k_1, k_2) > U_\mathrm{I}(k_1, k_2)$: The group effect is more than the critical effect threshold. Thus, the group is considered as an important group and it will be divided into two small group for further testing.
4. $\overline{QE}^\alpha(k_1, k_2) \le U_\mathrm{I}(k_1, k_2)$: The group effect does not exceed $U_\mathrm{I}(k_1, k_2)$. However, since we haven't done sufficient experiments to study the effect thoroughly, the process goes to stage Ⅱ. In stage Ⅱ, more experiments will be conducted.

Note that $N(k_1, k_2)$ is the maximum number of experiments we spend on the group with factors $k_1, \cdots, k_2$. If the number of experiments has reached $N(k_1, k_2)$, then we will take the effect as the true effect of the group. $N(k_1, k_2)$ is calculated as below.

$$N(k_1, k_2) = \lceil h^2 S^2(k_1, k_2)/(\Delta_1 - \Delta_0)^2 \rceil \tag{22}$$

$h$ is a constant satisfying $\mathrm{P}\left(T_{n-1} \le t_{\sqrt{1-\delta}, n-1} - h\right) = (1+\gamma)/2$, where $T_{n-1}$ is the $t$-distributed random variable with $(n-1)$ degree of freedom.

As the process goes to stage II, more experiments will be run to estimate the effect more accurately. The additional number of iterations is $(N(k_1,k_2)-n)$. As a result, the group will run $N(k_1,k_2)$ iterations in total. The $N(k_1,k_2)$ effects will be utilized to calculate the updated $\overline{QE}^{\alpha}(k_1,k_2)$ and $S^2(k_1,k_2)$. $U_{\mathrm{II}}(k_1,k_2)$ will be calculated by the new $S^2(k_1,k_2)$. The comparing process can be continued after the additional experiments and value updating. We compare the updated $\overline{QE}^{\alpha}(k_1,k_2)$ with $U_{\mathrm{II}}(k_1,k_2)$.

1. $\overline{QE}^{\alpha}(k_1,k_2) \le U_{\mathrm{II}}(k_1,k_2)$: The quantile effect is less than $U_{\mathrm{II}}(k_1,k_2)$. Since we have done $N(k_1,k_2)$ iterations for group but the effect is still less than the threshold, we conclude that all the factors in the group are unimportant factors.
2. $\overline{QE}^{\alpha}(k_1,k_2) > U_{\mathrm{II}}(k_1,k_2)$: The quantile effect is more than $U_{\mathrm{II}}(k_1,k_2)$. We declare that the group effect is important. There may be important factors in the group. As a result, the group will be divided into two subgroups to identify the important factors.

With the two-stage classifying procedure, the group will be labeled as important or unimportant. For the unimportant group, all the factors in the group will be classified as unimportant factors. As for the important group, the factors in the group will be divided into two subgroups and both will go back to phase one for testing the group effect. The process will terminate until there is only one factor in the group. Thus, by this framework, all the factors can be classified as important or unimportant.

## 4. CONCLUSION

In this paper, we propose a framework for quantile-based factor screening method. This new method combines CSB method with quantile estimation method by Pandey (2000) to specify the important factors. There are three phases in the framework. First, experiments are conducted at high level and low level respectively to identify the factors' effect. The quantile effect will be estimated by applying Pandey (2000) which estimate the quantile functions by using the maximum entropy principle. In the second phase, the thresholds and test statistics for the third phase will be calculated. In the third phase, the group of factors will be classified as important or unimportant. For the important group, factors in the group will be divided into two subgroups and the subgroups will return to the first phase for classifying the factors. The process will terminate until there is only one factor in the group. For the unimportant group, all factors in the group will be classified as unimportant factors. By these three phases, all factors can be classified as important or unimportant.

This methodology framework is designed to specify the important factors for problems with quantile performance. It should be noticed that as this method is developed by CSB, it will be more useful for large scale problems. Moreover, it will cost much more computation resources than other screening strategies to obtain the quantile effect. Though the method costs much computation resources, it can still save resources in some cases. For quantile-based problems, such as optimization problems with quantile performance, adopting this quantile-based screening methods can help users to study the problems more efficiently by spending reputation resources on the important factors. When the solving process, such as optimizing, will cost a large number of resources, it is worth to adopt our methodology first.

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