Optimization Resource-Constrained Project Scheduling Problem with Stochastic Activity Durations

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Abstract. Project scheduling under resource and time constraints always has raised attention of researchers and project managers. In this paper, the Resource Constrained Project Scheduling (RCPS) problem was considered in both small and large scale projects. A Mixed Integer Linear Programming (MILP) model was developed and for solving resource constrained project scheduling problem (RCPSP) with small size project (less than 60 activities). In order to solve RCPSP for large size project (more than 60 activities), Constraint Programming model was employed. Furthermore, to have more realistic results, simulation is used to deal with stochastic project activity durations. The RCPSP model can also be developed for the case of multi mode resources and flexible resources.

Keywords: resource-constrained project scheduling, stochastic duration activities, constraint programming, mixed integer linear programming.

1. INTRODUCTION

Construction project is a set of related activities which are planned to achieve the goal through using the defined resources in a specified period. Construction project has a starting and ending, and construct a building, bridge or factory... For each pair (i, j) activities, there are there relations such as: conjunction relation, paralleled relation and disjunction relation. The process of construction project includes five steps: initiating, planning, executing, monitoring and controlling, closing. All project managers as: civil engineer or architect must have knowledge about: cost, time, scope, quality, procurement, communication… to control their project delivering on time, on budget. Therefore project scheduling is an important factor of project management. In real life, the resources of activities are not unlimited; unexpected events such as: adverse weather, damaged equipment can always occur. These factorials often delay the project completion time so that project managers have difficulty for scheduling project. For the reason above, resource-constrained project scheduling problem under stochastic duration activities is studied.

2. LITERATURE REVIEW

2.1 Deterministic Resource constraint project scheduling problem

Resource constraint project scheduling problem (RCPSP) is a flavor in operation research which consists of assigning resources - constrained to completion activities in an effective way (Bhaskar & Pal, 2011). The objective is to optimize the completion time of project (makespan) with scheduling some activities over time in scare resource. Peter Brucker at. al (1998) review exact and heuristic solution for the single-mode, the multi-mode and stochastic activity durations for problems with other objectives than makespan minimization and, last but not least, for problems with stochastic activity durations. RCPSP is nondeterministic polynomial-time hard (NP-hard) in the strong case (Blazewicz et al. 1983), following (Alcaraz and Maroto, 2001) and (Herroelen et al, 1998) there are two methods to solve RCPSP such as: exact solution and heuristic solution. Each of the method has disadvantages and advantages, the exact method can only solve RCPCP with small scale (less than 60 activities).

2.2 Exact solution

Mohamed Haouari , Anis Koolia and Emmanuel Nérond (2012) studied three-classes of lower- bounds that depend on idea of energetic reasoning. They found that an effective shearing procedure enables to grow an excellent lower bound that regularly outflanks the best bound from the literature while significantly simpler.

Thomas S. Kyriakidis, Georgios M. Kopanos and Michael C. Georgiadis (2012) studied mixed-integer linear programming model for solving single and multi-mode RCPSP with objective minimization make-span.

Oumar Koné , Christian Artigues, Pierre Lopez and Marcel Mongeau (2011) compared several mixed integer linear programming model (MILP) for RCPSP such as: Time index, Flow-base continuous-time and On / off event base formulation.

Anis Kooli, Mohamed Haouari, Lotfi Hidri and Emmanuel Nérond (2010) studied RCPSP. They presented Integer programming model with new feasibility tests for the energetic reasoning. They took data set from PSPLIB.

2.3 Heuristic algorithm

Wail Menesi, Behrooz Golzarpoor; and Tarek Hegazy (2013) studied a constraint programming (CP) model to solve scheduling problem with large scale project. After that they compare results from CP approach with another heuristic algorithms (Ant colony , genetic algorithm..) at same examples. CP model is much faster than heuristic algorithms in some cases.

Agustín Barrios, Francisco Ballestín and Vicente Valls, (2011) studied a genetic algorithm (heuristic solution procedure) for resource-constrained project scheduling problem (RCPSP). The objective determines start time and mode resource of each activities to schedule all activities with minimization completion time of project (makspan).

Koorush Ziarati, Reza Akbaria and Vahid Zeighamib (2011) proposed bee algorithms (such as: bee algorithm, artificial bee colony and bee swarm optimization) to apply the RCPSP by utilizing intelligent behaviors of honeybees. Each of the algorithms includes three main stages: initialization, update, and termination.

W. T. Chan and Hao Hu (2002) studied constraint programming to solve scheduling problem with small size. After that they compared the Constraint programming approach is compared against commonly with some heuristic rules (such as EDD rule, the as soon as possible (ASAP) rule, and our CBE rule) example problem.

2.4 Probability distribution

Duration activity of the project is often estimated. The actual duration of the task is not quite right with the scheduled time. It is sometime smaller or larger than the estimated time. Due to the instability of duration, researchers describe it such a probability distribution. There are many probability distribution forms to describe random duration activity. Stork (2011) studied stochastic resource-constrained project scheduling. He assumed many probability distribution forms to describe random duration activity such as distribution of all: (Uniform), triangular distribution (Triangle), distributed approximately Normal, Gamma distribution and Exponential distribution (Exponential).

Sobel et al. (2009) studied scheduling projects with stochastic activity duration to maximize expected net present value. They assumed exponential distribution for the stochastic activity duration to survey the desired value largest net present value of the project.

Ballestin (2009) studied resource-constrained project scheduling with stochastic activity durations, which is given normal distribution and exponential distribution.

3. MATHEMATICAL MODEL

3.1 Deterministic RCPSP model

The resource-constrained project scheduling problem is studied with a set of activities connected by end-to-start precedence relations with resource in scare. The minimization of completion time of project (makespan) is objective.

Given are n jobs (activities), we add two dummy activities as starting activity 0 and ending activity n+1. The dummy activities have resource and processing ti me equal 0. We have notation like as:

Indices

$i \in A$	activities (jobs), i = 1 … n+1
$r \in R$	resources
$t \in [1, \ldots, T]$	horizon time

Parameters

di	duration time of activity i
n+1	number of activities
r_{ik}	amount of resource k required by activity i
R_k	resource capacity k, k = 1,2 ,…,K
E	Set of precedence relationship (i , j)

Decision variable

$C_{max} = S_{n+1}$	Project completion time (makespan)
S_i	starting of activity i
Xit = 1	if activity start at time t
Xit = 0	otherwise

Objective :

$$\mathrm{Min}\, C_{max} \qquad (1)$$

Subject to:

$$S_i + d_i \le S_j, \forall (i, j) \in E \qquad (2)$$

$$\sum_{i \in S_t} r_{ik} \le R_k, S_t = \{ i \mid S_i \le t < S_i + d_i \} \qquad (3)$$

3.2 Mixed Integer Linear Programming (MILP)

RCPSP using Mixed Integer Linear Programming (MILP) is exact algorithm. In 1969, Pritsker et al. developed a mathematical model of the RCPSP using binary decision variable, X_{it} indexed by both activities and time. The variable X_{it} is defined like this:

bjective:

$$\min C_{max} \qquad (4)$$

Subject to:

$$S_i = \sum_{t \in S_t} t \times X_{it}, \forall i \in A \qquad (5)$$

$$\sum_{t \in S_t} X_{it} = 1 \qquad (6)$$

$$X_{it} = \{0,1\}, \ \forall i \in A \cup \{0, n+1\}, \forall t \in S_t \qquad (7)$$

$$S_i + d_i \le S_j, \ \forall (i,j) \in E \qquad (8)$$

$$\sum_{i=1}^{n} r_{ik} \sum_{\tau = t - di + 1}^{t} X_{i\tau} \le R_k, \ \forall t \in S_t, \forall k \in R \qquad (9)$$

3.3 Discrete Time (DT) formulation with Earliest Start (ES) and Latest Finish (LF) of each task.

bjective:

$$\min C_{max} \qquad (4)$$

Subject to:

$$S_i = \sum_{t \in S_t} t \times X_{it}, \forall i \in A \qquad (5)$$

$$\sum_{t \in S_t} X_{it} = 1 \qquad (6)$$

$$X_{it} = \{0,1\}, \ \forall i \in A \cup \{0, n+1\}, \forall t \in S_t \qquad (7)$$

$$S_i + d_i \le S_j, \ \forall (i,j) \in E \qquad (8)$$

$$\sum_{i=1}^{n} r_{ik} \sum_{\tau = t - di + 1}^{t} X_{i\tau} \le R_k, \ \forall t \in S_t, \forall k \in R \qquad (9)$$

$$ES_i \le S_i \le LF_i \ , \forall i \in A \qquad (10)$$

3.4 Constraint programming

3.4.1 Definition

Constraint programming (CP) is a declarative modeling paradigm in which the relations between variables are specified through a set of constraints. On the contrary to mixed integer linear programming, which only address linear objectives and constraints, Constraint Programming can deal with overall (linear and non-linear) objectives and constraints. Constraint programming uses constrain propagation and constructive search to solve the problem.

Constraint propagation: is a technique which deduces new constraints from certain ones. This method may be used to tighten the search space in connection with branch-and-bound methods or local search heuristics.

Constructive search: Depth-First Search: test and generate (Backtrack algorithm): check a constraint violation after instantiation of all the variables inherent to the constraint

3.4.2 Constraint programming for optimization problems: Scheduling Problem

Given a project contains of a set A = {0, 1, ... ,n +1} of activities, where activities 0 and n +1 are dummies, and respectively represent the start time and the completion time of the project. A set of precedence relationships between activity pairs must be specified for the project.

CP Model for RCPSP

Objective

$$\mathrm{Min}\, \mathrm{Cmax} \qquad (1)$$

Subject to:

$$S_i + d_i \le S_j, \forall (i, j) \in E \qquad (2)$$

+ Resource Constraints:

$$\mathrm{Cumulative}\,(S, D, r, R) \qquad (10)$$

S: set of start time Si

d: set of duration time di

r: set of resource requirement r_{ik}

R: resource capacity.

4. COMPUTATION EXPERIMENTS

We use data from library PSPLIB which contains 4 types: J30 (n = 30 activities), J60 (n = 60 activities), J90 (n = 90 activities) and J120 (n = 120 activities). There are 480 instances for each J30, J60, J90 and J120. These instances are divided into 02 kinds: highly cumulative and highly disjunctive. Highly cumulative (mean that many activities can be executed in parallel on same resource) is more complicated than highly disjunctive (mean that many pair of activities cannot process in parallel on same resource). [24] Almost highly cumulative instances have not been solved optimal solution. In my article will solve both highly cumulative and highly disjunctive.

These instances are run on Intel Core 2 Duo CPU P8600 @ 2.40 GHz and 4.0 GB Ram. The code was written in IBM ILOG CPLEX 12.6.2. All of instances J30, J60, J90 and J120 with 4 limited resources and each activity with a maximum 3 successors.

Table 1. Result of highy disjunctive

No.	Statistic	Number of instance	Optimal value	Average CPU time	Notes
1	J30	20	100%	6.48	DT
2	J60	30	100%	9.2	DT
3	J90	20	100%	254.8	DT
4	J90	20	100%	91,05	DT with reducing constraint and variable
5	J90	20	100%	60	HPC without reducing Period

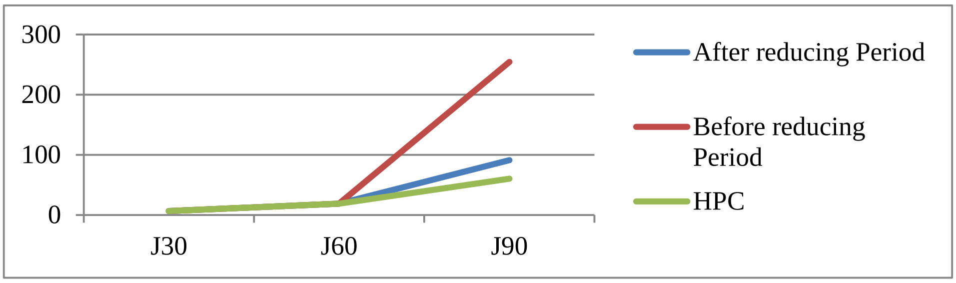

Figure 1. Comparison of CPU time before and after reducing constraints and variables

Table 2. Result of J30 - J60 highly cumulative

No.	Model	J30	J60	Notes
1	Discrete time (seconds)	1683	-	
2	Discrete time with ES-LF (seconds)	350.56	2550	
3	Constraint programming (seconds)		650	

Table 3. Result of J120

No.	Model	J120	Notes
1	Discrete time (seconds)	-	
2	Discrete time with ES-LF (seconds)	-	
3	Constraint programming (seconds)	8.04	

Table 4. Result of Mathematical model

No.	Statistic	Mathematical model		
		Discrete time	Discrete time with ES-LF	Constraint programming
1	J30 – J90 disjunctive	x	x	-
2	J30 cumulative	-	x	-
3	J60 – J90 cumulative	-	-	x
4	J120	-	-	x

Regard to J30, J60 and J90 highly disjunctive instances, we can use discrete time (DT) model to solve. The CPU time is few seconds but the result is optimal value.

With J30 highly cumulative, we should use discrete time with ES – LF for solving. Otherwise, when we use discrete time with ES – LF for solving the CPU time is very high or do not get solution some cases so that we use constraint programming (CP) model to solve. Although the value which is solved by CP is near optimal but the CPU time is acceptable.

5. CASE STUDY: GARDEN COURT II – PHU MY HUNG PROJECT, VIET NAM

5.1 Introduction about project

Garden Court 2 which is located in District 7, includes

+ Land area of 5.500m2,

+ The construction density of 54.64%,

+Total floor area 28.053m2 with 3 blocks from 7 to 15 floors

+ Total amount: 35 million USD

+ Owner: Phu My Hung

+ Contractor: Vien Dong Company Lt.d

5.2 Determine probability distribution duration activity

To describe probability distribution duration activity, we survey the opinions of the experts directly construction and collect more than 60 project schedules. We use Arena software to fit probability distribution duration activity.

$$D = \frac{Q}{a \times m} \tag{17}$$

Where:

D: duration activity

Q: amount of quantity

a: capacity of labors

m: number of labors

According to literature review, they studied probability distribution for construction activity duration with assumption as: normal distribution, triangle distribution and exponential distribution.

Table 5. Result of probability distribution

No.	ACTIVITY	PROBABILITY DISTRIBUTION
1	Start	
2	Site Preparation	NORM(26,4)
3	Excavation	TRI(8,9,16)
4	Foundation	TRI(15,20,30)
5	Reinforced Concrete -2F	TRI(8,9,14)
6	Reinforced Concrete -3F	TRI(8,9,14)
7	Reinforced Concrete -4F	TRI(8,9,14)
8	Reinforced Concrete -5F to 15F and roof floor	TRI(96,108,135)
9	Brick work 1 Floor	NORM(10,2)
10	Brick work 2 Floor	NORM(10,2)
11	Brick work 3 Floor	NORM(10,2)
12	Brick work 4 Floor to 15 Floor	NORM(120,10)
13	Alum window frame install & External plastering	TRI(40,45,56)
14	External Paint	NORM(45,6)
15	Miscellaneous exterior finishing	NORM(30,2)
16	Interior plaster & ME conduit-piping 15 F	NORM(12,2)
17	Interior plaster & ME conduit-piping 14 F	NORM(12,2)
18	Interior plaster & ME conduit-piping 13 F	NORM(12,2)
19	Interior plaster & ME conduit-piping 12 F	NORM(12,2)
20	Interior plaster & ME conduit-piping 11 F	NORM(12,2)
21	Interior plaster & ME conduit-piping 10F to 1F	NORM(120,14)
22	Internal paint	NORM(152,8)
23	Metal work	NORM(150,8)
24	Ceiling work 15F	NORM(10,2)
25	Ceiling work 14F	NORM(10,2)
26	Ceiling work 13F	NORM(10,2)
27	Ceiling work 12F	NORM(10,2)
28	Ceiling work 11F	NORM(10,2)
29	Ceiling work 10F	NORM(10,2)
30	Ceiling work 9F to 1F	NORM(90,10)
31	Tiling work	TRI (100,120,131)
32	Timber door install	TRI (45,65,80)
33	Aluminum door install	TRI (45,65,80)
34	Sanitary ware install	TRI (45,65,75)
35	Miscellaneous interior	NORM(30,2)

	finishing	
36	Inspection & Remedy work	NORM(20,2)
37	Handover	

Table 6. Result of probability distribution

No.	Statistic	Value	
1	N	30	
2	Minimum	551	
3	Maximum	749	
4	Mean	645	
5	Variance	1554	
6	Std, Deviation	40	
7	95% Confidence Interval	Lower: 631	Upper: 659

According to the above results, the project completion time range is 631 – 659 days with confidence interval as 95%. Comparison to actual completion time - 647 days, the result is very reasonable.

6. CONCLUSION

Project scheduling under resource and time constraints always has raised attention of researchers and project managers. In this paper, the Resource Constrained Project Scheduling (RCPS) problem was considered in both small and large scale projects. A Mixed Integer Linear Programming (MILP) model was developed and for solving resource constrained project scheduling problem (RCPSP) with small size project and highly disjunctive). In order to solve RCPSP for large size project or highly cumulative, Constraint Programming model was employed. Furthermore, to have more realistic results, simulation is used to deal with stochastic project activity durations. The RCPSP model can also be developed for the case of multi mode resources and flexible resources.

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