Car Rental Optimization under Effect of Upgrade Policy

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Abstract. In the paper, the application of revenue management in the context of car rental business is introduced. In the reality, the arrival request is uncertainty over time, the operator is not aware of the future request. As the result, it is very hard to accept or reject the current request. In particular, the customer makes a request of car rental, given the car type, length of rent as well as the starting time. When the request arrives, the car rental company will satisfy the customer request. Based on the acceptance policy of company, the operator will accept the request or reject it and propose another car belonging the superior group. It calls upgrade. For this purpose, the model of approximation liner programming is introduced. Because the demand is uncertainty in the reality, the model will be implemented with the dynamic way. Besides the revenue-based opportunity cost acceptance policy is also proposed and their effectiveness is derived by comparing between opportunity cost and first come first serve.
Keywords:revenue management, car rental optimization, approximation linear programming.

1. INTRODUCTION

Revenue management is the approach to maximize the revenue, obtained by managing the resource, each product will be sold to right customer at the right time with the right price [1]. The revenue management began in the airline industry, in [2] author proposed an approach considered the seat's request fulfilled when its value exceeded the average value of the seat in future. After the Airline Deregulation Act of 1978, the control of airline prices was loosening, it caused big changes and a rash of innovation in this industry. As the result, the dawn of revenue management development happened. For more detail consideration, readers can refer [2, 3, 4].

Since the beginning of revenue management application in the airline industry in [5, 6], revenue management application has been extended in many industries and many aspects of car rental problem have been studied deeply, such as pool segment, size of vehicle fleet. In [7] authors studied pool segmentation approach to utilized the fleet and improve the logistics management efficiency by introducing the dynamic model and heuristics algorithm for the leasing network. In [8] a two-stage stochastic programming is considered in with the uncertainty demand. About the fleet planning [9, 10] a tactical fleet planning model has been introduced to compute the number of cars will be allocated at each station and the moves between two locations with the objective is minimize the transfer costs and maximize the revenue.

In the literature, there are several studies about the revenue management application in car rental problem such as [11] considered the allocation of capacity management is accessed by two classes of customers. In [12] authors presented the stochastic model in a car rental company with two classes of customers, premium and classic service. Because the common pool of cars is accessed by both demands, the company must decide which one would be accepted or denied.

There are some features that revenue management application in car rental context is different versus its application in other business such as hotel. In the network case, the car rental company can easily transfer their fleet between their stations [8]. Revenue management has been applied in truck rental problem. In particular, there are some paper addressed some aspects of truck rental problem such as manage the truck fleet with different capacity, sharing and repositioning the empty trucks [13,14]. The paper by Guerriero, F., G. Miglionico, and F. Olivito [13] proposed the formulation via dynamic programming in case of managing the fleet of truck with different capacities to serve the random requests of different customers over time.

The dynamic prices model of revenue management in car rental was proposed in [15]. Recently, a dynamic approach to control capacity with planned upgrade has been introduced [16]. In [16], authors proposed 2 dynamic programming decomposition approaches to simultaneously consider upgrades and capacity control decisions. The first approach is daily dynamic programming decomposition and the second one is single resource dynamic programming decomposition. Very recently, [14] authors proposed the formulation based on linear programming approximation to get the revenue management decision policies, primal and dual decision, for the operators. The formulation is solve by updating the demand and capacity information at the beginning of each time period.

The contribution of this paper is followed: first of all, the dynamic formulation of car rental problem is proposed. Based on the formulation, the approximation linear programming model for two scenarios non-upgrade and upgrade are derived. In addition, the application of first come first serve policy and opportunity cost policy are provided respectively. Finally, the numerical experiment is conducted to assess the improvement among two polices.

The rest of paper is organized as follows. In section 2, the problem is taken consideration. Section 3 is devoted to represent the dynamic programming model for car rental problem. Section 4 derives deterministic linear programming formulation for non-upgrade scenario and upgrade. Section 5 contains the revenue – based acceptance policy. Section 6 presents the experiment and conclusion reports are found in Section 7.

2. PROBLEM DISCRIPTION

In the section the car rental typical process and core decision are represented.

A rental is begun from the customer pick up the car at the station where the contract has been signed to the car rental company and ends with check-in at the same place where the car is returned. Before check-in the car, customer must provide some data such as length of rent and the location where customer pick up the car. The revenue will be calculated daily rental rate multiply by number of rental days.

Customer must provide the check-in time, group of car. Since request arrives, the car rental operators have to make the decision accept or deny the customer's requests and the operators don't know type of upcoming requests will be in the future. The operators cannot accept all the customer requests, because the potential is the car rental company will lose many request which are more benefit. However, the operators cannot reject all the customer's requests also because there are many unused resources remaining. This is the characteristic of decision making in accordance of revenue management in the car rental context.

In the paper, no-show – it means customers book car but they disappear to use or rent that car, and cancellation is not taken consideration. In reality, the decision of operators is influenced by some other factors such as the season, day of week, the special contract with companies. However, these factors are out of paper scope.

3. DYNAMIC PROGRAMMING (DP) MODEL

A dynamic programing model to the case of round trip rental problem (RTRp) for the upgraded case is proposed. In this car rental process, the pick-up point is the same as the destination point.

In the model, time is assumed discrete and time horizon is isolated into T period, $t = 1, 2 \ldots T$. One period one request comes. At each time period, the customer makes a reservation. The car rental company will accept or deny the specific car group from month i to month j according to the customer request for optimizing the revenue management.

Time horizon is the periods that booking request will occur. The time horizon is isolated into T period. A booking request is determined by pair of i and j, where month i is the pick-up month and month j is the return month with $i = 1,2,3..T-1$ and $j = i+1,..,T$. Length of rentis calculated by $(j - i)$. Demand of customer is random and time-dependent.

K denotes the group of vehicle. In each group, $Q^k$is total capacity of group, with $k = 1, 2...K$. P^k is the monthly rental rate corresponding to k-th car group. The higher car group, the higher monthly rental rate $p^1<p^2<\ldots<p^k$. If a request is accepted, the revenue can be $R_{ij}^k = (j-i)*P^k$.

A customer is classified into k-th group when the request's customer is k-th car group. If that request is satisfied with the advance group, the upgraded case happened. The term 'product' means the car belonging to a certain car group to be rented from month $i$to month j.

$[A^1|A^2|A^K]$ with $A \in R^{K*(2K-1)}$ is the 0-1 matrix. It is set of potential car groups can satisfy demand of class k customer. Sub-matrix A^1 is the set of potential car group can satisfy demand of class 1 customer, within the second group in sub-matrix A^1 is the higher group than the first group in the sub-matrix. The last sub-matrix A^K contains the highest car group so there is only one column in this sub-matrix A^K.

Each column of A is denoted $A\, v_k$ where $v_k = v_{min},\ldots,v_{max}$. With $v_{min} = (2k-1)$, $v_{max} = 2k$, $k = 1, 2... K$ and $v_{min} = v_{max}$for $k = K$. Each $a_{v_k}^k$, $k = 1,2...K$ and $v_k = v_{min},\ldots,v_{max}$ be the element of A. $a_{v_k}^k$is equal to 1 if k-th car group is rented. Otherwise, $a_{v_k}^k$ is equal to 0.

$$\begin{cases} a_{vk} = 1, \text{if } k \text{ car is used for product } vk \\ \ldots \end{cases}$$

In the system the state of $Q = [Q_1|Q_2|\ldots|Q_T]$ where $Qt = (q_t^1 \ldots . q_t^K)$$\forall$t = $1, 2 \ldots . T$ and $\forall k = 1, 2 \ldots . K$ means number of k-th group car is available to satisfy the request at month t.

Let λ_{ij}^{tk}denotes the probability of request for k-th car group from time I to time j is arrived at the t time period. Let λ_0^t be the probability of no request at the t time period. $u_{ij}^{vk,t}$ =1 if a rental request from time $i$to time j for a k-th car group is accepted at the t time period and $u_{ij}^{vk,t}$ = 0, otherwise.

The Bellman equation for$V'_t(Q)$ is followed:

$V'_t(Q)$

$$= \sum_{k=1}^{K'} \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} \lambda_{ij}^{t'k} \quad \max_{\substack{u_{i'j}^{k,t} \in \{0,1\} \\ vk = vmin(k) \dots . vmax(k)}} [$$

$$(j-i)\text{pk}u_{ij}^{vk,t'} + V_{t'+1}(Q')] + \lambda_0^{t'} V_{t'+1}(Q')$$

The limitation conditions of the equation are

$V_{t'}(Q) = 0, \forall t'$;

$V_t'(Q) = -\infty \; if \; q_t^k < 0, for \; some \; t, n, k, \forall t'$

$V_{t'+1}(Q) = 0 \; if \; q_t^k \geq 0, \forall t, n, k.$

$V_t'(Q) = -\infty \; if \; q_t^k < 0, for \; some \; t, n, k.$

- $Q'_{i'} = (Q_{i'} - A_{vk} u_{ij}^{vk,t'})$,$\forall i' = i \dots T$: defines the updated capacity at time *i'* when rental request at the pickup time *i* is accepted.
- $Q'_j = (Q_{j'} - A_{vk} u_{ij}^{vk,t'})$,$\forall j' = j \dots T$: defines the updated capacity at time *j'* when rental request at the return time *j* is accepted.
- $Q'_l = Q_l, \; \forall l \neq [i, j]$ defines the updated remaining capacity from time *i* to time *j*.

When the request comes, if the operator accepts the request, the inventory and revenue will be fluctuated. Otherwise, if the operator rejects the request, the inventory will move on the next period.

4. DETERMINISTIC LINEAR PROGRAMMING (DLP) MODEL

Although the dynamic programming model application to accepted or rejected decision is real, the model increases the difficult computation. For that reason, the linear programming approximation is proposed. Although the proposed model is linear, it is solved in dynamic way by updating the state of vehicle capacity and demand at every beginning of period. In the follow, the round trip rental problem is considered in non-upgraded scenario and upgrade, respectively.

4.1 DLP model in non-upgrade scenario

Firstly, we introduce the detail explanation to non-upgrade scenario as follow. In this scenario, when customer places an order and if the requested car is available, the request will be accepted exactly. Otherwise, that will be denied.

Secondly, the main parameters and variables of the model are represented as followed.

C^g: The rental rate of car belonging to car group g.

x_{az}^{gv}: The number of car belonging to type *v* to be used to meet the request of *g* class customer from time *a* to time *z*.

D_{az}^g: The average demand for cars belonging to car group *g* from time *a* to time *z*, where *g = 1,2,..,G*; *a = 1,2,…,T-1; z = a+1,…, T.*

Q_a^g: Total number of car belonging to group *g* at the time $a.Q_a^g$ is proposed in **Table 4.1.1**

$R^{RTRp}(Q) =$

$$Max \left[\sum_{a=1}^{T-1} \sum_{z=a+1}^{T} \sum_{g=1}^{G} \sum_{v=1}^{G} C^g * (1 + \pi_{z-a}) * x_{az}^{gv} \right] \quad (1)$$

$$\sum_{v=1, v=g}^{G} x_{az}^{gv} \leq D_{az}^g \; , \forall a = 1,2, \dots, T-1,$$
$$z = a+1, \dots, T; \; g = 1, .., G \quad (2)$$

$$\sum_{t=1}^{a} \sum_{z=t+1}^{T} \sum_{v=1}^{G} \alpha_v^g * x_{tz}^{gv} \leq Q_a^g + \sum_{c=1}^{a-2} \sum_{z=2}^{a-1} \sum_{v=1}^{G} \alpha_v^g * x_{cz}^{gv} + \sum_{z=1}^{a-1} \sum_{v=1}^{G} \alpha_v^g * x_{za}^{gv} \quad (3)$$

$$x_{az}^{gv} \geq 0, integer \; \forall v = 1, \dots G, a = 1,2, \dots, T-1, \; z = a+1, \dots, T \quad (4)$$

α_v^g is proposed in **Table 4.1.2**

C^g is proposed in **Table 4.1.3**

Function (1) is the objective function, which establishes the total revenue. In (1), the non-negative scalar π_{z-a} is introduced in **Table 4.1.4**, such that $\pi_0 = 0, \pi_1 < \pi_2 < \cdots < \pi_T$. Function (2) establishes the number of car to be satisfied the *g* class customer cannot exceed the demand of class *g*. Function (3) defines the number of car to be satisfied the demand cannot exceed the available capacity at every period.

The proposed model allows determining the number of vehicles and the upgrades to be rented. The objective is to maximize the average revenue while the demand and capacity constraints are satisfied.

4.2 DLP model in upgrade scenario

In this scenario, the car rental company will propose customer the higher category with the same rental rate. For example, we take consideration the car rental company with three car rental group. If requested car belongs to group 1 and capacity of group 1 exceeds the demand of customer over horizontal time, the car rental company can offer the car of group 2 additionally but the monthlyrental rate is the same. Similarly, the proposed formulation of upgrade scenario is below

T: Time horizon of booking request.

G: Number of car group.

C^g: The rental rate of car belonging to car group g.

a: The starting time (pick-up time) of the booking request , *a = 1, 2…T-1.*

z: The ending time (return time) of the booking request, *z = a+1,…,T.*

g: The index of car group, *g = 1,2,..,G.*

α_v^g is proposed in **Table 4.2**

$$R^{RTRp}(Q) = Max\left[\sum_{a=1}^{T-1}\sum_{z=a+1}^{T}\sum_{g=1}^{G}\sum_{v=v_{min}}^{v_{max}} C^g * (1+\pi_{z-a}) * x_{az}^{gv}\right] \quad (5)$$

$$\sum_{v=v_{min}}^{v_{max}} x_{az}^{gv} \leq D_{az}^{g}, \forall a = 1,2,\ldots,T-1, \quad z = a+1,\ldots,T;\ g = 1,..,G \quad (6)$$

$$\sum_{t=1}^{a}\sum_{z=t+1}^{T}\sum_{v=1}^{2*G-1}\alpha_v^g * x_{tz}^{gv} \leq Q_a^g + \sum_{c=1}^{a-2}\sum_{z=2}^{a-1}\sum_{v=1}^{2*G-1}\alpha_v^g * x_{cz}^{gv} + \sum_{z=1}^{a-1}\sum_{v=1}^{2*G-1}\alpha_v^g * x_{za}^{gv} \quad (7)$$

$$x_{az}^{gv} \geq 0, integer\ \forall v = 1,\ldots 2*G-1, \quad a = 1,2,\ldots,T-1, \quad z = a+1,\ldots,T \quad (8)$$

5. REVENUE – BASED ACCEPTANCE POLICY

5.1 First come first serve (FCFS) policy

In both scenarios, the policy is considered for long run. At each scenario, the model always consider the status of requested car. If the requested car is available, the request is accepted. Else, it is rejected. Readers can refer the **Figure 5.1.1** and **Figure 5.1.2**

5.2 Opportunity cost policy

For this policy, the revenue level corresponding to request is always calculated. The request is accepted if its revenue threshold is greater than or equal the previous recorded one. Else, the request is rejected. Readers can find in **Figure 5.2.1** and **Figure 5.2.2**

6. NUMERICAL EXPERIMENT

The round trip rental problem (RTRp) is introduced. The purpose is to assess the performance of model in two scenarios: non-upgrade and upgrade. Each of scenario, we consider which policy is suitable: first come first serve or the opportunity cost. We use model in the small-size problem with time horizon is 4 months and number of vehicle groups are 3.

After simulating non-upgrade model and upgrade model are conducted with both FCFS policy and opportunity cost policy respectively, the result is drawn as below:

- In the non-upgrade scenario, the improvement of opportunity cost increase modestly at 7 per cent versus FCFS policy. Similarly, the improvement of opportunity cost is at 6 per cent for upgrade scenario.
- Considering improvement of two models with applying opportunity cost policy, the improvement of upgrade case is higher than non-upgrade by 8 per cent. Under applying FCFS policy, the improvement is 9 per cent as higher application of upgrade case as non-upgrade's.

The average profit of two scenarios are derived in **Table 6.1**

7. CONCLUSION

The paper was undertaken to apply the revenue management in the car rental context. The mathematical model is introduced and the solution is derived to help the car rental firm in term of accept or reject the car rental request. For the purpose, the approximation linear programming model is developed under the one-way rental situation. For the scenario, the objective of paper is to maximize the revenue of the car rental firm based on expected demand and quantity satisfaction. Also, we define the bid-price limit mechanism calling opportunity cost to accept or reject the car rental request. In order to consider the effectiveness, we compare the model under first com first serve with opportunity cost mechanism. In our best knowledge, we also consider the non- upgrade case and the upgrade case. For the numerical experiment, the small-size problem is taken consideration. Finally, the author used Mat-lab to perform the analysis.

Due to time limitation, the author doesn't take the consideration the below aspects in the model

- No-show: Customers made reservation already, but they don't show up or don't pay for their reservation.
- We don't count on the factors which impact the rate such as the season, the day of week or special contracts with certain customers.
- In case of pick-up period is delayed or returned car need to repair.
- Walk-in customer situation.

Although the revenue management is the classic problem, it is still interesting and there are many useful applications in practice. Because of lack of time, the author developed the model theoretically. In the further research, the model can be developed to become the large size, get more car group, the time horizon extends longer. In addition, the model can be combined some above aspects such as no-show, seasonal index, walk-in or round trip rental problem. Besides, the reinforcement learning approach can be integrated into the model.

8. FIGURES AND TABLES

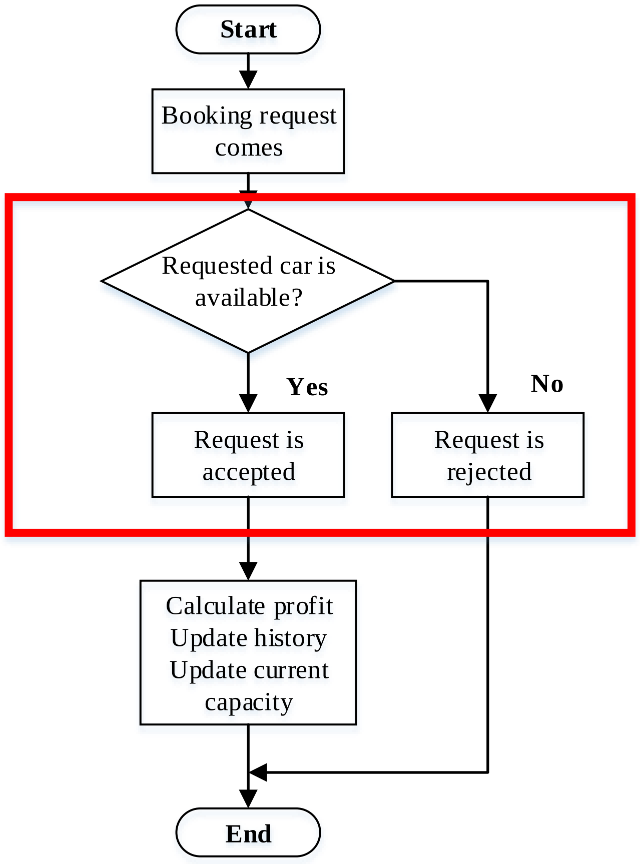

Figure 5.1.1 FCFS policy in non-upgrade scenario

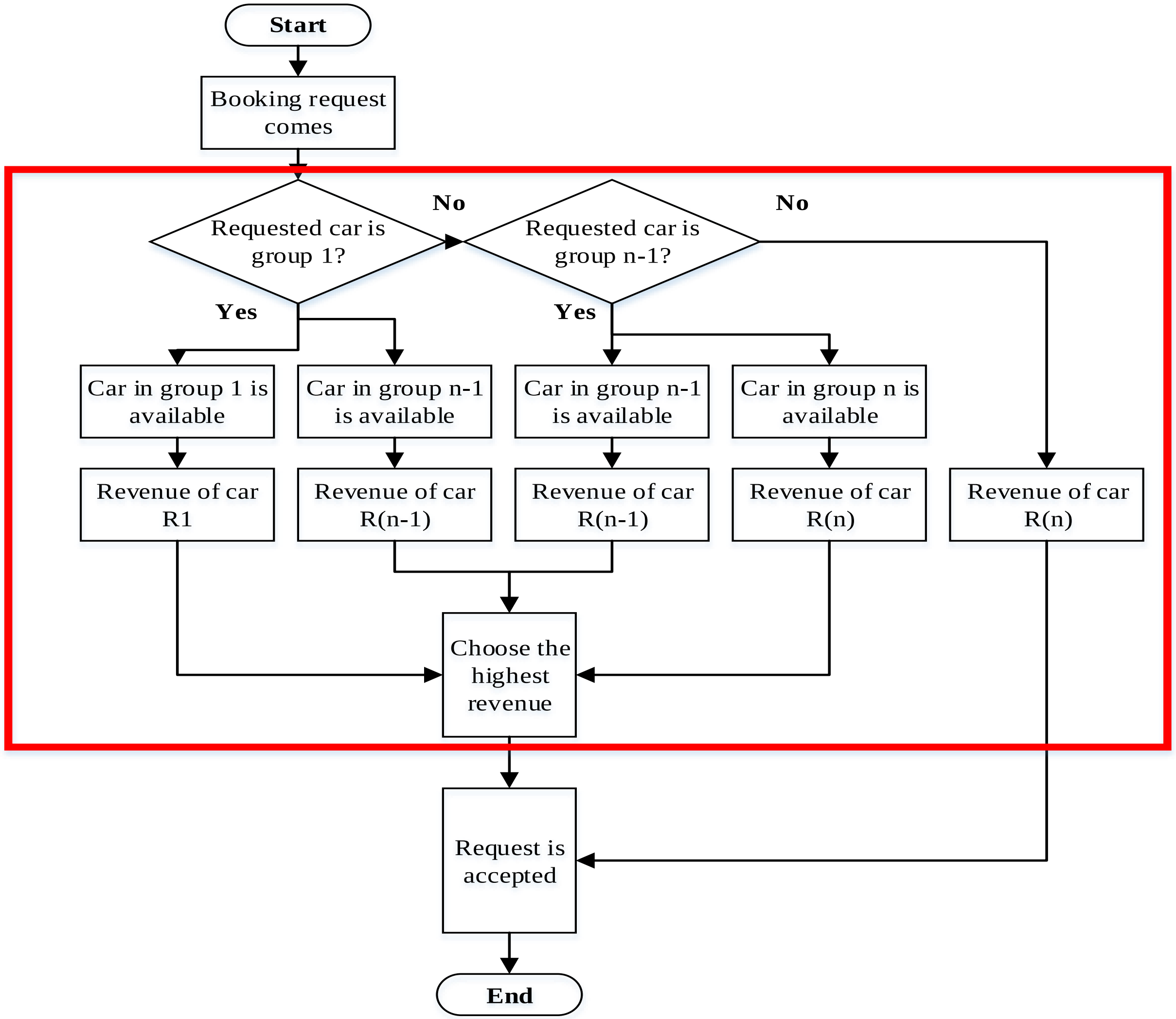

Figure 5.1.2 FCFS policy in upgrade scenario
(n is the number of car rental groups)

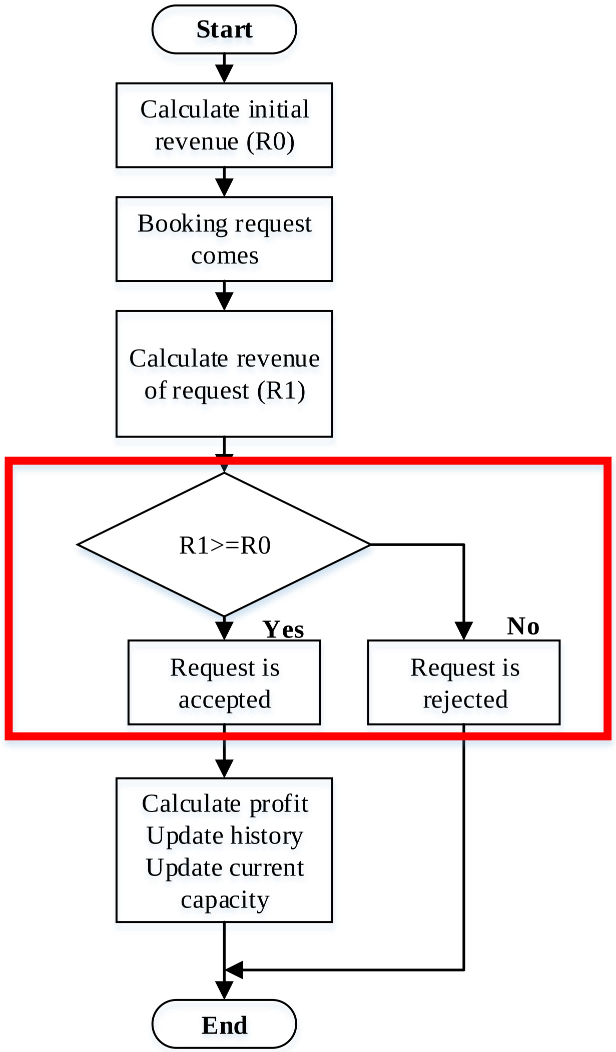

Figure 5.2.1 Opportunity cost policy in non-upgrade scenario

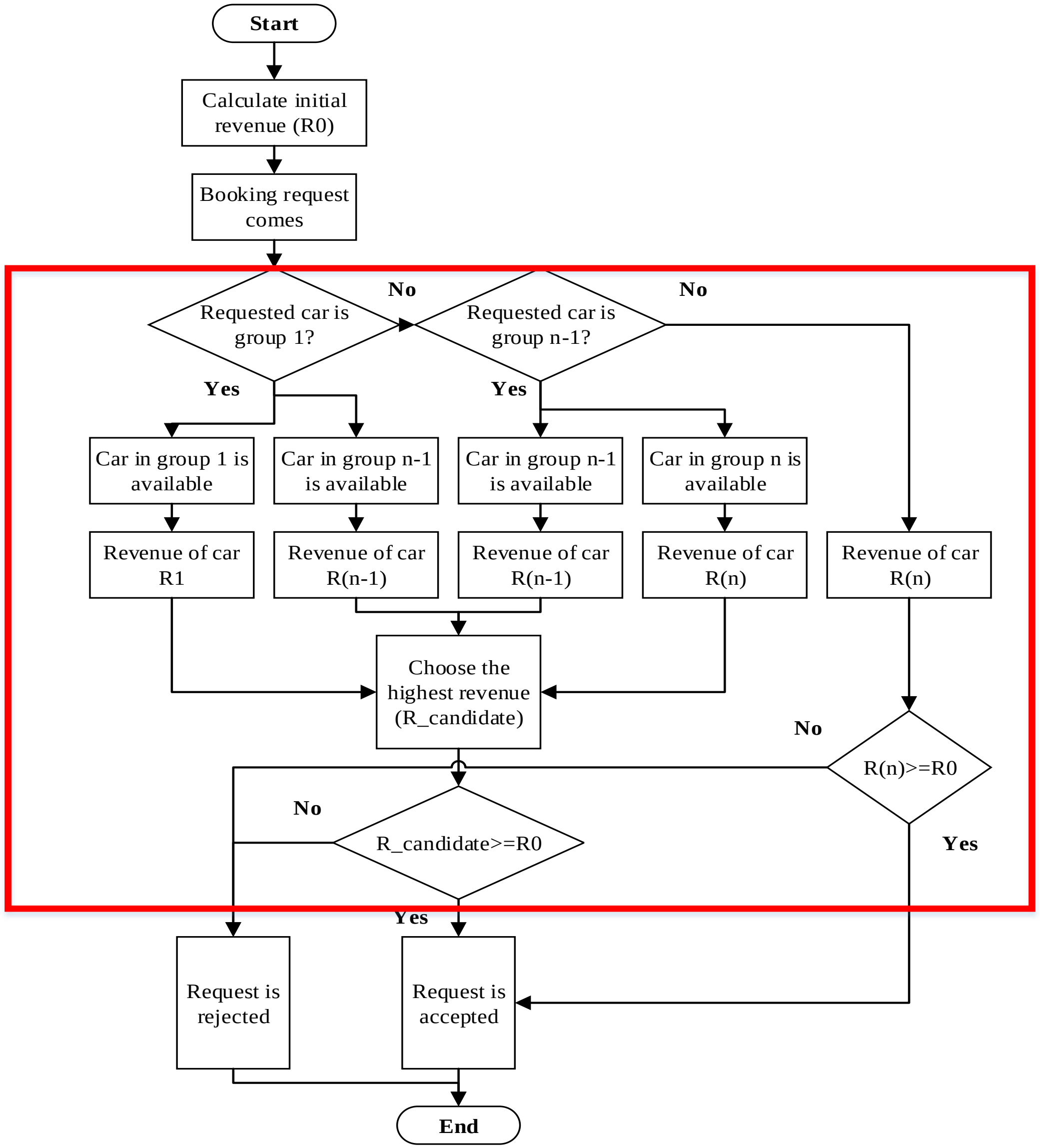

Figure 5.2.2 Opportunity cost policy in upgrade scenario (n is the number of car rental groups)

Table 4.1.1 Q_a^g for both scenarios

	a		
g	1	2	3
1	10	10	10
2	13	13	13
3	8	8	8

Table 4.1.2 $\boldsymbol{\alpha_v^g}$ for non-upgrade scenario

$\boldsymbol{\alpha_v^g}$	Car group is rented potentially (v)		
Customer's request or car group (g)	1	2	3
1	1	0	0
2	0	1	0
3	0	0	1

Table 4.1.3 $\boldsymbol{C^g}$ for both scenarios

Car group	Rental rate of car per month ($)
1	913
2	1,050
3	1,186

Table 4.1.4 π_{z-a} for both scenarios

Length of rent	π_{z-a}
1	0
2	0.91
3	1.18
4	1.67
5	1.824
6	1.91

Table 4.2 $\boldsymbol{\alpha_v^g}$ for upgrade scenario

$\boldsymbol{\alpha_v^g}$	Car group is rented potentially (v)				
Customer's request or car group (k)	1	2	2	3	3
1	1	1	0	0	0
2	0	0	1	1	0
3	0	0	0	0	1

Table 6.1 Comparison average profit (USD) between two polices of two scenarios

	FCFS	Opportunity	Improvement
Non-upgrade	7,097	7,631	7%
Upgrade	7,798	8,296	6%
Improvement	9%	8%	

ACKNOWLEDGMENTS

I would like to express my gratitude to my advisor Dr. Phan Nguyen KyPhuc for the continuous support of my master research, for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research.

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