A Robust Optimization Model for Lot Sizing with Dynamic Demands

Aurora Rostenila
Department of Industrial Engineering
Universitas Katolik Parahyangan (UNPAR), Bandung, Indonesia
Tel: (+62) 203- 2655, Email: rostenilaaurora@gmail.com

Carles Sitompul †
Department of Industrial Engineering
Universitas Katolik Parahyangan (UNPAR), Bandung, Indonesia
Tel: (+62) 203- 2655, Email: carles@unpar.ac.id

Abstract. In today's competitive world, it is imperative that a manufacturer possesses a good production and inventory planning model because customer demands are becoming more dynamic and difficult to forecast. To model the uncertainties of demands at each period, we use three different scenarios of demands mimicking a symmetric triangle distribution. In theory, a larger number of scenarios would bring about a closer look to the real world. Unfortunately, it would also exacerbate the problem exponentially. We propose a lot sizing model with stochastic and dynamic demands using a well-known robust optimization approach. We implement the model using AMPL (A mathematical programming Language) program and test it for different parameters. The result shows that the robust optimization approach provides small variances of cost functions even when we used only three scenarios at each period of demands.

Keywords: Lot sizing, dynamic demands, robust optimization, scenarios.

1. INTRODUCTION

In an ever changing environment, a good production and inventory planning requires some sort of knowledge about the customer demand even if it possesses uncertainties. Although forecasting models are widely investigated, the results always subject to a certain level of uncertainties.

When demands are projected to have a trend, inventory models assuming stationary demands are bound to fail (Wagner and Whitin, 1958). In reality, demands are often not stationary. Aguirregabiria and Nevo (2010) states that dynamic demands can be the results of today's demand which affect future demand. They can also be the results of customer's expectation of the future. For example, a customer expects that in the future price will increase then order will be placed today rather than in the future. Another factor which affects the customer order decision is product availability. When a customer thinks that a product will be scarce in the future, it is more likely to place order in the present rather than waiting for the next periods. Non stationary demands are also a logical consequence of product life cycles. Products in the introduction, growth and declining stages are normally trending upwards or downwards.

It is therefore necessary to deal with non-stationary demands during a production planning. We believe that these dynamic demands together with the fact that they are uncertain should be dealt with in the tactical level in order to give enough time for inventory managers to revise their plans when needed. When demands are deterministic, Wagner and Whitin (1958) proposed an algorithm to solve lot sizing problem aiming at lower total costs.

Demands are however most often uncertain than deterministically known in advance. Even when sophisticated forecasting models are used, errors are still present which brings about stock-outs and inventories. If production level is higher than anticipated demand then inventory cost increases. In reverse, the stock out cost increases if production level is lower than demands. Long range consequences of stock outs are rather difficult to measure such as the decreasing of customer's satisfaction.

Therefore, it is necessary to develop a lot sizing model which includes demands uncertainties early in the planning stages. The term robust optimization approach attempts to proactively include uncertainties in the planning models (Mulvey et al, 1995). The solution obtained from such approach is robust, i.e. insensitive to changing environment, which stays close with the optimal solution. Vanderbei and Zenios (1995) stated that robust approach is basically using goal programming formulas as a result of scenarios reprsented the uncertain parameters. This approach is also more general in term of applicability than stochastic linear programming because it does need knowledge on the distribution of the parameters.

We propose a robust optimization model for lot sizing problems in order to obtain solutions that is robust or insensitive to uncertainties. In doing so, the production planner can avert risks in the future.

2. MATHEMATICAL FORMULATION

2.1 Deterministic Formulation

We studied a make-to-stock production system where inventories appear in the form of finished products. The manufacturing plant produces a variety of products to fulfill independent non stationary demands. Relevant costs to be minimized in the problem are setup and holding costs, as well as stock out costs which is represented by backorder costs (e.g. Gonzales and Tullous, 2004). A single type of product was considered by Gonzales and Tullous (2004) which will be generalized into multiple products manufactured in one single facility.

Let consider the following parameters of the problem

d_{it}= Demands of product *i* in period *t (units)*

A_{it}= Setup costs in of manufacturing product *i* in period *t*

C_{it}= Variable costs of manufacturing product *i* in period *t* per unit

h_{it}= Holding costs per unit product *i* in period *t*

b_{it}= Back order costs per product *i* in period *t*

T = Number of planning periods

P = Number of product types

where $i \in \{1,2,\dots,P\}$ and $t \in \{1,2,\dots,T$

Decision variables of the problem can also be described as follows:

Y_{it}= 1 if production of product *i* takes place in period *t*
0, otherwise.

Q_{it}= The quantity of product *i* manufactured in period *t*

I_{it}= The inventory level of product *i* at the end of period *t*

B_{it}= The backorder level of product *i* at the end of period *t*

The deterministic model can then be formulized as follows.

Minimize

$$\sum_{i=1}^{P}\sum_{t=1}^{T} A_{it}Y_{it} + \sum_{i=1}^{P}\sum_{t=1}^{T} C_{it}Q_{it} + \sum_{i=1}^{P}\sum_{t=1}^{T} h_{it}I_{it} + \sum_{i=1}^{P}\sum_{t=1}^{T} b_{it}B_{it} \qquad (1)$$

Subject to:

$$\sum_{t=1}^{T} Q_{it} = \sum_{t=1}^{T} d_{it}\,;\ \forall\ i \qquad (2)$$

$$Q_{it} + I_{it-1} - I_{it} + B_{it+1} - B_{it} = d_{it} \qquad \forall\ i,\mathrm{j} \qquad (3)$$

$$Q_{it} - \mathrm{M}Y_{it} \le 0;\ \forall\ i,j \qquad (4)$$

$$Y_{it}\ \text{binary},\ Q_{it},\ I_{it},\ B_{it} \ge 0$$

Equation (1) shows the total costs to be minimized comprising of setup costs, variable manufacturing costs, holding costs and back order costs. In Equation (2), total production during planning horizon equals to the total demand. This means that if inventory and backorder at the beginning of planning horizon equals to zero then they will be zero too at the end of the planning horizon. The generalizations of these inventory and backorder levels are rather straightforward. The inventory and backorder balance equations are described in Equation (3) as explained in Figure 1 for each product.

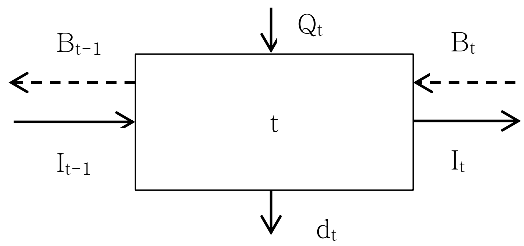

Figure 1: Inventory and backorder balance constraints

Equation (4) shows the setup cost realization when production in period *t* takes place.

2.2 Robust Formulation

We consider a scenario based model to develop a robust plan taking into account that demand is uncertain. Let us assume that demand scenario for each product in a period is taking a triangle distribution with three realizations in the minimum value, mean value and maximum value with probability 0.16, 0.68, and 0.16 respectively as given in Figure 2. This discrete distribution is symmetric and widely used when the knowledge on distribution function is very limited. In a two products problem, for example, the number of scenario will be 3^2 = 9 for each period.

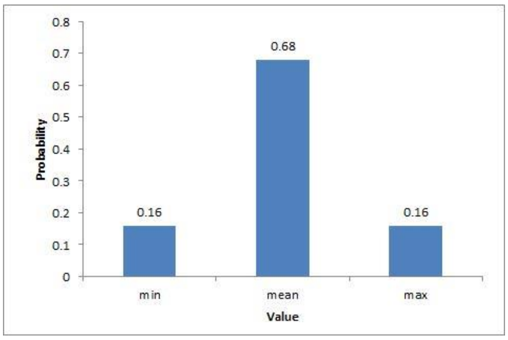

Figure 2: Demand scenario

We used the same concept as Mulvey et al., (1995) to describe the robust definition of a plan. Robust plan is defined as one that is close to optimal solution for any realization of scenarios. It is also near to feasible for any realizations. Let us consider that S is the number of scenario for each period, where demand of product i is realized at the value d_{is} with probability p_s. We set that the decision variables that cannot be changed are the setup variables and the quantity of production even when demand is realized. In contrarily, inventory and back order levels are floating as the demand is realized, which are denoted with I_{its} and B_{its} respectively.
In a multi period planning horizon, unfortunately, the number of scenarios is exponentially expanded in the order s^t which makes it intractable to solve due to enormous number of variables.

We propose an approximation method using the average of floating variables of inventory and back order levels as given in Figure 3 where demands are realized into 3 different scenarios.

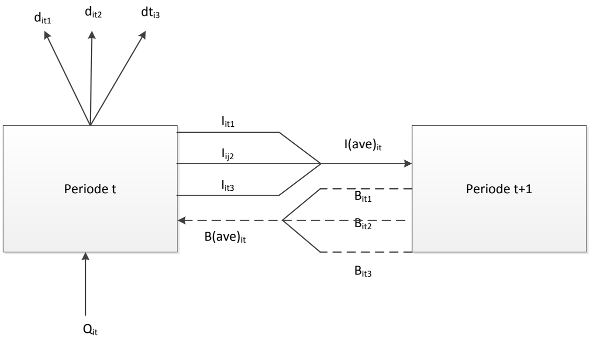

Figure 3: Floating variables

The robust optimization problem can then be formulized as follows.

Minimize

$$\bar{\xi}_{tot} + \lambda \sum_{i=1}^{P} \sum_{t=1}^{T} \sum_{s=1}^{S} p_s (\xi_{its} - \bar{\xi}_{it})^2 + \omega \sum_{i=1}^{P} \sum_{t=1}^{T} \sum_{s=1}^{S} p_s E_{its} \quad (5)$$

Subject to

$$\bar{\xi}_{tot} = \sum_{i=1}^{P} \sum_{t=1}^{T} \bar{\xi}_{it} \quad (6)$$

$$\xi_{its} = A_{it} Y_{it} + C_{it} Q_{it} + E_{its} \quad (7)$$

$$\bar{\xi}_{it} = \sum_{s=1}^{S} p_s \xi_{its} \quad (8)$$

$$E_{its} = h_{it} I_{its} + b_{it} B_{its} \quad (9)$$

$$Q_{it} + \bar{I}_{it-1} - \bar{B}_{it-1} = d_{its} + I_{its} - B_{its} \quad (10)$$

$$\bar{I}_{it} = \sum_{s=1}^{S} p_s I_{its} \quad (11)$$

$$\bar{B}_{its} = \sum_{s=1}^{S} p_s B_{its} \quad (12)$$

$$I_{its} \leq M Z_{its} \quad (13)$$

$$B_{its} \leq M U_{its} \quad (14)$$

$$Z_{its} + U_{its} = 1 \quad (15)$$

$$Q_{it} \leq \mathrm{MY}_{it} \quad (16)$$

Y_{it} , Z_{its}, U_{its} binary,

Q_{it}, I_{its}, B_{its}, $\bar{I}_{it}$, $\bar{B}_{it}$, $\bar{\xi}_{tot}$, ξ_{its}, $\bar{\xi}_{it}$, E_{its} ≥ 0 .

Equation (5) shows the total costs to be minimized comprising of the average costs, the variance of the costs which is weighted with parameter λ and the sum of floating costs which is weighted with parameter ω. The total costs for all periods is given by Equation (6), while Equation (7) shows the scenario costs consisting of the setup and variable production costs (unchanged) and the floating costs. The average of costs in period t is given in Equation (8), while Equation (9) shows the floating costs. Equation (10) is a balance constraint as explained in Figure 3. Equation (11) and Equation (12) shows the average of inventory levels and back order levels respectively. Equation (13), (14) and (15) makes sure that inventory levels and back order levels are complementary to avoid both variables become positives. Equation (16) is the usual setup realization when production takes place in period t. This robust optimization model is non-linear because the objective function consists of a quadratic function. However, all constraints are linear.

3. IMPLEMENTATION

We implement the model into an AMPL (A Mathematical Programming Language) model and then run it in NEOS Server available in the following http://www.neos-server.org/neos/solvers/minco:FilMINT/AMPL.html

We tested the model using 12 illustrative examples in Table 1, where demands for two products are following normal distributions with average μ and variance σ^2. As approximation, the minimum value, mean value, and maximum values take place at $(\mu - \sigma)$, μ, and $(\mu + \sigma)$ respectively for each product for each period. For example, in period t scenario 1 is resulted from the realization of minimum value for product 1 and also the minimum value for product 2. The probability of such scenario is 0.16(0.16) = 0.0256.

The results of the experiments are given in Figure 4, Figure 5 and Figure 6. In Figure 4, the average costs increase when the parameter ω increases to the point that they reach steady states. In other hand, the cost variance decreases when parameter ω increases as shown in Figure 5. The effect of parameter λ is rather the opposite of parameter ω. The robust optimization model is basically using the goal programming concept, where multiple objective functions are weighted by some parameters.

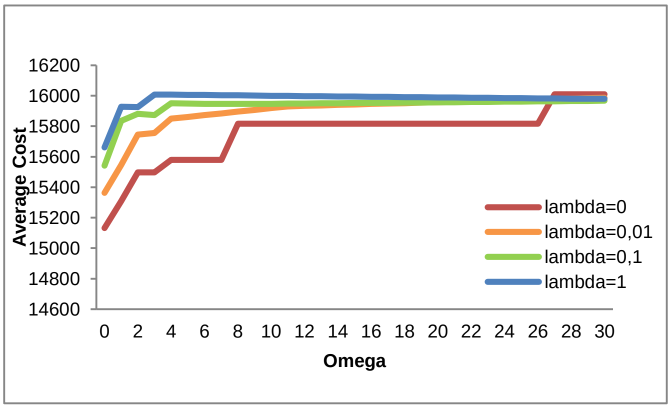

Figure 4: Average costs in function of Omega and Lambda

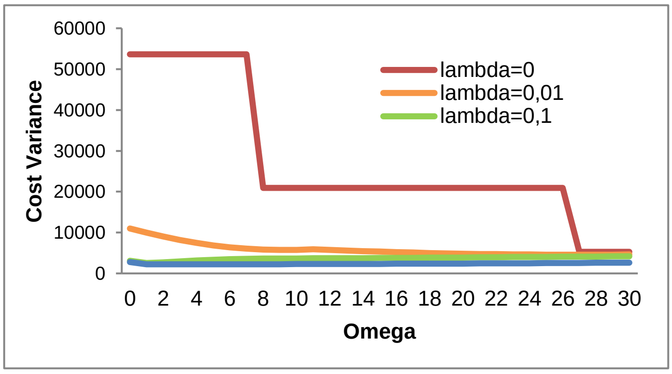

Figure 5: Cost Variances in function of Omega and Lambda

We also tested the solution where demands are realized following a normal distribution. The resulted solution shows that total costs as the performance measure are distributed following a truncated normal distribution. The variance of this distribution is rather small which is expected since the objective function has taken this into account at the beginning of the model. Figure 6 shows the objective function distribution.

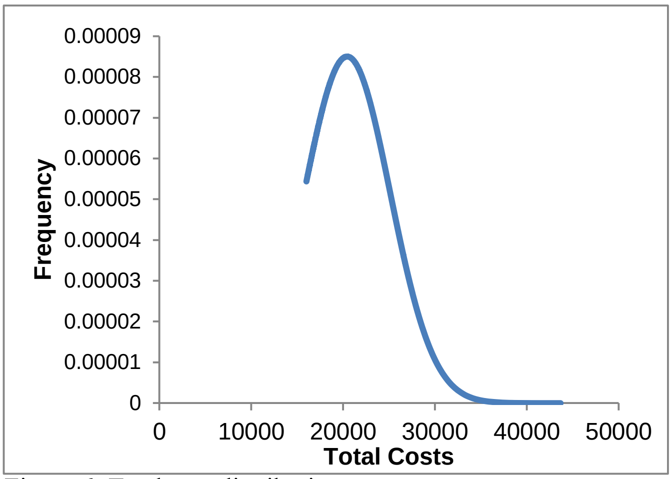

Figure 6: Total cost distribution

Table 1: Illustrative examples

t	Product 1					Product 2				
	d $(\mu_{ij}, \sigma_{ij}^2)$	A	C	h	b	d $(\mu_{ij}, \sigma_{ij}^2)$	A	C	h	b
1	(60,400)	20	4	0,5	10	(80,900)	18	3	0,2	9
2	(65,400)	20	4	0,5	10	(85,900)	18	3	0,3	10
3	(70,400)	21	5	0,6	11	(90,900)	19	4	0,4	11

t	Product 1					Product 2				
	d $(\mu_{ij}, \sigma_{ij}^2)$	A	C	h	b	d $(\mu_{ij}, \sigma_{ij}^2)$	A	C	h	b
4	(75,400)	22	5	0,7	12	(95,900)	19	4	0,5	12
5	(80,400)	22	6	0,8	13	(100,900)	20	5	0,6	12
6	(85,400)	23	6	0,8	14	(100,900)	21	5	0,6	14
7	(90,400)	23	7	0,9	15	(105,900)	22	6	0,7	14
8	(95,400)	24	7	1	15	(105,900)	22	7	0,8	15
9	(100,400)	25	8	1	16	(110,900)	23	7	0,9	15
10	(105,400)	25	8	2	16	(120,900)	23	8	0,9	16
11	(110,400)	26	9	2	17	(125,900)	24	8	1	16
12	(115,400)	26	10	3	18	(130,900)	24	9	2	17

4. CONCLUSION AND FURTHER RESEARCH

We conclude that lot sizing problem with dynamic demands can be modeled using the robust optimization model. The performance of robust model is seen in the form of the variance of performance measures, i.e. total costs. The result confirmed previous studies that when addressed earlier in the model, uncertainties effect to the plan is less visible. In other words, the plan is robust or less risk against demand uncertainties. Furthermore, the robust optimization approach can be very beneficial in real life practices.

Further research is directed toward the use of other demand distribution, especially non-symmetric distributions such as exponential distribution. We also need to investigate deeper on the performance of pre-emptive goal programming instead of weighted goal programming. For example, we can say that the variance of the total costs has greater importance more than the average.

In the modeling process, we assumed that setup and production quantities are variables that cannot be changed once they are decided. In the future, it is necessary to look into the problem where only setup decisions are unchanged, while production quantities can be changed into some degree depending on the realization of demands.

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