Evaluating the Need of Dynamic Stock Balancing for Bike Sharing

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Abstract. In bike sharing systems, the flow of bikes is dynamic and uneven across bike stations and in different times of the day. An operation problem is rebalancing of bike stocks between stations. This operation requires a fleet of transportation vehicles and teams of staff. It incurs very significant cost and has direct effects on quality of services. In this paper, a model based on Poisson process is first presented to analyze the supply and demand characteristics of single stations. A mathematical programming formulation is then used to determine the optimal quantity of bike transfers.

Keywords: Bike sharing systems, Dynamic balancing, Quality of services

1. INTRODUCTION

Bike sharing systems have been increasing all over the world in recent years. A bike sharing system is a facility that provides public bikes to users who need a bike to make a short journey. Bike sharing systems (BSS) are usually located near mass transportation stations, parks, schools, residential areas and business centers in order to offer urbanites alternative modes of transport, especially for the short route.

To use bike sharing systems, a user picks up a bike in one station and typically will return it to another station. The flow of bikes is dynamic across bike stations and in different times of the day. It is common that during some time periods some stations have a large demand but have insufficient quantity of bikes. Also, there are times that all parking posts at a station might be fully occupied by bikes and a user who wants to return a bike cannot find an empty post. These problems of stock-out and blocking of returns bring about customer complaints. Thus, one fundamental problem of system operation is periodical reallocation of bikes between stations that have too many bikes and stations that are in short supply of bikes. In principle, increasing the bike stock at all stations or critical stations could mitigate service quality problems, but the associated costs are quite high and most BSSs already rely on public subsidy. This is a problem of trading off between service quality and costs (Lu, 2013).

The design and operation of BSSs have been studied in many research words. Research topics include bike station location (García-Palomares et al., 2012; Midgley, 2009; Hu & Liu, 2013), and integration of mass transportation and bike sharing system (Midgley, 2011).) Normally, the location of bike stations must take into consideration existing points of public transportation stations and desired destinations of customers in order to provide the missing links (Midgley, 2009).

Re-allocation of bikes is known in the literature as the bike rebalance (BR) problem. The costs of bike rebalance is quite high. BSSs are usually operated by city government directly or indirectly through contracts. The operating cost including maintenance, staff, insurance, office space and so on are born by the operator. Demaio (2009) reports that the reallocation of bikes is about 3 US dollars per bike on average.

The BR problem has two versions: static or dynamic optimization. Static rebalance is also called periodical rebalancing. Rebalancing operation is mostly executed during non-busy periods of the day. Ho and Szeto (2014) studied static the BR problem. They consider a procedure of selecting a set of stations to visit, sequencing them and determining the number of bikes to pick up and drop off on each station. They utilize the iterated tabu search heuristic for minimizing a total penalty. Vogel et al. (2014) apply mixed integer linear programming to minimize the total expected costs of bike reallocation and unsatisfied demand. The result also yields relocation operation and fill level at each station in whole day. Di Gaspero et al. (2014) apply two Constraint Programming models to the BR problem: a routing model base on classical Vehicle Routing Problem, and a step model that takes a planning perspective of the problem. Raviv et al. (2013) constructed two Mix Integer Linear Programming formulations that include stochastic and dynamic factors of demand in the objective function.

A few authors address the dynamic reallocation problem. Contardo et al. (2012) have introduced a dynamic public bike-sharing balancing problem (DPBSBP) from the daily operations of BSS during peak hours. They not only provide mathematical formulation but also develop a scalable methodology that provides lower and upper bounds in short computing times. Caggiani & Ottomanelli (2012) propose a decision support system for dynamic bike redistribution process to minimize vehicle repositioning costs while keeping high-level of user satisfaction. In this work, a neural network is used to forecast the bike demand at stations. Sayarshad et al. (2012) construct a multi-period mathematical formulation to maximize the profit of rented bikes by including allocation cost, operating cost, cost of holding bikes at station, capital cost per period and penalty cost of unmet demand.

The literature review shows that both static rebalancing and dynamic rebalancing can improve the efficiency of BSSs. Static rebalancing has been implemented in most BSSs. But dynamic rebalancing needs to be economically justified. The objective of this paper is to evaluate the potential contribution of dynamic rebalancing.

2. PROBLEM DESCRIPTION

We consider a BBS based on the 400-station “Ubike” system in Taipei, Taiwan. Each station, indexed by subscript *i*, has a capacity k_i of bike racks. The quantity of available bikes in each station is dynamic information that is accessible in real-time. Let y_i be the quantity of available bikes. Then the number of vacant racks equals $\omega_i - y_i$. The arrival and departure of bikes are assumed to follow Poisson processes. Dynamic balancing has a short time frame, such as 20 minutes, as opposed to periodical rebalancing. We assume the arrival and departure rates to be constant in the time horizon and the net arrival rate is represented as λ_i.

Stations will be classified as surplus, deficient, or normal stations, based on their stock level of bikes. Stations that have more bikes than expected are called surplus stations. In contrast, stations that have fewer bikes than expected are called deficient stations. They have requirements of additional bikes. Normal stations are those that are not surplus or deficient stations. Whether the status of a station is surplus, deficient or normal is not fixed or static. Instead, the status changes dynamically. Our proposed method utilizes two thresholds in bike quantity to monitor the status: $\underline{y}_i$ and $\overline{y}_i$. If the bike quantity of a station is fewer than $\underline{y}_i$, then the station is deemed a deficient station. A station is a surplus station if its bike quantity is greater than or equal to $\overline{y}_i$.

The problem addressed in this paper has the following characteristics:

1. Customer can pick-up the bike at a station and return it either at the pick-up station or other stations.
2. A customer can only rent one bicycle in each journey.
3. If no bicycle is available at the rental station at the time customer arrive, the customer will leave system immediately.
4. Dynamic rebalancing takes place in the next time period.

Figure 1 illustrates three categories of bike stations in the BSS in Taipei. The objective of dynamic rebalancing is to determine the quantity of bikes $x_{i,j}$ that are transferred from station *i* to station j to optimize some functions of service quality and utilization. The problem is characterized by uncertain supply and uncertain demand. A one station analysis will be presented in the next section. A multiple station model of mixed integer linear program will be presented in section 4.

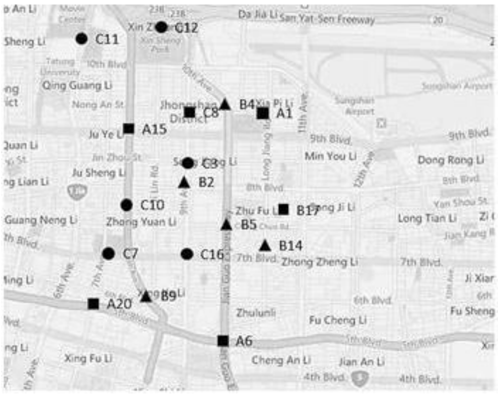

Figure 1: the deficient and surplus bike stations

3. An analysis of one station

In this section, it is assumed that the net arrival of bikes to a station follows a Poisson process. Given a demand function $f_D(y)$ and a supply function $f_s(x)$, the decision on hand is to determine the quantity of supply q. A prominent characteristic of this problem is that both demand and supply are uncertain. It should be noted that, because the supply is also uncertain, the actual delivery might be less than any optimized q.

Suppose the planned supply is a variable q. Let $F_D(y)$ and $F_s(x)$ be the CDF of $f_D(y)$ and $f_s(x)$, respectively. Since a supplied bike might not be utilized, we are concerned with the utility of each bike that is to be supplied. We are also concerned with service quality.

1. The q bikes will be fully utilized with a total probability $1 - F_D(q)$. This is also the probability of stock-out in inventory theory.
2. Average utilization (u) = E(min(Y, q))/q
3. The expected value of the shortage (l) is given by the partial expectation E(Y/Y>q).

As q increases, both average utilization and shortage decrease. We shall find tradeoffs between these effects of q. The average utilization is = E(min(Y, q))/q.

$$E(\min(Y,q)) = \sum_{y=0}^{q-1} y \cdot P(y) + \sum_{y=q}^{\infty} q \cdot P(y)$$

$$u = \frac{1}{q}\sum_{y=0}^{q-1} y \cdot P(y) + \frac{1}{q}\sum_{y=q}^{\infty} q \cdot P(y) = \frac{1}{q}\sum_{y=0}^{q-1} y \cdot P(y) + 1 - F_y(q-1) \tag{1}$$

The expected loss is

$$l = E(Y \mid Y > q) = E(\max\{Y-q, 0\}) = \sum_{y=q+1}^{\infty} (y-q)P(y) \tag{2}$$

The average utilization and expected loss are shown in smoothed curves in Figure 2 for λ=1,2,…,5. They will be called the u and l functions. The solution q and the corresponding expected loss and utilization will be denoted as q_0, l_0 and u_0, and it can be seen that

$$l_0 = \frac{\lambda}{q_0 + 1} \quad \text{and} \quad u_0 = \frac{\lambda - l_0}{q_0} \tag{3}$$

The intersection between each pair of u and l functions is a characteristic of this problem. It can be determined by solving the equality $u(q)=l(q)$:

$$\frac{1}{q}[\lambda - \sum_{y=q+1}^{\infty} (y-q) \cdot P(y)] = \sum_{y=q+1}^{\infty} (y-q)P(y)$$

$$(q+1)\sum_{y=q+1}^{\infty} (y-q)P(y) = \lambda \tag{4}$$

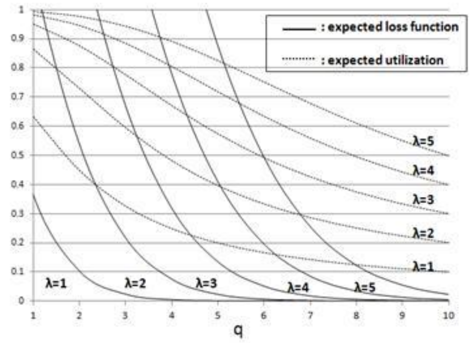

Figure 2: The average utilization and expected loss

Consider an optimization problem to determine q under the two criteria of utilization and lost customers. The objective function can be expressed as $Z = \omega \cdot u - l$, where w is a weight. Geometrically, the objective value is the gap between the curves of u and l (Figure 2). Whereas unit weight is used in plotting Figure 2, using a non-zero weight merely changes the altitude of ωu and does not change the characteristics of the intersection between the two curves. Optimization over q can be treated as integer nonlinear programming problem.

Since the supply is also uncertain, the above analysis on the equality $u(q)=l(q)$ can be analogously extended to the expressions: $\omega \cdot u(q)P(X = q)$ and $l(q)P(X = q)$. The optimization over q is also an integer nonlinear programming problem.

4. Optimization over multiple stations

While each station has its characteristic of supply or demand, a mathematical program can be used to determine the quantity of bikes that are to be transferred. Let q_j be the quantity of insufficient bikes at stations *j and* s_i be the quantity of available supply from surplus station *i*. Let $d_{j,j}$ be the distance between stations *i* and stations *j and let* d_{max} be the maximum distance that is imposed.

$$\text{Maximize} = \sum_j \sum_i X_{i,j}$$

$$\text{s.t.} \quad \sum_i x_{i,j} \le \sum q_j, \ \forall j \tag{5}$$

$$\sum_j x_{i,j} \le s_i, \forall i \tag{6}$$

$$\text{If } y_i \le \underline{y}, \text{ then } q_j = \underline{y} - y_j, \text{ else } q_i = 0 \ \forall j \tag{7}$$

$$\text{If } y_i \le \underline{y}, \text{ then } s_i = y_i - \underline{y}, \text{ else } s_i = 0, \ \forall i \tag{8}$$

$$\text{If } d_{i,j} \ge d_{\max} \text{ then } x_{i,j} \le 0, \forall ij \tag{9}$$

The model is to find the maximum number of allocation bicycle $X_{i,j}$. Constraint (5) ensures that the total quantity of transfers does not exceed the total quantity of demand.

Constraint (6) ensure that the total quantity of transfers does not exceed the total quantity of supply. Constraints (7), and (8) quantity the demand and supply, respectively. Constraint (9) is the maximum distance constraint.

5. NUMERICAL EXAMPLE

A numerical example is used to demonstrate the above procedure for dynamic rebalancing. The basic data is shown in Table 1. The arrivals and departures are generated following the Poisson process. Bike stock is shown in Table 2, wherein insufficient stations are highlighted in gray background.

Using the proposed procedure, the solution of the allocation is listed in Table 3. The bike from surplus stations *i* was transferred to deficient stations *j* with its required amount. The total quantity of transfers is 52 bikes.

Table 1: Starting bike stock and expected arrival and departure rate

station		1	6	15	19	20	2	4	5	9	13	14	17	18	3	7	8	10	11	12	16
1700		14	2	18	22	20	4	18	22	33	33	47	59	13	4	18	4	21	1	14	33
1710	in	2	2	2	1	4	2	0	0	3	3	1	3	1	0	1	1	1	1	0	3
	out	1	0	1	0	0	1	2	0	3	1	1	2	0	2	5	4	3	2	4	4
1720	in	1	3	6	4	1	2	1	0	2	2	2	2	2	2	1	1	0	1	1	2
	out	0	1	2	1	1	2	2	2	1	1	0	0	0	2	3	8	3	6	3	4
1730	in	3	4	4	3	3	1	3	3	3	2	4	3	0	0	0	2	2	1	1	1
	out	0	0	1	1	0	1	1	1	4	2	2	1	3	3	6	3	6	2	3	3
1740	in	3	2	6	2	3	4	1	2	2	1	2	2	2	1	0	0	0	0	1	0
	out	0	0	0	0	3	2	0	1	0	2	2	1	0	3	6	3	6	3	4	2
1750	in	5	2	4	6	6	5	3	1	2	3	2	2	1	3	0	2	3	1	1	3
	out	2	0	1	0	0	2	3	1	2	2	2	4	3	5	7	3	3	5	5	4

Table 2: Bike stock before reallocation

station		1	6	15	19	20	2	4	5	9	13	14	17	18	3	7	8	10	11	12	16	shortage
Bikecycle's stock befor allocate	5:00 PM	14	2	18	22	20	4	18	22	33	33	47	59	13	4	18	4	21	1	14	33	3
	5:10 PM	15	4	19	23	24	5	16	22	33	35	47	60	14	2	14	1	19	0	10	32	6
	5:20 PM	16	6	20	26	24	5	13	19	34	36	49	62	16	3	12	-4	16	-2	8	30	12
	5:30 PM	19	10	18	28	27	5	8	21	33	36	51	64	13	0	6	2	12	2	6	28	5
	5:40 PM	22	12	23	30	27	7	8	19	35	35	51	65	15	1	0	0	6	0	3	26	11
	5:50 PM	25	14	23	36	33	10	5	19	32	36	51	63	13	1	-4	2	4	-1	-1	25	18

Table 3: Rreallocated bike quantity from station i to station j

Time 17:10			Time 17:20			Time 17:30			Time 17:40			Time 17:50		
i	j	bikes	i	j	bikes	i	j	bikes	i	j	bikes	i	j	bikes
4	8	2	4	8	7	4	8	1	4	8	3	2	8	1
15	11	3	15	11	5	15	11	1	15	11	3	15	11	4
5	3	1				5	3	3	9	7	3	15	12	4
									10	3	2	4	3	2
												9	7	7
Total		6	Total		12	Total		5	Total		11	Total		18

6. CONCLUTION

In this paper, we address the dynamic stock rebalancing for bike sharing. The analysis of uncertain supply and demand of one-to-one station was determined, the analysis is focused on the total utilization and total expected loss. A mathematical programming formulation is used to optimize the quantity of bike transfers. The numerical example shows the feasibility of dynamics rebalancing in the short-time period.

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