

Bicycle-sharing with reallocation trucks and private exchange

– Taipei YouBike system as example

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Abstract. In bicycle-sharing systems, because the number of bicycles is limited, an unbalanced distribution of bicycles at docks can significantly decrease the system utilization. We consider trucks for bicycle reallocation and smart phone APPs for private exchange. For bicycle reallocation, trucks are hired to dynamically redistribute bicycles among unbalanced stations. For private exchange, smart phone APPs are used to transfer bicycles among users without docks. This allows bicycle exchange even when all docks are full. Three core objectives are considered in this study. The first is to maximize the total trips of bicycles, which is highly related to the private vehicle replacement ratio. The second is to maximize the net profit of system, which is highly related to the sustainability of bicycle-sharing systems. The net profit includes the income of bicycle trips and the costs of truck operation, dock construction, and APP system maintenance. The third is to optimize the fleet sizes of bicycles and trucks, and determines the best initial distribution of bicycles and docks. Integer programming models are built and real data from the Taipei YouBike system is used in the computational study. Two methods, linear relaxation and Particle Swarm Optimization are adopted to solve and verify our models.

Keywords: Bicycle-sharing, Bicycle reallocation, Dock distribution, Private exchange

1. INTRODUCTION

In recent years, environmental issues have encouraged cities to expand their public transportation systems in order to replace private vehicles, save energy, and reduce carbon emissions. There are many types of mass public transportation systems that have stations for passengers to start their journey, such as trains, Mass Rapid Transit (MRT) and buses. To increase their efficiency, stations are positioned with long distances between them. A public bicycle-sharing system (BSS) is adopted in many cities as a way to move passengers to mass transit stations of public transportation systems. This not only extends accessibility, but also reduces construction costs. The YouBike system is an example of a BSS in Taipei city, Taiwan. In order to promote bicycles for short distance transitions, the Department of Transportation of the Taipei city government launched a program to construct the YouBike system in 2009.

Due to the limited numbers of bicycles stations and

docks, an unbalanced distribution of bicycles is an important issue that decreases the system utilization. To address this issue, we build integer programs to model trucks for bicycle reallocation and APPs for private exchange. The decision is to find the best initial allotment of bicycles and docks. We consider two different but important objectives. The first objective is to maximize the net profit of a BSS. The net profit includes the income of bicycle trips and the costs of truck operation, dock construction, and APP system maintenance. The other objective is to optimize the fleet sizes of bicycles and trucks, and to decide the best initial distribution of bicycles and docks. Integer programming models are constructed and solved by Particle Swarm Optimization (PSO). Also, the linear relaxation of the three integer programming models is solved by CPLEX. Numerical studies are implemented with the real data of the YouBike system in 2013.

This paper is organized as follows. We review some import BSS literature in Section 2. In Section 3, we define

our problem and construct the integer programming models. In Section 4, computational studies are presented that study the impact of truck reallocation and private exchange. Conclusions and directions of future research are discussed in Section 5.

2. LITERATURE REVIEW

DeMaio (2004) reviews the history of bicycle-sharing, from the first generation programs to the most recent third generation programs, including the examination of provision models with benefits and operating costs. DeMaio (2004) also considers the future with a discussion about what a fourth generation bicycle-sharing program might look like. Raviv and Kolka (2013) propose an inventory model to study the bicycle management of stations in bicycle-sharing system. Their study is based on a single station, which helps decision making on docking capacity and bicycle redistribution. Nair and Miller-Hooks (2011) study the truck redistribution problem and construct a stochastic mixed-integer programming model with joint chance constraints. Schuijbroek et al. (2013) further develop a model to optimize routes for bicycle redistribution. Contardo et al. (2012) investigate the dynamic scenario where rebalancing is continuing while the bicycle-sharing system is in use. For randomly created instances, this approach is able to find feasible solutions to problems with up to 100 stations and 60 time periods. However, significant gaps between lower and upper bounds still remain. Chemla et al. (2012) consider only one redistribution truck to pickup and delivery bicycles among stations. They describe a branch-and-cut approach utilizing an embedded tabu search procedure for locally improving incumbent solutions.

Shu et al. (2013) propose a deterministic linear programming model to mimic the bicycle-sharing system and compare the result of simulation on real data. Their notation and model are as follows.

- S : Set of stations, where $S = \{1, 2, \dots, s\}$
- N : Set of time periods, where $N = \{1, 2, \dots, n\}$
- $r_{ij}(t)$: Travel demand from station i to station j , for $i, j \in S, t \in N$
- $y_i(0)$: Initial bicycle allotment at station i , for $i \in S$
- $y_i(t)$: Number of bicycles at station i at the beginning of time period t , for $i \in S, t \in N$
- $y_{ii}(t)$: Number of bicycles remaining at station i during time period t , for $i \in S, t \in N$
- $x_{ij}(t)$: Number of bicycles traveling from station i to station j during time period t , for $i, j \in S, t \in N$.

$$\text{Max} \quad \sum_{t \in N} \sum_{i \in S} \sum_{j \in S: j \neq i} x_{ij}(t)$$

(O) s.t.

$$\frac{x_{ij}(t)}{x_{il}(t)} = \frac{r_{ij}(t)}{r_{il}(t)}, \quad (1)$$

for $i, j, l \in S, t \in N$

$$x_{ij}(t) \leq r_{ij}(t), \quad (2)$$

for $i, j \in S, t \in N$

$$y_i(1) = y_i(0), \quad (3)$$

for $i \in S$

$$y_i(t) = y_{ii}(t) + \sum_{j \in S: j \neq i} x_{ij}(t), \quad (4)$$

for $i \in S, t \in N$

$$y_i(t+1) = y_i(t) - \sum_{j \in S: j \neq i} x_{ij}(t) \quad (5)$$

$$+ \sum_{j \in S: j \neq i} x_{ji}(t)$$

for $i \in S, t \in N$

$$x_{ij}(t), y_i(t), y_{ii}(t) \geq 0,$$

for $i, j \in S, t \in N \cup \{0\}$.

Constraint (1) is proportionality constraint, which requires that the proportions of traveling-out bicycles and travel demand should be equal. Constraint (2) ensures traveling bicycles must less than or equal to the travel demand. Constraint (3) provides the initial allotment of bicycles. Constraint (4) describes that bicycles either remain at the same station or travel to another station in each time period. Constraint (5) are flow balance equations, which ensures that the number of available bicycles at station i in the beginning of time period $t+1$ equals the number of unused bicycles plus the number of arriving bicycles minus the number of departing bicycles, at station i during time period t . The goal of Model O is to maximize total bicycle trips.

3. PROBLEM FORMULATIONS

In this section, we first define our problem and provide the assumptions used in this study. Based on Model O, there are three integer programming models proposed for our problem. The first and second is for bicycles

reallocation without and with flow balance of trucks, respectively. The third is for APPs of private bicycles exchange.

3.1 Problem definition and assumptions

Consider a BSS. At the beginning of each time period, passengers arrive at some station to use the bicycles to travel to the other stations. A decision maker needs to decide the initial allotment of bicycles at each station. The objective is to maximize the income of total bicycle trips.

The following assumptions are used in this paper:

- (i) Bicycles travel begins at the beginning and ends at the conclusion of each time period. Travel that requires two or more time periods is not considered.
- (ii) Bicycles do not leave and return to the same station.
- (iii) Bicycles and reallocation trucks are two different types of vehicles.
- (iv) Travel demand from one station to another station is given.
- (v) Private bicycles exchange is allowed only if all docks are full. Otherwise, passengers must return and rent bicycles at docks.

3.2 Bicycles reallocation with trucks

In this subsection, we consider trucks to reallocate bicycles in a BSS. The trucks can move idle or surplus bicycles from one station to another station to balance the distribution of bicycles. As a result, more travel demand can be met.

Our objective is to maximize the net profit of system. The net profit includes income of bicycle trips and operation cost of truck trips. The following notation used.

- b : Index of vehicle types, where $b = 1$ for bicycles and $b = 2$ for trucks
- q_b : Number of total b vehicles in the system, for $b \in \{1,2\}$
- c : Operation cost of trucks per trip
- h : Capacity of a truck
- $y_{ib}^B(0)$: Initial allotment of b vehicles at station i , for $i \in S, b \in \{1,2\}$
- $y_{ib}^B(t)$: Number of b vehicles at station i at the beginning of time period t , for $i \in S, t \in N, b \in \{1,2\}$

$y_{ib}^R(t)$: Number of b vehicles remaining at station i at the middle of time period t , for $i \in S, t \in N, b \in \{1,2\}$

$x_{ijb}(t)$: Number of b vehicles traveling from station i to station j during time period t , for $i, j \in S, t \in N, b \in \{1,2\}$

$u_{ij}(t)$: Number of bicycles carried by trucks from station i to station j during time period t , for $i, j \in S, t \in N$.

We generalize Model O by Shu et al. (2013) as follows.

$$\text{Max} \quad \sum_{t \in N} \sum_{i \in S} \sum_{j \in S: j \neq i} x_{ij1}(t) - c \sum_{t \in N} \sum_{i \in S} \sum_{j \in S: j \neq i} x_{ij2}(t)$$

(T) s.t.

$$\frac{x_{ij1}(t)}{x_{i11}(t)} = \frac{r_{ij}(t)}{r_{i1}(t)}, \quad (1)$$

for $i, j, l \in S, t \in N$

$$x_{ij1}(t) \leq r_{ij}(t), \quad (2)$$

for $i, j \in S, t \in N$

$$y_{i1}^B(1) = y_{i1}^B(0), \quad (3)$$

for $i \in S$

$$\sum_{i \in S} y_{i1}^B(0) \leq q_1, \quad (6)$$

$$y_{i1}^R(t) = y_{i1}^B(t) - \sum_{j \in S: j \neq i} x_{ij1}(t) \quad (7)$$

$-\sum_{j \in S: j \neq i} u_{ij}(t),$

for $i \in S, t \in N$

$$y_{i1}^B(t+1) = y_{i1}^R(t) + \sum_{j \in S: j \neq i} x_{ji1}(t) \quad (8)$$

$+\sum_{j \in S: j \neq i} u_{ji}(t),$

for $i \in S, t \in N$

$$\sum_{i \in S} y_{i2}^B(0) \leq q_2, \quad (6')$$

$$y_{i2}^B(1) = y_{i2}^B(0),$$

for $i \in S$

$$y_{i2}^R(t) = y_{i2}^B(t) - \sum_{j \in S: j \neq i} x_{ij2}(t), \quad (7')$$

for $i \in S, t \in N$

$$y_{i2}^B(t+1) = y_{i2}^R(t) + \sum_{j \in S: j \neq i} x_{ji2}(t), \quad (8')$$

for $i \in S, t \in N$

$$u_{ij}(t) \leq h \cdot x_{ij2}(t), \quad (9)$$

for $i, j \in S, t \in N$

$$x_{ijb}(t), y_{ib}^B(t), y_{ib}^R(t) \geq 0,$$

for $i, j \in S, t \in N \cup \{0\},$
 $b \in \{1,2\}$

$$u_{ij}(t) \geq 0,$$

for $i, j \in S, t \in N.$

Constraints (1)-(3) appear in Model **O**. Constraint (6) allocates all q_1 bicycles to stations at the beginning of time 0. Constraint (7) keeps the bicycle balance at the middle of each time period. That is, the number of bicycles at the middle of time period t is equal to the beginning bicycles minus the leaving bicycles (by passengers and trucks) at the beginning of time period t . Constraint (8) shows the bicycle flow balance cross time periods. That is, the number of bicycles at the beginning of time period $t+1$ is equal to the number of bicycles at the middle of time period t plus the arriving bicycles (by passengers and trucks) at the end of time period t . See Figure 1. Similarly, Constraints (6'), (7), and (8') keep the flow

balance of trucks. Constraint (9) ensures that trucks cannot overload.

Note that, we can remove Constraints (6')-(8') and the flow balance of trucks is ignored. The simplified model (without Constraints (6')-(8')) is called Model **T-simple**.

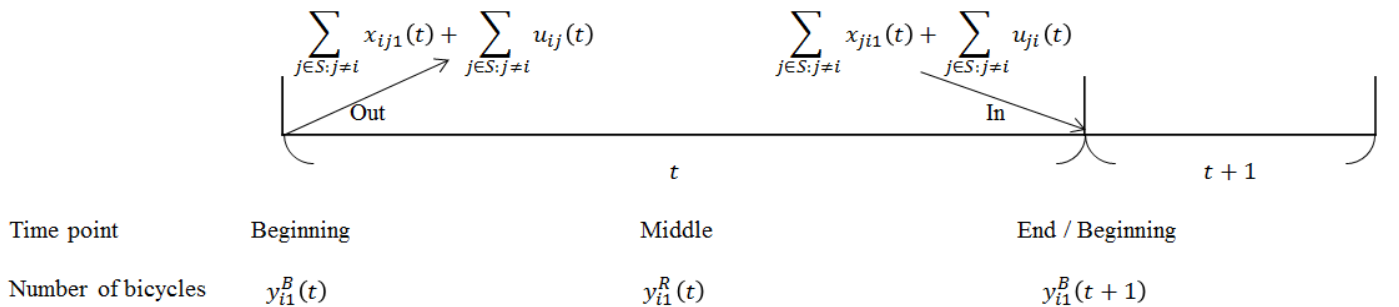
3.3 Private exchange

We also consider the smart phone APPs, which can transfer the registration of bicycles face-to-face without docks. When all docks are full at some station, passengers of returning bicycles can privately exchange bicycles by APPs with other passengers of renting bicycles directly. When no passenger rents a bicycle, we assume that there exists a staff at each station to accept returning bicycles by APPs. As a result, private exchange by APPs not only saves construction cost of docks but also improves passenger satisfaction. Based on Model **O**, we propose Model **PE** to consider the private exchange. Our objective is to maximize the net profit of system. The net profit includes income of bicycle trips and cost of docks and private exchange. The following additional notation is needed.

- d : Number of docks
- c_d : Construction cost per dock
- c_e : Private exchange cost per bicycle
- $y_i^B(0)$: Initial bicycle allotment at station i , for $i \in S$
- $y_i^B(t)$: Number of bicycles at station i at the beginning of time period t , for $i \in S, t \in N$
- $y_i^R(t)$: Number of bicycles remaining at station i at the middle of time period t , for $i \in S, t \in N$
- d_i : Initial dock allotment at station i , for $i \in S$
- $e_i(t)$: Number of private exchange at station i during time period t , for $i \in S, t \in N.$

Model **PE** is as follows:

Figure 1: Time period of Model **T**.



$$\text{Max} \quad \sum_{t \in N} \sum_{i \in S} \sum_{j \in S: j \neq i} x_{ij}(t) - c_d \sum_{i \in S} d_i - c_e \sum_{t \in N} \sum_{i \in S} e_i(t)$$

(PE) s.t.

$$\frac{x_{ij}(t)}{x_{il}(t)} = \frac{r_{ij}(t)}{r_{il}(t)}, \quad (1)$$

for $i, j, l \in S, t \in N$

$$x_{ij}(t) \leq r_{ij}(t), \quad (2)$$

for $i, j \in S, t \in N$

$$y_i^B(1) = y_i^B(0), \quad (3)$$

for $i \in S$

$$\sum_{i \in S} y_{i1}^B(0) \leq q_1, \quad (6)$$

$$y_i^R(t) = y_i^B(t) - \sum_{j \in S: j \neq i} x_{ij}(t), \quad (7)$$

for $i \in S, t \in N$

$$y_i^B(t+1) = y_i^R(t) + \sum_{j \in S: j \neq i} x_{ji}(t), \quad (8)$$

for $i \in S, t \in N \setminus \{n\}$

$$\sum_{i \in S} d_i \leq d, \quad (10)$$

$$e_i(t) \geq y_i^B(t+1) - d_i, \quad (11)$$

for $i \in S, t \in N \setminus \{n\}$

$$x_{ij}(t), y_i^B(t), y_i^R(t) \geq 0, \quad (9)$$

for $i, j \in S, t \in N \cup \{0\}$.

Constraints (1)-(3) appear in Model **O**. Constraints (6)-(8) appear in Model **T**. Constraint (10) decides initial dock allotment. Constraint (11) computes the number of private exchange bicycles.

4. COMPUTATIONAL STUDY

In this section, we first introduce the data from the Taipei YouBike system. Then, we estimate the cost of truck operation and APPs. Next, we consider some parameters that have an impact on the performance of our models. Finally, experimental results are provided.

4.1 Real data from the Taipei YouBike system

The data from the Taipei YouBike system from January, of 2013 to April of 2013 is adapted for our numerical studies. We focus on passengers for short distance communication. Therefore, bicycle trips longer than 30 minutes (about 16% of total bicycle trips) are ignored. Also, traveling from and returning to the same station (less than 1%) is not considered. We study the daily operation from 5:00 a.m. to 1:00 a.m. of next day (total 20 hours) as 40 time periods. The statistical data from January of 2013 to April of 2013 is shown as Table 1.

Table 1: Statistical data of the Taipei YouBike system from Jan-13 to Apr-13.

Month	Jan-13	Feb-13	Mar-13	Apr-13
# of stations	58	58	58	58
Total travel demand	287680	340400	445358	329280

We sum up all the travel demand between each pair of stations in the whole month for each time interval. The estimated income for the BSS is \$10 per bicycle trip. For the total number of bicycles, three scenarios are considered. First, the total number of bicycles in 2013 is around 2000. Second, since that the average travel time is about 13 minutes and each time period is 30 minutes. We calculate turnover rate and estimate the total number of bicycles can be used in a 30-minute period. Third, the total number of bicycles in 2016 is around 7500. The detail data is shown in Table 2.

Table 2: Average travel time and total numbers of bicycles in our models from Jan-13 to Apr-13.

Month	Jan-13	Feb-13	Mar-13	Apr-13
Average usage time (mins)	12.58	13.43	12.01	12.55
Bicycle turnover rate in 30 minutes	2.38	2.23	2.49	2.39
Total number of bicycles	1844	2004	2132	2196
Estimated total number of bicycles	4394	4475	5324	5249

4.2 Cost analysis of truck operation and APPs

In this subsection, we estimate the cost of reallocation trucks, docks, and private exchange for our models. The depreciation period is 7 years according to the Department of Transportation of the Taipei city government. According to Table 3, each truck needs two dispatchers and the total cost is about NT\$140 per truck trip in Model **T**.

Each truck trip can move up to 20 bicycles at a time. For Model PE, the private exchange cost is about NT\$0.5 per bicycle per private exchange. Dock cost is about NT\$20 per day.

4.3 Design of experiments

There are several factors studied: the total number of bicycles, the truck cost per trip, the private exchange cost, and the dock cost. For three integer programming models, we use CPLEX to solve linear relaxation problems and PSO to solve the integer programs. For PSO, we generate 30 particles, implement 5000 iterations, and set the stopping criteria of no improvement in 200 iterations.

In Case 0, we consider different total number of bicycles in Model **O** from January 2013 to April 2013. We

set Case 0 as a benchmark to compare with other models. The, we consider different truck costs for Models T-Simple and T in Cases 1 and 2, respectively. In Case 3 and 4, we study the impact of different private exchange costs and dock costs in Model **PE**.

Table 3: Cost analysis of the YouBike system.

Model	Item	Cost	Cost per unit	Remarks
T	Truck	NT\$700,000 / truck	NT\$7 / time interval	Truck capacity is 20 bicycles
	Dispatcher	NT\$30,000 / month	NT\$64 / time interval	Two dispatchers are needed in a truck
	About NT\$140 / truck trip			
PE	APP	NT\$240,000 / team	NT\$0.5 / private exchange	A 6-member team to develops APPs in around 20 days
	Dock	NT\$59,322 / dock	NT\$20 / day	

Table 4: Design of experiments for Model T-Simple and Model T.

Case	Model	Month	Number of total bicycles	Cost per truck trip (NT\$)	Number of scenarios
0	O	Jan-13~Apr-13	2000, Estimated, 7500	Not considered	12
1	T-Simple	Jan-13~Apr-13	2000, Estimated, 7500	80,100,...,200,500,1000,...,3000	156
2	T	Jan-13~Apr-13	2000, Estimated, 7500	80,100,...,200,500,1000,...,3000	156

Table 5: Design of experiments for Model **PE**.

Case	Model	Month	Number of total bicycles	Private exchange cost per bicycle (NT\$)	Cost per dock (NT\$)	Number of scenarios
3	PE	Jan-13~Apr-13	2000, Estimated, 7500	0, 0.1,..., 1, 5,10,...,30	20	204
4	PE	Jan-13~Apr-13	2000, Estimated, 7500	0.5	10,15,...,30	60

4.4 Experimental results

We first show the results of Case 0 for comparison.

Result – Case 0

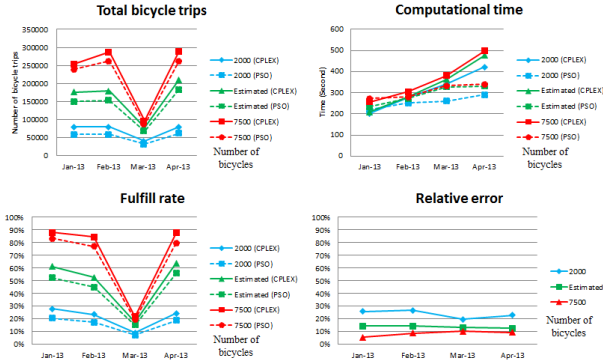


Figure 3: Experimental results of Case 0.

Figure 3 shows the total bicycle trips, fulfill rates, the relative errors, and computational times between the solutions of CPLEX and PSO in Case 0. In Figure 3, we observe that the number of bicycles trips increases with the total number of bicycles. Among the 4 months, March 2013 has unexpectedly high demand and the lowest fulfill rate. This means March 2013 is more critical than other 3 months. Thus, we use the results of March 2013 for brief demonstration.

Recall that the CPLEX is adopted for the linear relaxation and PSO is used for the integer program. As a result, the objective value of PSO is about 20~30% lower than that of CPLEX for all scenarios.

Next, we show the results of Case 1 (Model T-Simple) and Case 2 (Model T) in Figures 4 and 5, respectively.

Case 1 – Mar-13

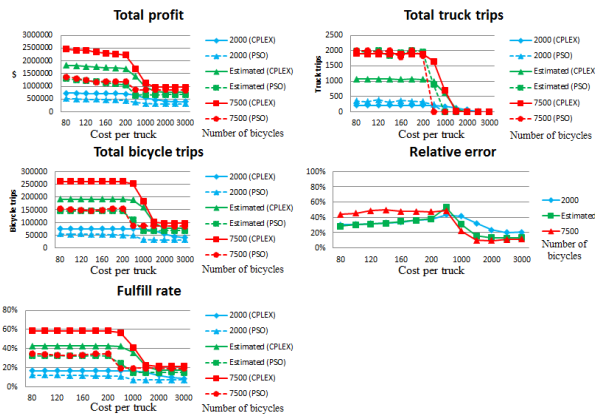


Figure 4: Experimental results of Case 1.

Case 2 – Mar-13

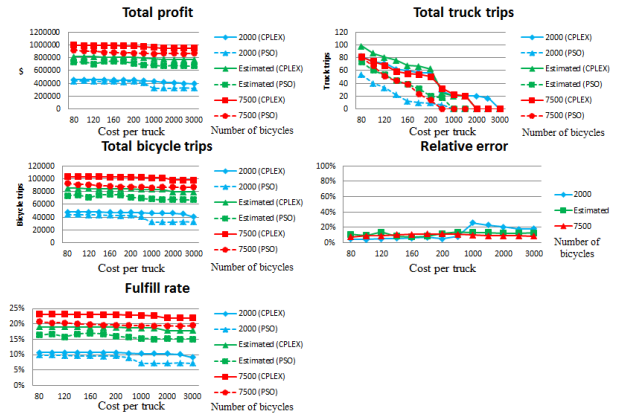


Figure 5: Experimental results of Case 2.

In Figures 4 and 5, if truck cost per trip decreases, then the number of truck trips increases. Hence, surplus bicycles can move to stations with high demand. As a result, more travel demand can be met. Both number of bicycle trips and fulfill rate increase.

Finally, we show the results of PE in Case 3 (Model PE with different private exchange costs) and Case 4 (Model PE with different dock costs) in Figures 6 and 7, respectively. There are two important observations.

Observation 1. *The number of docks increases with the private exchange cost.*

Observation 2. *The number of private exchange bicycles increases as the dock cost increases.*

In Figure 6, if private exchange cost increases, then more docks are suggested for long term to avoid private exchange. Moreover, the optimal numbers of private exchange bicycles and docks are sensitive when the private exchange cost is between NT\$0.5 and NT\$0.6 per bicycle.

In Figure 7, if dock cost increases, then fewer docks are provided. Instead, more private exchange is encouraged to maintain the fulfill rate and net profit. Moreover, the numbers of private exchange bicycles and docks are sensitive when the dock cost is between NT\$15 and NT\$20.

Case 3—Mar-13

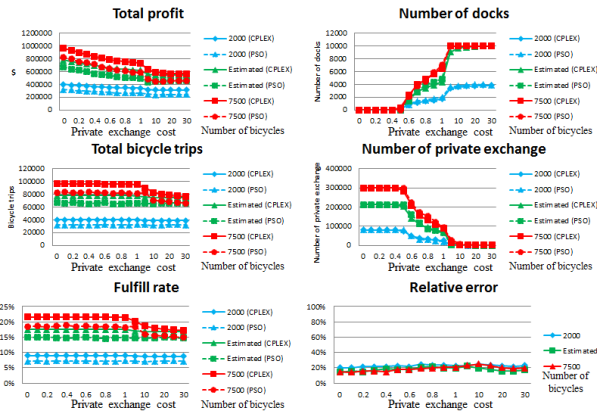


Figure 6: Experimental results of Case 3.

Case 4—Mar-13

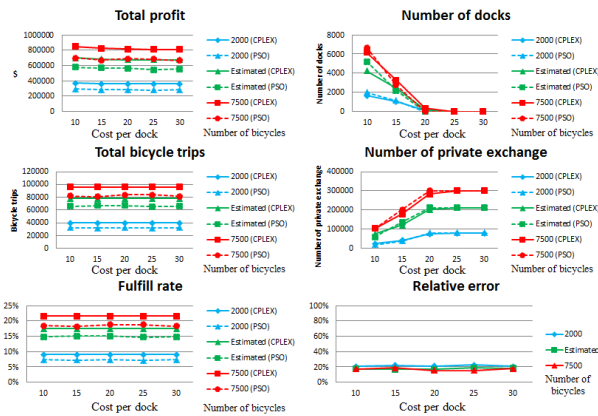


Figure 7: Experimental results of Case 4.

5. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we study a BSS, and consider trucks for bicycle reallocation and APPs for private exchange. Our decision is to find the best initial allotment of bicycles and docks. The objective is to maximize the net profit of the BSS. The net profit includes the income of bicycle trips and the costs of truck operation, dock construction, and APP system maintenance. Based on Shu et al. (2013), we propose three integer programming models. We adopt PSO to solve the integer programs and CPLEX to solve the linear relaxations.

The real data of Taipei YouBike system in 2013 is adopted for our computational study. We also estimate the total number of bicycles, the costs of trucks and APPs. Our experiments are designed to study the impact of the

total number of bicycles, truck cost, private exchange cost, and dock cost. Finally, experimental results show that the relative errors are less than 30% between CPLEX and PSO. Six important observations show the relationships among different parameters.

For future research, other meta-heuristic methods could be tried to reduce the gap between the integer programming and the linear relaxation. Also, our model could be expanded to other situations, such as dynamic pricing on the bicycle renting price.

REFERENCES

- Chemla, D., Meunier, F., & Calvo, R. W. (2013). Bike sharing systems: Solving the static rebalancing problem. *Discrete Optimization*, 10(2), 120-146.
- Contardo, C., Morency, C., & Rousseau, L. M. (2012). Balancing a dynamic public bike-sharing system. Technical Report CIRRELT-2012-09, CIRRELT, Montreal, Canada.
- DeMaio, P. (2009). Bike-sharing: History, impacts, models of provision, and future. *Journal of Public Transportation*, 14(4), 41-56.
- DeMaio, P., & Gifford, J. (2004). Will smart bikes succeed as public transportation in the United States?. *Journal of Public Transportation*, 7(2), 1-15.
- Eberhart, R. C., & Kennedy, J. (1995). A new optimizer using particle swarm theory. In *Proceedings of the sixth international symposium on micro machine and human science* 1(1), 39-43.
- Nair, R., & Miller-Hooks, E. (2011). Fleet management for vehicle sharing operations. *Transportation Science*, 45(4), 524-540.
- Raviv, T., Tzur, M., & Forma, I. A. (2013). Static repositioning in a bike-sharing system: models and solution approaches. *EURO Journal on Transportation and Logistics*, 2(3), 187-229.
- Shu, J., Chou, M. C., Liu, Q., Teo, C. P., & Wang, I. L. (2013). Models for effective deployment and redistribution of bicycles within public bicycle-sharing systems. *Operations Research*, 61(6), 1346-1359.
- Schuijbroek, J., Hampshire, R., & van Hoes, W. J. (2013). Inventory rebalancing and vehicle routing in bike sharing systems. Technical report 2013-E1, Tepper School of Business, Carnegie Mellon University.