Mean-Variance Analysis for Optimal Operation for Green Supply Chain with Uncertainties in Product Demand and Collectable Quantity of Used Products

Naoto Takahashi †
Course of Electrical and Electronic Systems, College of Engineering
Osaka Prefecture University, Sakai, Osaka, 599-8531, Japan
Tel: (+81) 72-254-9349, Email: sv106021@edu.osakafu-u.ac.jp

Etsuko Kusukawa
Course of Electrical and Electronic Systems, College of Engineering
Osaka Prefecture University, Sakai, Osaka, 599-8531, Japan
Tel: (+81) 72-254-9349, Email: kusukawa@eis.osakafu-u.ac.jp

Abstract. This paper discusses risk management regarding the uncertainties in product demand and collectable quantity of used products occurring in green supply chain(GSC)'s operations and clears how two uncertainties as risks affect the optimal operation in a GSC with a retailer(R) and a manufacturer(M). R pays an incentive for collection of used products and sells a single type of products in a market during a single period. M produces an order quantity of products using recyclable parts with acceptable quality levels and covers a part of R's incentive as to the recycled parts. R faces the uncertainties in the product demand and the collectable quantity. Mean-variance analysis is conducted for three risk attitudes on two uncertainties: risk neutral attitude, risk-averse attitude, risk-prone attitude. The optimal decisions for product order quantity, sale price, maximum collection quantity, unit collection incentive and lower limit of quality level are determined under the decentralized GSC(DGSC) and the integrated GSC(IGSC). DGSC optimizes each member's utility function, meanwhile IGSC does the whole system's. The analysis numerically illustrates how three risk attitudes affect the optimal operations in a GSC. The benefit of supply chain coordination adopting Nash Bargaining solution to shift from DGDC to IGSC is discussed.

Keywords: green supply chain, uncertainties in product demand and collection quantity of used products, mean-variance analysis, supply chain coordination, game theory

1. INTRODUCTION

From social concerns about 3R (reduce, reuse, recycle) activity worldwide, it is urgently-needed to construct a green supply chain (GSC) from collection of used products through recycling of them to sales of the products using the recycled parts (Schenkel et al., 2015; Cannella et al., 2016). In general, it is considerable for the system operation in a GSC to face the uncertainties in demand of a single type of products and quality of a single type of used products collected from a market. The following previous papers: Aras et al. (2004), Ferguson et al. (2009), Kusukawa and Alozawa (2015) and Zikopoulos and Tagaras (2015) verified how the uncertainties in product demand and the quality of the used products affected the optimal operation and the expected profits in a GSC. The incorporation of the game theory into supply chain coordination in a GSC have been discussed by Nagarajan and Sosic (2008), Hong et al. (2015), Ghosh and Shah (2015), Kusukawa and Alozawa (2015). Those previous papers mentioned above determined the optimal operations in a GSC, which maximized the expected profits of a retailer, a manufacturer, and the whole system in the GSC. This implies that the above papers don't consider the effect of variance of the individual profit on the optimal operation in a GSC.

In order to solve this above problem, mean-variance analysis (Choi et al., 2008; Chiu and Choi, 2016) is incorporated into the optimal decision for a single type of products to sell in a single period. The motivation of this paper is to incorporate mean-variance analysis on the uncertainties in product demand and the collectable quantity of the used products into the modeling and the theoretical analysis in a GSC, and determine the optimal operation. Concretely, this paper discusses risk management regarding the uncertainties in product demand and collectable quantity of the used products occurring in GSC's operations and clears how two uncertainties as risks affect the optimal operation in a GSC with a retailer (R) and a manufacturer (M). R pays an incentive for collection of the used products from customers and sells a single type of products in a market during a single period. M produces an order quantity of the products using the recyclable parts with acceptable quality levels and covers a part of the R's incentive as to the quantity of the recycled parts. R faces the uncertainties in the product demand and the collectable quantity. Mean-variance analysis is conducted for three risk attitudes regarding two uncertainties: risk neutral attitude, risk-averse attitude, risk-prone attitude. The optimal decisions for the product order quantity, the sale price, the maximum collection quantity, the unit collection incentive and the lower limit of quality level are determined under the decentralized GSC (DGSC) and the integrated GSC (IGSC). DGSC optimizes each member's utility function, meanwhile IGSC does the whole system's. The analysis numerically illustrates how three risk attitudes affect the optimal operations in a GSC. The benefit of supply chain coordination (SCC) adopting Nash Bargaining solution to shift from DGDC to IGSC is discussed.

The contribution of this paper provides the following managerial insights that (i) the optimal operation in a GSC should be determined as to risk attitudes introduced into not only the product demand but also the collectable quantity of the used products: risk-neutral attitude which maximizes the expected profits in a GSC, risk-averse attitude and risk-prone attitude which maximize the utility function with the expectation and variance of profits in a GSC, (ii) supply chain coordination should be conducted by taking balance between the expected profits of R and M using Nash bargaining solution.

2. MODEL DESCRIPTIONS

First, the operational flow of a GSC is discussed. A GSC consists of a retailer (R) and a manufacturer (M). The operational flow of the GSC consists of the transfer from the collection of a single type of used products through recycling the used products into a single type of recycled parts to sales of a single type of products produced from both the recycled parts and new parts in a single period. This paper focuses on a single type of products such as consumer electronics (mobile phone, personal computer), semiconductor, and electronic component.

(1) R pays the unit collection incentive t to collect the used products from consumers under the maximum collection quantity S_r of the used products.

(2) Unless the collectable quantity x_r of the used products exceeds S_r, all the used products x_r are delivered to M at the unit cost c_t. R incurs the opportunity loss cost c_c per used products which exceeds S_r and are not collected. R incurs the unit shortage penalty cost s_w of the used products which does not satisfy S_r.

(3) M disassembles the used products and inspects all the recyclable parts with the unit cost c_a. After that, M classifies the recyclable parts into quality level $\ell\ (0 \le \ell \le 1)$. The lower limit of quality level $u\ (0 \le u \le 1)$ for the recyclable parts is optimally determined. M remanufactures all the recyclable parts with quality level ℓ more than u. M disposes all the recyclable parts with the lower quality level than u with the unit cost c_d.

(4) As a reward for R's cooperation to M's recycling activity, M pays compensation $R(t)$ to R's collection costs of the used products based on the unit collection incentive t as to the quantity of the recycled parts.

(5) The unit collection incentive t and the product order quantity Q of a single type of products to M are optimally determined under the uncertainties in the collectable quantity and the product demand. R orders the order quantity Q from M.

(6) M produces the products at the unit cost c_m to satisfy the order quantity Q from R. R orders the order quantity Q of a single type of products from M. If the required quantity of parts to produce Q is unsatisfied with the quantity of the recycled parts, M procures the required quantity of new parts at the unit cost c_n from an external supplier.

(7) M sells the quantity Q of the products to R at the unit wholesale price w.

(8) R sells the products in a market with the unit sale price p during a single period. R incurs the unit inventory holding cost h_r of the unsold products, while R incurs the unit shortage penalty cost s_c of the unsatisfied product demand.

Next, model assumptions in a GSC is shown.

1) The product demand from consumers x is uncertain. x is modeled as $x = D(p) + \varepsilon_d$. $D(p)$ is the expected amount of the product demand and is a monotone decreasing function in terms of p. ε_d is the additional variation and follows the normal distribution $N\left(0, \sigma_d^2\right)$.

2) The collectable quantity x_r is uncertain. x_r is modeled as $x_r = A(t) + \varepsilon_a$. $A(t)$ is the expected amount of the collectable quantity and is a monotone increasing function in terms of t. From the aspect of the R's profit, the feasible range of t is $0 \le t \le t_U < p$. ε_a is the additional variation and follows the normal distribution $N\left(0, \sigma_a^2\right)$.

3) ε_d and ε_a are independent each other.

4) The unit of a single type of recyclable parts is extracted from the unit of a single type of used products. M remanufactures a single type of products using a single type of recyclable parts with acceptable quality levels.

5) The variability of quality level ℓ of the recyclable parts is modeled as a probabilistic distribution with the PDF $q(\ell)$.

6) The unit remanufacturing cost $c_r(\ell)$ of the recyclable parts with the quality level ℓ varies as to the quality level $\ell\ (0 \le \ell \le 1)$. The lower quality level ℓ is, the higher the unit remanufactured cost $c_r(\ell)$ is. Here, $\ell = 0$ indicates the worst quality level of the recyclable parts, meanwhile $\ell = 1$ indicates the best quality level of the recyclable parts. Thus, $c_r(\ell)$ is a monotone decreasing function in terms of ℓ.

Note that each quality of the recycled parts produced from the recyclable parts is as good as that of new parts produced from new materials.

7) The unit wholesale price w is calculated from the unit procurement cost c_n of new parts, the unit production cost c_m of the products, and the unit margin m_a from wholesales.

3. EXPECTATION AND VARIANCE OF PROFITS IN A GSC

From section 2, the retailer(R)'s profit consists of the collection cost of the used products from consumers, the delivery cost of the used products to the manufacturer (M), the opportunity loss cost of the used products which exceeds the maximum collection quantity, the shortage penalty cost of the used products which is unsatisfied with the maximum collection quantity, the compensation revenue of collection of the used products, the product sales, the procurement cost the of products, the inventory holding cost of the unsold products, and the shortage penalty cost for unsatisfied product demand in a market. Taking expectations of the product demand x and the collectable quantity of used products x_r, the R's expected profit $E[\pi_R(Q,p,S_r,t,u)]$ for Q, p, S_r, t and u can be derived as

$$E\left[\pi_R\left(Q,p,S_r,t,u\right)\right]=\left\{-t-c_t+R\left(t\right)\int_u^1 q\left(\ell\right)d\ell\right\}S_r$$
$$-\left\{-t-c_t+R\left(t\right)\int_u^1 q\left(\ell\right)d\ell+s_w\right\}E\left[S_r-x_r\right]^+-c_cE\left[x_r-S_r\right]^+$$
$$+\left(p-w\right)Q-\left(p+h_r\right)E\left[Q-x\right]^+-s_cE\left[x-Q\right]^+. \qquad (1)$$

Here, $E\left[Q-x\right]^+$ indicates the expected excess quantity of the products when $Q>x$, and can be derived as

$$E\left[Q-x\right]^+=\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d\ . \qquad (2)$$

$E\left[x-Q\right]^+$ indicates the expected shortage quantity of the products when $Q\le x$, and can be derived as

$$E\left[x-Q\right]^+=D\left(p\right)-Q+\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d\ . \qquad (3)$$

$E\left[S_r-x_r\right]^+$ indicates the expected excess quantity of the used products when $S_r>x_r$, and can be derived as

$$E\left[Q-x\right]^+=\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d\ . \qquad (4)$$

$E\left[S_r-x_r\right]^+$ indicates the expected shortage quantity of the used products when $S_r\le x_r$, and can be derived as

$$E\left[x_r-S_r\right]^+=A\left(t\right)-S_r+\int_{-A(t)}^{S_r-A(t)}G_a\left(\varepsilon_a\right)d\varepsilon_a\ . \qquad (5)$$

From Eq. (1), variance of the R's profit for Q, p, S_r, t and u can be derived as

$$V\left[\pi_R\left(Q,p,S_r,t,u\right)\right]=2\left\{Q-D\left(p\right)\right\}$$
$$\times\left\{\left(p+h_r\right)^2-s_c^2+s_c\left(p+h_r+s_c\right)\right\}\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d$$
$$+2\left\{s_c^2-\left(p+h_r\right)^2\right\}\int_{-D(p)}^{Q-D(p)}\varepsilon_dF_d\left(\varepsilon_d\right)d\varepsilon_d$$
$$-\left(p+h_r+s_c\right)^2\left\{\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d\right\}^2+s_c^2\cdot V\left[\varepsilon_d\right]$$
$$+2\left\{S_r-A\left(t\right)\right\}\left\{k_1^2-c_c^2+c_c\left(k_1+c_c\right)\right\}\int_{-A(t)}^{S_r-A(t)}G_a\left(\varepsilon_a\right)d\varepsilon_a$$
$$+2\left\{c_c^2-k_1^2\right\}\int_{-A(t)}^{S_r-A(t)}\varepsilon_aG_a\left(\varepsilon_a\right)d\varepsilon_a$$
$$-\left(k_1+c_c\right)^2\left\{\int_{-A(t)}^{S_r-A(t)}G_a\left(\varepsilon_a\right)d\varepsilon_a\right\}^2+c_c^2\cdot V\left[\varepsilon_a\right]\ , \qquad (6)$$

$$k_1=-t-c_t+R\left(t\right)\int_u^1 q(\ell)d\ell+s_w\ . \qquad (7)$$

From section 2, the M's profit consists of the disassembly and inspection costs of the used products, the remanufacturing cost of the recyclable parts, the disposal cost of un-recycled parts, the compensation cost, the procurement cost of new parts, the production cost of the products, and the product wholesales. Taking expectations of x and x_r, the M's expected profit $E[\pi_M(Q,p,S_r,t,u)]$ for Q, p, S_r, t and u can be derived as

$$E\left[\pi_M\left(Q,p,S_r,t,u\right)\right]=-\left\{c_a+\int_u^1 c_r\left(\ell\right)q\left(\ell\right)d\ell\right.$$
$$\left.+R\left(t\right)\int_u^1 q\left(\ell\right)d\ell+c_d\int_0^u q\left(\ell\right)d\ell-c_n\int_u^1 q\left(\ell\right)d\ell\right\}S_r$$
$$+\left\{c_a+\int_u^1 c_r\left(\ell\right)q\left(\ell\right)d\ell+R\left(t\right)\int_u^1 q\left(\ell\right)d\ell\right.$$
$$\left.+c_d\int_0^u q\left(\ell\right)d\ell-c_n\int_u^1 q\left(\ell\right)d\ell\right\}\cdot E\left[S_r-x_r\right]^+$$
$$-c_nQ-c_mQ+wQ\ . \qquad (8)$$

From Eq. (8), variance of the M's profit for Q, p, S_r, t and u can be derived as

$$V\left[\pi_M\left(Q,p,S_r,t,u\right)\right]=\left\{c_a+\int_u^1 c_r\left(\ell\right)q\left(\ell\right)d\ell\right.$$
$$\left.+R\left(t\right)\int_u^1 q\left(\ell\right)d\ell+c_d\int_0^u q\left(\ell\right)d\ell-c_n\int_u^1 q\left(\ell\right)d\ell\right\}^2$$
$$\times\left[2\left\{S_r-A\left(t\right)\right\}\int_{-A(t)}^{S_r-A(t)}G\left(\varepsilon_a\right)d\varepsilon_a\right.$$
$$\left.-2\int_{-A(t)}^{S_r-A(t)}\varepsilon_aG\left(\varepsilon_a\right)d\varepsilon_a-\left\{\int_{-A(t)}^{S_r-A(t)}G\left(\varepsilon_a\right)d\varepsilon_a\right\}^2\right]. \qquad (9)$$

As the sum of both members' expected profits in Eqs. (1) and (8), the expected profit of the whole system (S) $E[\pi_S(Q,p,S_r,t,u)]$ for Q, p, S_r, t and u is obtained as

$$E\left[\pi_S\left(Q,p,S_r,t,u\right)\right]$$
$$=E\left[\pi_R\left(Q,p,S_r,t,u\right)\right]+E\left[\pi_M\left(Q,p,S_r,t,u\right)\right] \qquad (10)$$
$$=pQ-c_mQ-c_nQ$$
$$-\left(p+h_r\right)\cdot E\left[Q-x\right]^+-s_c\cdot E\left[x-Q\right]^+$$
$$+k_2S_r-\left(s_w+k_2\right)\cdot E\left[S_r-x_r\right]^+-c_c\cdot E\left[x_r-S_r\right]^+\ , \qquad (11)$$

$$k_2=-t-c_t-\int_u^1 c_r\left(\ell\right)q\left(\ell\right)d\ell-c_a$$
$$-c_d\int_0^u q\left(\ell\right)d\ell+c_n\int_u^1 q\left(\ell\right)d\ell\ . \qquad (12)$$

From Eqs. (1), (8), (11) and (12), variance of the S's profit for Q, p, S_r, t and u can be derived as

$$
\begin{aligned}
&V\left[\pi_S\left(Q,p,S_r,t,u\right)\right]\\
&=2\left\{Q-D(p)\right\}\left\{\left(p+h_r\right)^2-s_c^2+s_c\left(p+h_r+s_c\right)\right\}\\
&\times\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d\\
&+2\left\{s_c^2-\left(p+h_r\right)^2\right\}\int_{-D(p)}^{Q-D(p)}\varepsilon_d F_d\left(\varepsilon_d\right)d\varepsilon_d\\
&-\left(p+h_r+s_c\right)^2\left\{\int_{-D(p)}^{Q-D(p)}F_d\left(\varepsilon_d\right)d\varepsilon_d\right\}^2+s_c^2\cdot V\left[\varepsilon_d\right]\\
&+2\left\{S_r-A(t)\right\}\left\{\left(k_2+s_w\right)^2-c_c^2+c_c\left(k_2+s_w+c_c\right)\right\}\\
&\times\int_{-A(t)}^{S_r-A(t)}G_a\left(\varepsilon_a\right)d\varepsilon_a\\
&+2\left\{c_c^2-\left(k_2+s_w\right)^2\right\}\int_{-A(t)}^{S_r-A(t)}\varepsilon_a G_a\left(\varepsilon_a\right)d\varepsilon_a\\
&-\left(k_2+s_w+c_c\right)^2\left\{\int_{-A(t)}^{S_r-A(t)}G_a\left(\varepsilon_a\right)d\varepsilon_a\right\}^2+c_c^2\cdot V\left[\varepsilon_a\right]. \quad (13)
\end{aligned}
$$

4. MEAN-VARIANCE ANALYSIS OF PROFITS IN A GSC FOR TWO UNCERTAINTIES

Mean-variance analysis (Choi et al., 2008; Li and Cai, 2009) is conducted for individual profits in a GSC with the uncertainties in the product demand and the collectable quantity of the used products. Concretely, by using utility functions of a retailer (R), a manufacturer (M), and the whole system (S) in a GSC, the risk analysis is conducted for three risk attitudes regarding two uncertainties mentioned above : risk-neutral attitude (*N*), risk-averse attitude (*A*), and risk-prone attitude (*P*). The attitude *N* makes a decision without consideration of variance of profit in a GSC. The attitude *A*, with negative consideration of variance of profit in a GSC, hopes to stabilize the profit. The attitude *P*, with positive consideration of variance of profit in a GSC, weighs heavily improvement in chances to generate large profit rather than stability of the profit. Therefore, utility functions of *member* $(\in\{\mathrm{R},\mathrm{M},\mathrm{S}\})$ in attitude $j\ (\in\{N,A,P\})$ for Q, p, S_r, t and u are defined as

$$
\begin{aligned}
&U\left[\pi_{member}^j\left(Q,p,S_r,t,u\right)\right]\left(j\in\{N,A,P\},member\in\{\mathrm{R},\mathrm{M},\mathrm{S}\}\right)\\
&=E\left[\pi_{member}^j\left(Q,p,S_r,t,u\right)\right]+\gamma_d\,\mathrm{V}\left[\pi_{member}^j\left(Q,p\right)\right]\\
&+\gamma_a\,\mathrm{V}\left[\pi_{member}^j\left(S_r,t,u\right)\right]. \quad (14)
\end{aligned}
$$

Here, $\gamma_k\,(k\in d,a)$ denotes a degree of risk attitude. Accordingly, $\gamma_d=\gamma_a=0$ indicates the risk-neutral attitude (*j*=*N*), $\gamma_d<0,\ \gamma_a<0$ indicates the risk-averse attitude (*j*=*A*), and $\gamma_d>0,\ \gamma_a>0$ indicates the risk-prone attitude (*j*=*P*).

5. OPTIMAL OPERATIONS UNDER DGSC

In decentralized GSC (DGSC), the optimal decision approach for the Stackelberg game (Nagarajan and Sosic, 2008) is adopted. This paper regards a retailer (R) as the leader of the decision-making under DGSC and a manufacturer (M) as the follower of the decision-making of R under DGSC. R determines the optimal order quantity $Q_D^j\ \left(j\in\{N,A,P\}\right)$, the optimal sale price p_D^j, the optimal maximum collection quantity S_{rD}^j and the optimal unit collection incentive t_D^j so as to maximize the R's utility function in Eq. (14) in risk attitude $j\ (\in\{N,A,P\})$. M determines the optimal lower limit of quality level u_D^j so as to maximize the M's utility function in Eq. (14) under Q_D^j, p_D^j, S_{rD}^j and t_D^j. The procedures for the optimal decisions $(Q_D^j,p_D^j,S_{rD}^j,t_D^j,u_D^j)$ under DGSC in attitude $j\ (\in\{N,A,P\})$ are shown hereinafter.

5.1 Optimal Operation in Attitude N

[Step 1] The R's expected profit in Eq. (1) is the concave function in terms of Q under p. Determine the provisional order quantity $Q_D^N(p)$ under p as

$$Q_D^N\left(p\right)=D\left(p\right)+F_d^{-1}\left(\frac{-w+p+s_c}{p+h_r+s_c}\right). \quad (15)$$

[Step 2] From Eq. (1), $Q_D^N(p)$ under p are unaffected by S_r, t and u. Find the optimal combination of the order quantity and the sale price (Q_D^N,p_D^N) to maximize the R's expected profit in Eq. (1) though numerical search. By changing p, satisfying conditions $p>0$ and $D(p)>0$, $Q_D^N(p)$ and p are substituted into Eq. (1) under S_r, t and u. The optimal combination $\left(Q_D^N\left(p_D^N\right),p_D^N\right)$ can be determined as $\left(Q_D^N(p),p\right)$ which maximizes the R's expected profit $E\left[\pi_R(Q_D^N(p),p|S_t,t,u\right]$ in Eq. (1) under S_r, t and u.

[Step 3] The R's expected profit in Eq. (1) is the concave function in terms of S_r under t and u. Determine the provisional maximum collection quantity $S_{rD}^N(t,u)$ under t and u as

$$S_{rD}^N\left(t,u\right)=A\left(t\right)+G_a^{-1}\left(\frac{R\left(t\right)\int_u^1 q\left(\ell\right)d\ell-t-c_t+c_c}{R\left(t\right)\int_u^1 q\left(\ell\right)d\ell-t-c_t+c_c+s_w}\right) \quad (16)$$

if the condition $R\left(t\right)\int_u^1 q\left(\ell\right)d\ell>t+c_t-\left(c_c+s_w\right)$ is satisfied.

[Step 4] The first order derivative of the M's expected profit in Eq. (8) in terms of u under Q_D^N, p_D^N, $S_{rD}^N(t,u)$ and t is

$$
\begin{aligned}
&dE\left[\pi_M\left(u\right)|\,Q_D^N,p_D^N,S_{rD}^N\left(t,u\right),t\right]\big/du\\
&=\left[S_r-\int_{-A(t)}^{S_r-A(t)}G_a\left(\varepsilon_a\right)d\varepsilon_a\right]q\left(u\right)\left\{c_r\left(u\right)+R\left(t\right)-c_d-c_n\right\}.
\end{aligned}
$$

From model assumption 6) in section 3, $c_r(u)$ is a monotone decreasing function in terms of u. Determine the provisional lower limit of quality level $u_D^N(t)$ under t so as to satisfy the following condition

$$c_r\left(u\right)+R\left(t\right)-c_d-c_n=0. \quad (17)$$

u satisfying Eq. (17) is obtained as $u_D^N(t)$. From Eq. (17), $u_D^N(t)$ is affected by t.

[Step 5] $S_{rD}^N(t,u)$, $u_D^N(t)$ and t are unaffected by Q and p. Find the optimal combination of the unit collection incentive, the maximum collection quantity and the lower limit of quality level $\left(t_D^N,S_{rD}^N,u_D^N\right)$ to maximize the M's expected profit by changing t in Eq. (8) though numerical search. By varying t within the range where $0 \le t \le t_U$, t, $S_{rD}^{N}(t,u)$, and $u_D^N(t)$ are substituted into Eq. (1) under Q_D^N and p_D^N. The optimal combination $\left(t_D^N, u_D^N\left(t_D^N\right), S_{rD}^{N}\left(t_D^N, u_D^N\left(t_D^N\right)\right)\right)$ can be determined as $\left(t, u_D^N(t), S_{rD}^{N}\left(t, u_D^N(t)\right)\right)$ which maximizes the R's expected profit $E\left[\pi_R\left(t, u_D^N(t), S_{rD}^{N}\left(t, u_D^N(t)\right)\right) \middle| Q_D^N, p_D^N\right]$ in Eq. (1) under Q_D^N and p_D^N.

5.2 Optimal Operations in Attitudes A and P

The optimal decision procedures under DGSC in attitudes A and P are provided as follows:

[Step 1] The optimal decisions for Q and p in attitudes A and P are unaffected by S_r, t and u. Find the optimal combinations of the order quantity and the sale price $\left(Q_D^A, p_D^A\right)$, $\left(Q_D^P, p_D^P\right)$ so as to maximize the R's utility function in Eq. (14) when $\gamma_d < 0$ in attitude A and $\gamma_d > 0$ in attitude P by numerical calculation and the numerical search.

[Step 2] The optimal decisions for S_r, t and u in attitudes A and P are unaffected by Q and p. Find the optimal combination of the unit collection incentive, the maximum collection quantity and the lower limit of quality level $\left(S_{rD}^{A}, t_D^A, u_D^A\right)$, $\left(S_{rD}^{P}, t_D^P, u_D^P\right)$ when $\gamma_a < 0$ in attitude A and $\gamma_a > 0$ in attitude P by numerical calculation and numerical search. Concretely, first, find and record u under each S_r, and t so as to maximize the M's utility function. Next, find the S_r and t from recorded combinations so as to maximize the R's utility function.

6. OPTIMAL OPERATION UNDER IGSC

In integrated GSC (IGSC), the optimal decisions for Q_I^j, p_I^j, S_{rI}^{j}, t_I^j and u_I^j are made so as to maximize the utility function of the whole system(S) in Eq. (14) in risk attitude $j\ (\in \{N, A, P\})$. The procedures for the optimal decisions $(Q_I^j, p_I^j, S_{rI}^{j}, t_I^j, u_I^j)$ under IGSC in attitude j are shown below.

6.1 Optimal Operation in Risk Attitude N

[Step 1] The S's expected profit in Eqs. (11) and (12) is the concave function in terms of Q under p. Determine the provisional order quantity $Q_I^N(p)$ under p as

$$Q_I^N(p) = D(p) + F_d^{-1}\left(\frac{-c_m - c_n + p + s_c}{p + h_r + s_c}\right). \tag{18}$$

[Step 2] From Eqs. (11) and (12), $Q_I^N(p)$ under p are unaffected by S_r, t and u. Find the optimal combination of order the quantity and the sale price $\left(Q_I^N, p_I^N\right)$ to maximize the S's expected profit in Eqs. (11) and (12) though numerical search. By changing p, satisfying conditions $p > 0$ and $D(p) > 0$, $Q_I^N(p)$ and p are substituted into Eqs. (11) and (12) under S_r, t and u. The optimal combination $\left(Q_I^N\left(p_I^N\right), p_I^N\right)$ can be determined as $\left(Q_I^N(p), p\right)$ which maximizes the S's expected profit $E\left[\pi_S(Q_I^N(p), p \middle| S_t, t, u\right]$ under S_r, t and u.

[Step 3] The S's expected profit in Eqs. (11) and (12) is the concave function in terms of S_r under t and u. Determine the provisional maximum collection quantity $S_{rI}^{N}(t,u)$ under t and u as

$$S_{rI}^{N}(t,u) = A(t) + G_a^{-1}\left(k_3 / \left(k_3 + s_w\right)\right), \tag{19}$$

$$k_3 = -t - c_t - \int_u^1 c_r(\ell) q(\ell) d\ell - c_d \int_0^u q(\ell) d\ell - c_a + c_n \int_u^1 q(\ell) d\ell + c_c \tag{20}$$

if the condition

$$c_n \int_u^1 q(\ell) d\ell > t + c_t + \int_u^1 c_r(\ell) q(\ell) d\ell + c_d \int_0^u q(\ell) d\ell + c_a - \left(c_c + s_w\right)$$

is satisfied.

[Step 4] The first order derivative of the S's expected profit Eqs. (11) and (12) for u under Q_I^N, p_I^N, $S_{rI}^{N}(t,u)$ and t is

$$dE\left[\pi_S(u) \mid Q_I^N, p_I^N, S_{rI}^{N}(t,u), t\right] / du = \left[S_r - \int_{-A(t)}^{S_r - A(t)} G_a(\varepsilon_a) d\varepsilon_a\right] q(u) \left\{c_r(u) - c_d - c_n\right\}.$$

As with DGSC, determine the optimal lower limit of quality level u_I^N so as to satisfy the following condition

$$c_r(u) - c_d - c_n = 0\,. \tag{21}$$

u satisfying Eq. (21) is obtained as u_I^N.

[Step 5] $S_{rI}^{N}\left(t, u_I^N\right)$, u_I^N and t are unaffected by Q and p. Find the optimal combination of the unit collection incentive and the maximum collection quantity $\left(t_I^N, S_{rI}^{N}\right)$ to maximize the S's expected profit by changing t in Eqs. (11) and (12) by numerical search. By varying t within the range where $0 \le t \le t_U$, t and $S_{rI}^{N}(t)$ are substituted into Eqs. (11) and (12) under Q_I^N, p_I^N and u_I^j. The optimal combination $\left(t_I^N, S_{rI}^{N}\left(t_I^N\right)\right)$ can be determined as $\left(t, S_{rI}^{N}(t)\right)$ which maximizes the S's expected profit $E\left[\pi_R\left(t, S_{rI}^{N}(t)\right) \middle| Q_I^N, p_I^N, u_I^N\right]$ under Q_I^N, p_I^N and u_I^N.

6.2 Optimal Operation in Attitudes A and P

The optimal decision procedures under IGSC in attitudes A and P are provided as follows:

[Step 1] The optimal decisions for Q and p in attitudes A and P are unaffected by S_r, t and u. Find the optimal combinations of the order quantity and the sale price $\left(Q_I^A, p_I^A\right)$, $\left(Q_I^P, p_I^P\right)$ so as to maximize the S's utility function in Eq. (14) when $\gamma_d < 0$ in attitude A and $\gamma_d > 0$ in attitude P by numerical calculation and the numerical search.

[Step 2] The optimal decisions for S_r, t and u in attitudes A and P are unaffected by Q and p. Find the optimal combination of the unit collection incentive, the maximum collection quantity and the lower limit of quality level $\left(S_{rI}^{A}, t_I^A, u_I^A\right)$, $\left(S_{rI}^{P}, t_I^P, u_I^P\right)$ so as to maximize the S's utility function in Eq. (14) when $\gamma_a < 0$ in attitude A and $\gamma_a > 0$ in attitude P by numerical calculation and the numerical search.

7. SUPPLY CHAIN COORDINATION

As supply chain coordination (SCC) to guarantee the profit improvement for each member under IGSC, the effect of profit sharing approach on the expected profit of each member for the optimal decision as to attitude $j\left(\in\{N,A,P\}\right)$ under IGSC is discussed. In this paper, the unit wholesale price w^j and compensation per used product $R^j(t)$ are coordinated between both members as to attitude j under IGSC. w and $R(t)$ are set as $w=w(m_a)=c_n+c_m+m_a$ and $R(t)=(1+\alpha)\,t$. The degree α of compensation and the margin m_a for wholesale per product are coordinated as α^{jN} and m_d^{jN} by adopting the Nash bargaining solutions (Nagarajan and Sosic, 2008) as to attitude j under IGSC. w^j and $R^j(t)$ are calculated by substituting α^{jN} and m_d^{jN} into w^j and $R^j(t)$. α^{jN} and m_d^{jN} are determined so as to maximize Eq. (22) satisfying the constrained conditions in Eqs. (23) and (24):

$$\begin{aligned} Max\ T\left(\alpha^{jN},m_a^{jN}\right)&=\{E^{jN}[\pi_R(\alpha^{jN},m_a^{jN}\left|Q_I^j,p_I^j,S_{rI}^j,t_I^j,u_I^j)\right] \\ &\quad -E^j[\pi_R(\alpha,m_a\left|Q_D^j,p_D^j,S_{rD}^j,t_D^j,u_D^j)\right]\} \\ &\quad \times\{E^{jN}[\pi_M(\alpha^{jN},m_a^{jN}\left|Q_I^j,p_I^j,S_{rI}^j,t_I^j,u_I^j)\right] \\ &\quad -E^j[\pi_M(\alpha,m_a\left|Q_D^j,p_D^j,S_{rD}^j,t_D^j,u_D^j)\right]\}, \end{aligned} \tag{22}$$

subject to

$$\begin{aligned} &E^{jN}[\pi_R\left(\alpha^{jN},m_a^{jN}\left|Q_I^j,p_I^j,S_{rI}^j,t_I^j,u_I^j\right.\right)] \\ &\quad -E^j[\pi_R\left(\alpha,m_a\left|Q_D^j,p_D^j,S_{rD}^j,t_D^j,u_D^j\right.\right)]>0\ , \end{aligned} \tag{23}$$

$$\begin{aligned} &E^{jN}[\pi_M\left(\alpha^{jN},m_a^{jN}\left|Q_I^j,p_I^j,S_{rI}^j,t_I^j,u_I^j\right.\right)] \\ &\quad -E^j[\pi_M\left(\alpha,m_a\left|Q_D^j,p_D^j,S_{rD}^j,t_D^j,u_D^j\right.\right)]>0\ . \end{aligned} \tag{24}$$

Eqs. (23) and (24) are the constraint conditions to guarantee that the expected profit of each member in attitude j under IGSC with SCC is always higher than that under DGSC.

8. NEMERICAL EXPERIMENTS

The analysis numerically illustrates how the risk attitude affects the optimal decisions of the order quantity, the sale price, the maximum collection quantity of the used products, the unit collection incentive and the lower limit of quality level under DGSC and IGSC. The optimal operation and the expected profits under DSGC are compared with those under IGSC. Supply chain coordination to enable the shift of the optimal operation under IGSC from that under DGSC is discussed. The unit wholesale price and the compensation are coordinated between a retailer (R) and a manufacturer (M) under IGSC, by adopting Nash Bargaining solution.

8.1 Numerical Examples

The numerical examples are provided as $s_c=175$, $h_r=15$, $c_c=1$, $s_w=1$, $c_d=1$, $c_a=1$, $c_t=1$, $c_n=35$, $c_m=2$, $m_a=15$. $D(p)$ and $\varepsilon_d^{\,2}$ are set at $D(p)=1500-5p$ and $\varepsilon_d^{\,2}=300$. $A(t)$ and $\varepsilon_a^{\,2}$ are set at $A(t)=100+50t$ and $\varepsilon_a^{\,2}=30$. The unit remanufacturing cost of the recyclable parts with quality level ℓ is set as $c_r(\ell)=40(1-0.9\ell)$. $\alpha=0.7$ is set form the aspect of the profits of R and M.

The quality distribution of recycle parts with $\ell(0\le\ell\le1)$ is modeled by using the beta distribution $B(\ell\mid a,b)$ with parameters a and b. Here, the case $B(\ell|1,1)$ where each quality of the recyclable parts is uniformly distributed are used. When parameters a and b vary, a variety of quality distribution of recycle parts with $\ell\ (0\le\ell\le1)$ is expressed.

8.2 Results of Numerical Analysis

8.2.1 Effect of Risk Attitude on the Optimal Operations and the Expected Profits under DGSC and IGSC

The effect of the degree of risk attitude $\gamma_k(k\in d,a)$ on the optimal decisions for the order quantity, the sale price, the maximum collection quantity, the unit collection incentive, the lower limit of quality level and the expected profits under DGSC and IGSC are discussed. Table 1 shows the effect of γ_k on decision variables under DGSC and IGSC. Table 2 shows the effect of γ_k on the expected profits of R, M and the whole system(S) under DGSC and IGSC.

From Table 1, the following results can be seen.

- The higher γ_k is, the more optimal order quantities are under DGSC and IGSC. Accordingly, $\gamma_k<0$ indicates that a decision-maker with attitude A tends to decrease the optimal order quantity, while $\gamma_k>0$ indicates that she with attitude P tends to increase it under DGSC and IGSC.
- The optimal sale prices under DGSC and IGSC few change depending for γ_k.
- The higher γ_k is, the more optimal maximum collection quantities under DGSC, but that under IGSC are not affected by γ_k.
- The higher γ_k is, the less optimal unit collection incentives under IGSC, but that under DGSC are few affected by γ_k.
- γ_k few affects the optimal lower limit of quality level.

From Table 2, the following results can be seen.

- The higher γ_k is, the expected profits of R under DGSC, M and S under DGSC and IGSC tend to decrease.
- The expected profits of R under DGSC and IGSC, and S under IGSC are quite low when $\gamma_d=5.0\times10^{-6}$. Therefore, it is necessary for R under DGSC and IGSC, and a policy maker of S under IGSC with attitude P to estimate carefully the degree γ_k of risk attitude.

8.2.2 Comparisons of Optimal Operations and Expected Profits under DGSC and IGSC

The optimal decisions for the order quantity, the sale price, the maximum collection quantity, the unit collection incentive, and the lower limit of quality level under DGSC are compared

Table 1: Effect of the degree of risk attitude γ_k on decision variables

Risk attitude	Degree of risk attitude		Q		p		S_r		t		u	
	γ_d (Q,p)	γ_a (S_r,t,u)	DGSC	IGSC	DGSC	IGSC	DGSC	IGSC	DGSC	IGSC	DGSC	IGSC
A	-1×10^{-5}	-0.01	721	794	178	179	222	340	2.74	5.03	240	111
	-1×10^{-6}	-0.005	872	957	177	169	221	347	2.74	4.94	240	111
N	0	0	887	973	177	169	219	367	2.74	4.63	240	111
P	5×10^{-7}	0.0005	894	980	177	169	219	369	2.74	4.57	240	111
	5×10^{-6}	0.05	965	1051	177	169	207	317	2.93	2.50	249	111

Table 2: Effect of the degree of risk attitude γ_k on expected profits

Risk attitude	Degree of risk attitude		Retailer		Manufacturer		Whole System	
	γ_d (Q,p)	γ_a (S_r,t,u)	DGSC	IGSC	DGSC	IGSC	DGSC	IGSC
A	-1×10^{-5}	-0.01	45371	47824	12626	13158	57997	60982
	-1×10^{-6}	-0.005	48433	48584	14885	15893	63318	64477
N	0	0	48468	48019	15098	16497	63566	64516
P	5×10^{-7}	0.0005	48460	48150	15203	16426	63663	64576
	5×10^{-6}	0.05	47548	45968	16149	17686	63697	63654

Table 3: Effect of supply chain coordination on the expected profits under IGSC

Risk attitude	Degree of risk attitude		R's expected profit		M's expected profit		SCC	
	γ_d (Q,p)	γ_a (S_r,t,u)	DGSC	IGSC (SCC)	DGSC	IGSC (SCC)	m_a	α
A	-1×10^{-5}	-0.01	45371	46987(+1616)	12626	14247(+1621)	14.9	0.11
	-1×10^{-6}	-0.005	48433	49030(+597)	14885	15492(+607)	13.6	0.10
N	0	0	48468	48976(+508)	15098	15619(+521)	13.5	0.14
P	5×10^{-7}	0.0005	48460	48916(+456)	15203	15669(+466)	13.4	0.10

with those under IGSC.

(i) The optimal decisions for order quantity and sale price

From Table 1, it is verified that the optimal order quantity under DGSC tends to be larger than that under IGSC. There are few differences of the amount of change for $\gamma_k(k\in d,a)$ between DGSC and IGSC.

The optimal sale price under IGSC is higher than that under DGSC. The optimal sale price few change for γ_k.

(ii) The optimal decisions for maximum collection quantity, unit collection incentive and lower limit of quality level

From Table 1, the following results can be seen:

- The optimal maximum collection quantity under IGSC is larger than that under DGSC in any $\gamma_k(k\in d,a)$. Therefore, the shift to IGSC promotes the collecting activity.
- The optimal unit collection incentive under IGSC is higher than that under DGSC. Therefore, the shift to IGSC promotes the collecting activity.
- The optimal lower limit of quality level under IGSC is lower than that under DGSC in any γ_k. It means increment in remanufacturing ratio. Therefore, the shift to IGSC promotes the remanufacturing activity. Moreover, the optimal lower limit of quality level under IGSC is unaffected by $\gamma_k(k\in d,a)$.

(iii) The expected profits of R, M, and S

From Table 2, it can be seen the S's expected profit under IGSC is higher than that under DGSC except for $\gamma_d=5.0\times10^{-6}$ and $\gamma_a=0.05$. When $\gamma_d=5.0\times10^{-6}$ and $\gamma_a=0.05$, the S's expected profit under IGSC is lower than that under DGSC due to maximization of the S's utility function. In this case, the shift from the optimal operation under DGSC to that under IGSC shouldn't be conducted.

Also, under the situation where the S's expected profit under IGSC is higher than that under DGSC, even if the M's expected profit under IGSC is higher than that under DGSC, the R's expected profit under IGSC is lower or slightly higher than that under DGSC. Under the situation, supply chain coordination is incorporated into IGSC to guarantee the increase of the expected profit of R and M under IGSC.

8.2.3 Effect of Supply Chain Coordination on the Expected Profits under IGSC

Incorporating supply chain coordination (SCC) into IGSC, the unit wholesale price and the degree of compensation for the unit collection incentive are adjusted as Nash bargaining solutions by Eqs. (22)- (24) in Section 7. Table 3 shows the effect of SCC on the expected profits under IGSC. From Table 3, it can be seen that SCC can guarantee that the expected profits of R and M under IGSC are higher than those under DGSC when the S's expected profit under IGSC is higher than that under DGSC. Therefore, it is verified that the incorporation of SCC into the optimal operation under IGSC enables to shift from DGSC to IGSC, when S's expected profit under IGSC is higher than that under DGSC.

9. CONCLUSIONS

This paper discussed risk management regarding the uncertainties in product demand and collectable quantity of the used products occurring in green supply chain(GSC)'s operations and cleared how two uncertainties as risks affected the optimal operation in a GSC with a retailer(R) and a manufacturer(M). R paid an incentive for collection of the used products from customers and sold a single type of products in a market during a single period. M produced an order quantity of the products using the recyclable parts with acceptable quality levels and covered a part of R's incentive as to the quantity of the recycled parts. R faced the uncertainties in the product demand and the collectable quantity. Mean-variance analysis was conducted for three risk attitudes regarding two uncertainties: risk neutral attitude, risk-averse attitude, risk-prone attitude. The optimal decisions for the order quantity, the sale price, the maximum collection quantity, the unit collection incentive and the lower limit of quality level were determined under the decentralized GSC(DGSC) and the integrated GSC(IGSC). DGSC optimized each member's utility function, meanwhile IGSC did the whole system's. The analysis numerically illustrated how three risk attitudes affected the optimal operations in a GSC. The benefit of supply chain coordination adopting Nash Bargaining solution to shift from DGDC to IGSC was discussed. Results of theoretical analysis and numerical analysis in this paper verified the following managerial insights: (I) the optimal operation in a GSC should be determined as to three risk attitudes introduced into not only the product demand but also the collectable quantity of the used products: risk-neutral attitude which maximizes the expected profits in a GSC, risk-averse attitude and risk-prone attitude which maximize the utility function with the expectation and variance of profits in a GSC , (II) supply chain coordination should be conducted by taking balance between the expected profits of R and M using Nash bargaining solution. (III) the optimal decision for unit collection incentive and lower limit of quality level of the used products is unaffected by risk attitudes for the uncertainty in product demand when the uncertainty in product demand is independent of that in quality of the used products.

As future researches, it will be necessary to incorporate the following topics into a GSC model in this paper:

- Adding new frameworks of a GSC to encourage the collection and the remanufacturing of used products,
- The situation where the multiple types of the used products and the products are handled in the GSC.

ACKNOWLEDGMENT

This research has been supported by the Grant-in-Aid for Scientific Research C No. 25350451 from the Japan Society for the Promotion of Science.

REFERENCES

Aras, N., Boyaci, T., and Verter, V. (2004) The effect of categorizing returned products in remanufacturing, *IIE Transactions*, **36**, 319-331.

Cannella, S., Bruccoleri, M., Framinan, J.M. (2016) Closed-loop supply chains: What reverse logistics factors influence performance?, *International Journal of Production Economics*, **175**, 35-49.

Choi, T.-M., Li, D., Yan, H., and Chiu, C.-H. (2008) Channel Coordination in Supply Chain with Agents Having Mean-Variance Objectives, *Omega*, **36**, 565-576.

Chiu, C.-H., Choi, T.-M. (2016) Supply chain risk analysis with mean-variance models: a technical review, Annals of Operations Research, 240, 489-507.

Ferguson, M., Guide, V. D., Koca, E., and Souza, G. C. (2009) The value of quality grading in remanufacturing, *Production and Operations Management*, **18**, 300-314.

Ghosh, D. and Shah, J. (2015) Supply Chain Analysis under Green Sensitive Consumer Demand and Cost Sharing Contract, *International Journal of Production Economics*, 164, 319-329.

Hong, X., Xu, L., Du, P., Wang, W. (2015) Joint advertising, pricing and collection decisions in a closed-loop supply chain, *International Journal of Production Economics*, **167**, pp. 12-22.

Nagarajan, M., & Sosic, G. (2008) Game-theoretic analysis of cooperation among supply chain agents: review and extensions, *European Journal of Operational Research*, **187**, 719-745.

Kusukawa, E. and Alozawa, S. (2015) Optimal Operation for Green Supply Chain with Quality of Recyclable Parts and Contract for Recycling Activity, *Industrial Engineering and Management Systems*, **14**, 248-274.

Schenkel, M., Caniëls, M.C.J., Krikke, H. and Van Der Laan, E. (2015) Understanding Value Creation in Closed Loop Supply Chains - Past Findings and Future Directions, *Journal of Manufacturing Systems*, **37**, 729-745.

Zikopoulos, C., Tagaras, G. (2015) Reverse supply chains: Effects of collection network and returns classification on profitability, *European Journal of Operational Research*, 246, 435-449.