Performance Evaluation of Pull-type Control Mechanisms for Two-stage Production Systems with Advance Demand Information

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Abstract. In manufacturing systems the appropriate production and inventory control is needed to reduce the production cost and maintain consumers' satisfaction. When the amount of demand is informed to the systems, using this properly and managing the production system make the manufacturer increase the profit. Thus production and inventory control with advance demand information has been studied in literature. A production system usually consists of multiple stages, between which there is a lead time, and each stage has his own production facility. In such a production system advance demand information will be useful but it is difficult to analyze its effect theoretically because the system is complicated. In this paper, a two-stage production and inventory system with periodic review is considered. Both a base-stock policy system and an extended Kanban system with and without advance demand information are considered and formulated. Through numerical experiments the average costs and efficiencies are compared among these systems. For systems with advance demand information, sensitivity analysis is also developed with respect to the demand lead time, fluctuation of demand information and production capacity.

Keywords: Advance demand information, base stock, demand lead time, extended Kanban, Markov process

1. INTRODUCTION

In most of manufacturing companies it is important to control inventory systems appropriately. If the lost sale or delay to due date happens then consumer is unsatisfied, which leads to loss of consumers in future. On the other hand, more products in inventory increase inventory costs because of the deterioration of products, falling in the selling price and use of wide space of products.

Due to development of IT technology, the information on consumers becomes easily available. Use of it will be expected to decrease production and inventory costs. Advance demand information, which is abbreviated as ADI, is useful and the research on ADI has been developed. Gallego and Özer (2001) consider a single stage inventory model of a retailer and show that the base stock policy is optimal when replenish lead time is longer than demand lead time. Altendorfer and Minner (2014) consider a single stage production and inventory system with variable lead time and the optimality condition for base stocks and release lead time. Liberopoulos (2008) analyzes a single stage production and inventory system theoretically and show optimal base stock decreases in demand lead time. Hiraiwa and Nakade (2009) consider a single stage production and inventory system with ADI and derive the optimal amount of base stock and release lead time under base stock policy.

In practice, ADI may change in time, because of the sudden demand or cancel, and the change in production planning of the successive factory. Gayon et al. (2009) formulate the production inventory model with imperfect information and multiple-type customers into a Markov decision process and the optimality of base stock policy which is based on information of inventory position and advance demand. Benjaafar et al. (2011) formulate a single stage production system with imperfect information into a continuous time Markov decision process and show the optimality of base stock policy. In their models, however, the continuous-time model is applied, which implies the stop and restart of production at any time. This is unrealistic. Benbitour and Sahin (2015) consider a single stage production and inventory model in a discrete time with perfect and imperfect ADI and due date.

Generally, production system consists of multiple manufacturing factories, which orders parts periodically and ordered parts are delivered with demand lead time. In such multiple stage systems, many production order policies are applied: Base stock policies, CONWIP, Kanban, Extended Kanban, etc. Ohno (2011) compares these production order policies in multiple stage production systems.

In this paper, performance of pull-type control mechanism with and without time-variable ADI is discussed. The system is reviewed periodically. Two types of control mechanisms, a base stock policy and an extended Kanban policy, are applied. A simple Kanban policy is not included because basically Kanban is attached to a part and ADI is not directly applicable to Kanban control. The recursive equations are developed for these mechanism with and without ADI, and the relationships between these mechanisms and perfect/imperfect ADI are investigated.

2. TWO-STAGE PRODUCTION AND INVENTORY SYSTEMS

A two-stage single product production and inventory system with periodic review is considered. The time interval between successive observations is set as one period. The model is illustrated in Figure 1.

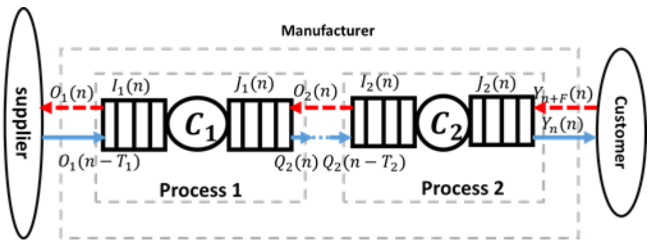

Figure 1: Production and Inventory System

The system consists of two processes (process 1, process 2) in the same manufacturer, an upstream supplier (process 0) and a customer in downstream of the manufacturer. Demand information on final products in advance arrives at the system before its due date. Based on the information, the manufacturer orders parts to the supplier and delivers finished products to the consumer. Demand information may vary in time, and it is confirmed one unit time before its due date. When the information does not change in time it is called perfect and it varies in time the information is called imperfect. The delivery time is required from process 0 to process 1 and from process 1 to process 2 because these processes are in different factory. Orders and delivery of products in processes 1 and 2 are made at the beginning of each period. Processes 0 and 1 must satisfy orders from processes 1 and 2 by delivering them with a fixed demand lead time, respectively. Process 0 is assumed to have enough inventory and thus there is no backlog between processes 0 and 1. On the other hand, backlogs may happen between processes 1 and 2, and process 2 and demand. Orders from process 2 are determined by considering the upper bound of work-in processes. The number of backlogs at the inventory of finished products has an upper bound, and the actual demand who arrives at the system and finds the current backlogs which reach the upper bound is lost.

Advance demand information for finished products arrives at the system F periods before (F=0,1,2..). This lead time F, which is a time interval from the arrival of demand information to its due date, is called demand lead time. $F=0$ implies that there is no ADI and demand directly arrives at the system to receive finished products.

The following notations are used in this paper.

$Y_m(n)$: the amount of demand, which is expected to receive a finished product in period m, at the beginning of period n ($n \le m \le n+F$),

$\delta_m(n)$: the amount of change of demand, which is expected to receive a finished product in period m, at the beginning of period n compared with period n-1 ($n+1 \le m \le n+F$),

$I_i(n)$: the amount of parts in buffer in front of process i at the beginning of period n (i=1,2),

$J_i(n)$: the amount of parts in a downstream buffer of process i at the beginning of period n (i=1,2),

$Q_i(n)$: the amount of parts delivered from process i-1 to i at the beginning of period n (i=1,2),

L_i: order lead time to process i-1 by process i (i=1,2),

T_i: delivery time to process i from process i-1 ($L_i > T_i$)(i=1,2),

$I_{i:max}$: buffer capacity in front of process i (i=1,2),

$J_{i:max}$: capacity of a downstream buffer of process i (i=1,2),

D_{min}: the minimum of demand,

D_{max}: the maximum of demand,

$C_i(n)$: the production capacity of process i in period n (i=1,2),

$C_{i:min}$: the minimal production rate of process i (i=1,2),

$C_{i:max}$: the maximal production rate pf process i (i=1,2),

B_{max}: the maximal number of backlogs,

P_i': the actual amount of products produced in process i in period n (i=1,2).

Decision variables

$P_i(n)$: the amount of products planned to manufacture in process i at the beginning of period n (i=1,2),

$O_i(n)$: the amount of products ordered to process i-1 from process i at the beginning of period n (i=1,2).

The production capacity follows the distribution function $p_k^i = P(C_i(n) = k)$ $(k = C_{i:min}, \dots, C_{i:max})$. Demand for finished products are mutually independent among periods and follow $q_k = P(Y_n(n) = k)$ $(k = D_{min}, \dots, D_{max})$. $\delta_{n+r}(n)$ is an amount of change in demand in unit period $(r = 1, \dots, F-1)$, and follows a distribution depending on $Y_{n+r}(n)$.

The sequence of orders, observation, decision and information in one period is illustrated in Figure 2.

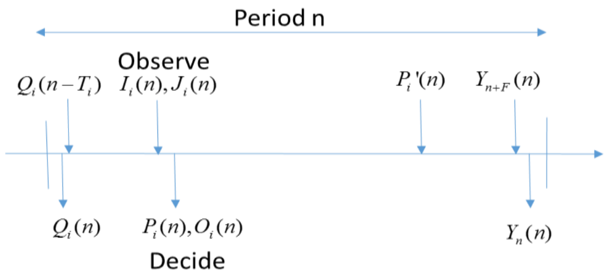

Figure 2. A Sequence of Parameters in One Period

Cost parameters are defined as follows.

C_i^I: an inventory cost per unit part per unit period in front of process i (i=1,2),

C_i^J: an inventory cost per unit part per unit period in the downstream buffer of process i (i=1,2),

C_i^Q: an inventory cost per unit part per unit period in delivery from process i-1 to process i (i=1,2),

C_i^B: a backlog cost per unit part per unit time in process i (i =1,2),

B_i: a backlog cost for each occurrence of backlog per unit time in process i (i =1,2),

C_l: the lost sale cost per unit demand in process 2.

In this paper, we derive the average costs under sub-optimal base stock policy and extended Kanban policy, which are compared under several parameter sets. These policies decide the numbers of orders and produced items in processes 1 and 2 after observing the state of the system at the beginning of each period.

Next we formulate our system as a Markov process. At the beginning of period n, the state of the system s_n is given by

$$\begin{gathered} s_n = (I_1(n), J_1(n), I_2(n), J_2(n), Y_n(n), \dots, Y_{n+F-1}(n), \\ O_1(n+1-L_1+T_1), \dots, O_1(n-1), \\ Q_1(n+1-T_1), \dots, Q_1(n), O_2(n+1-L_2+T_2), \\ \dots, O_2(n-1), Q_2(n+1-T_2), \dots, Q_2(n)). \end{gathered}$$

The space of possible states is defined as S. When the state is observed, the action is decided, which consists of pairs of amounts of orders and planned products in processes 1 and 2:

$$a_n = (O_1(n), P_1(n), O_2(n), P_2(n)).$$

For state $s_n \in S$, the sets of possible amounts of orders and products in process i, which are defined as $K_i^O(s_n)$ and $K_i^P(s_n), i = 1,2$ respectively, are given as follows.

$$K_1^P(s_n) = \{0, \dots, \min(I_1(n), C_{1:max}, J_{1:max} - J_1(n))\},$$

$$\begin{gathered} K_2^P(s_n) = \{0, \dots, \min(I_2(n), C_{2:max}, J_{2:max} \\ -J_2(n) + Y_n(n))\}, \end{gathered}$$

$$\begin{gathered} K_1^O(s_n) = \{0, \dots, I_{1:max} - I_1(n) - \textstyle\sum_{l=1}^{L_1-T_1-1} O_1(n-l) \\ -\textstyle\sum_{l=0}^{T_1-1} Q_1(n-l)\}, \end{gathered}$$

$$\begin{gathered} K_2^O(s_n) = \{0, \dots, \min(I_{2:max} - I_2(n) - [-J_1(n)]^+ \\ -\textstyle\sum_{l=1}^{L_2-T_2-1} O_2(n-l) - \textstyle\sum_{l=0}^{T_2-1} Q_2(n-l), \\ \min(I_1(n), C_{1:max}, J_{1:max} - J_1(n)) + J_1(n)\}. \end{gathered}$$

The Cartesian product of $K_i^O(s_n)$ and $K_i^P(s_n)$ $(i = 1,2)$ is given by $K(s_n)$. Then for each $s_n \in S$ and $a_n \in K(s_n)$ the transition to the next state is given as follows.

$$\begin{aligned} Y_{n+r}(n+1) &= Y_{n+r}(n) + \delta_{n+r}(n) \quad ,(r = 1, \dots, F-1) \\ I_1(n+1) &= I_1(n) + Q_1(n+1-T_1) - P_1'(n), \\ I_2(n+1) &= I_2(n) + Q_2(n+1-T_2) - P_2'(n), \\ J_1(n+1) &= J_1(n) + P_1'(n) - O_2(n+1-L_2+T_2), \\ J_2(n+1) &= \max(J_2(n) + P_2'(n) - Y(n), -B_{max}). \end{aligned}$$

The actual amounts of produced items, $P_i'(n)$, and the amounts of products to deliver from process i, $Q_i(n)$, for i=1,2, in period n, satisfy the following equations.

$$\begin{aligned} P_1'(n) &= min(P_1(n), C_1(n)), \\ P_2'(n) &= min(P_2(n), C_2(n)), \\ Q_1(n) &= O_1(n-L_1+T_1), \\ Q_2(n) &= min(O_2(n-L_2+T_2) + [-J_1(n-1)]^+, \\ &\quad P_1'(n-1) + [J_1(n-1)]^+). \end{aligned}$$

Thus the transition probability that the state becomes s_{n+1} when the action a_n is taken in state s_n, defined as $p(s_{n+1}|s_n, a_n)$, is given as follows.

$$p(s_{n+1}|s_n, a_n)$$

$$= \begin{cases} p_{c_1}^1 \times p_{c_2}^2 \times q_d \times \Delta(k_1, \dots, k_{F-1}|s_n) \\ if \;\; s_{n+1} = \begin{pmatrix} I_1(n) + Q_1(n+1-T_1) - \min(P_1(n), c_1), \\ J_1(n) + \min(P_1(n), c_1) - O_2(n+1-L_2+T_2), \\ I_2(n) + Q_2(n+1-T_2) - \min(P_2(n), c_2), \\ \max(J_2(n) + \min(P_2(n), c_2) - Y_n(n). -B_{max}), \\ Y_{n+1}(n) + k_1, \dots, Y_{n+F-1}(n) + k_{F-1}, d, \\ O_1(n+2-L_1+T_1), \dots, O_1(n), Q_1(n+2-T_1), \dots, \\ Q_1(n+1), O_!(n+2-L_2+T_2), \dots, O_2(n), \\ Q_2(n+2-T_2), \dots, Q_2(n+1) \end{pmatrix} \\ (C_{i:min} \le c_i \le C_{i:max}, i = 1,2 \quad D_{min} \le d \le D_{max}) \\ 0, \qquad otherwise. \end{cases}$$

Here, $\Delta(k_1, \dots, k_{F-1}|s_n)$ is the probability on demand fluctuation and given by

$$\begin{aligned} &\Delta(k_1, \dots, k_{F-1}|s_n) \\ &= P(\delta_{n+1}(n) = k_1, \dots, \delta_{n+F-1}(n) = k_{F-1}|Y_{n+1}(n), \dots, Y_{n+F-1}(n)). \end{aligned}$$

We define an indicate function on event e as

$$H(e) = \begin{cases} 1 & (\text{event } e \text{ happems}), \\ 0 & (\text{event } e \text{ does not happen}). \end{cases}$$

Then the expected cost in period n when action a_n is taken for state s_n, denoted by $c(s_n, a_n)$, is given by

$c(s_n, a_n) = \sum_{i=1}^{2} \{ C_i^I I_i(n) + C_i^J [J_i(n)]^+$
$+ C_i^Q \sum_{l=0}^{T_i-1} Q_i(n-l) + C_i^B [-J_i(n)]^+ + B_i H(J_i(n) < 0)\}$
$+ C_l \sum_{c_2=C_{2:min}}^{C_{2:max}} P\{C_2(n) = c_2\} \times [Y_n(n) - B_{max} - J_2(n)$
$- \min(P_2(n), c_2)].$ (2)

For given policy which determines action $a(s)$ for each state $s(\in S)$, the steady-state probability $\pi = \{\pi_s; s \in S\}$ can be obtained and the average cost g is given by

$$g = \sum_{s \in S} \pi_s c(s, a(s)). \quad (1)$$

3. PRODUCTION AND ORDERING POLICIES

3.1 Base Stock Policy with ADI

Here the base stock policy is discussed in our model. The base stock policy controls the inventory positions in each process as to maintain a fixed amount of echelon inventory. Here, the value of base stock in process i is denoted by s_i for $i = 1,2$, where

$$s_1 \geq s_2,$$

because the echelon inventory of process 1 includes the amount of products in process 2. In our model, the echelon inventory of process i is assumed to include the products placed in a front buffer of process i. In addition, when the demand information arrives, if it is used then the inventory position will decrease. On the other hand, if the demand information is not used the inventory position does not change by arrival of this information.

Under base stock and extended Kanban policies, the information on demand after order lead time L_2 or later is not used, because products are ordered based on the predetermined amount of base stocks, as to meet the future demand at its arrival. In addition, the use of such demand information will lead to the increase of base stock level and more products in inventory. Thus, in the following, it is assumed that if $F \leq L_2$, the advance demand information from period n to period $n+F-1$ is available, and the information from period $n+F$ to period $n + L_2$ is not used in period n, whereas if $F > L_2$, all advance information from period n to $n + L_2$ is available. Thus the total amount of available advance demand informed at period n to compute the inventory position in period n, which is denoted by $O_n^{L_2}$, is given by

$$O_n^{L_2} = \begin{cases} \sum_{i=0}^{F-1} Y_{n+i}(n) & (F \leq L_2), \\ \sum_{i=0}^{L_2} Y_{n+i}(n) & (F > L_2). \end{cases}$$

For the inventory positon of process 2 in period n, which is denoted by $x_2(n)$, is given by

$$x_2(n) \equiv I_2(n) + J_2(n) + \sum_{i=1}^{L_2-T_2-1} O_2(n-i) + \sum_{i=0}^{T_2-1} Q_2(n-i) - O_n^{L_2}. \quad (3)$$

For the inventory position of echelon inventory of process 1 in period n, denoted by $x_1(n)$, we have

$$x_1(n) \equiv I_1(n) + J_1(n) + \sum_{i=1}^{L_1-T_1-1} O_1(n-i) + \sum_{i=0}^{T_1-1} Q_1(n-i) + I_2(n) + J_2(n) - O_n^{L_2}. \quad (4)$$

When the demand information is imperfect and it is possible that $\delta_{n+i}(n)$ takes the negative value, the inventory position may exceed the amount of base stock. In this case, there is no order until the position is under the base stock level, and thus it follows that

$$O_i(n) = \max\{0, s_i - x_i(n)\}.$$

In particular, when the demand information does not decrease (that is, $\delta_{n+i}(n)$ is non-negative), the inventory position does not exceed the base stock level. Thus for each additional demand information $\delta_{n+i}(n)$ the order is made and thus

$$O_i(n) = \begin{cases} Y_{n+F-1}(n) + \sum_{k=0}^{F-1} \delta_{n+k-1}(n-1) & (F \leq L_2), \\ Y_{n+L_2}(n) + \sum_{k=0}^{L_2} \delta_{n+k-1}(n-1) & (F > L_2). \end{cases}$$

In any case, the amount of planned produced items is

$$P_i(n) = \min\{I_i(n), C_{i:max}\}.$$

Using the above values the action for each state is determined and transition probabilities and an expected cost per unit time is determined by equations shown in section 2.

3.2 Base Stock Policy without ADI

Inventory positions in period n are given as

$$x_2(n) \equiv I_2(n) + J_2(n) + \sum_{i=1}^{L_2-T_2-1} O_2(n-i) + \sum_{i=0}^{T_2-1} Q_2(n-i),$$

$$x_1(n) \equiv I_1(n) + J_1(n) + \sum_{i=1}^{L_1-T_1-1} O_1(n-i) + \sum_{i=0}^{T_1-1} Q_1(n-i) + I_2(n) + J_2(n).$$

(5)

By setting $I_1(0) = s_1 - s_2$, $I_2(0) = s_2$, $J_1(0) = J_2(0) = 0$ we have

$$O_i(n) = Y_{n-1}(n-1),$$
$$P_i(n) = min\{I_i(n), C_{i:max}\}.$$

3.3 Extended Kanban Policy

Extended Kanban policy is the combination of base stock and Kanbans. Parameters are the amounts of base stocks s_1, s_2, the amounts of withdrawal Kanbans M_1, M_2 and the amounts of production-ordering Kanbans N_1, N_2. The following must be satisfied.

$$s_2 \leq M_2 + N_2$$
$$s_1 - s_2 \leq M_1 + N_1.$$

The same equations (2) to (4) hold for $x_i(n)$ when the extended Kanban policy with ADI is applied and (5) holds when the policy without ADI is applied. Since extended Kanban policy takes the minimal value of orders determined by base stocks and withdrawal Kanbans as the amount of orders, it follows that

$$O_2(n) = \min\left\{[s_2 - x_2(n)]^+, M_2 - I_2(n) - [-J_1(n)]^+ - \sum_{l=1}^{L_2-T_2-1} O_2(n-l) - \sum_{l=0}^{T_2-1} Q_2(n-l)\right\},$$

$$O_1(n) = \min\left\{[s_1 - x_1(n)]^+, M_1 - I_1(n) - \sum_{l=1}^{L_1-T_1-1} O_1(n-l) - \sum_{l=0}^{T_1-1} Q_1(n-l)\right\}.$$

The amount of production order is determined by production-order Kanbans, and thus

$$P_2(n) = \min\{N_2 - [J_2(n)]^+, I_2(n), C_{2:max}\},$$
$$P_1(n) = \min\{N_1 - [J_1(n)]^+, I_1(n), C_{1:max}\}.$$

4. NUMEICAL EXPERIMENTS

4.1 Computations of State Space and Steady State Probabilities

For numerical comparison of base-stock and extended Kanban systems, it is desirable to derive the space of reachable states and steady state probabilities for given parameters, and derive optimal or sub-optimal sets of parameters in each system. In the following the procedures to derive them numerically are shown.

For given parameters under the policy, the action for each state is determined. The state space is derived by the following procedure.

Step 1: An initial state is set as the state which will be reached under successive no demand. For example, for F=2 and parameters (s_1, s_2) under the base stock policy,

$$s_0 = \left(I_1(0), J_1(0), I_2(0), J_2(0), Y_0(0), Y_1(0), Q_0(0)\right)$$
$$= (0, s_1 - s_2, 0, s_2, 0,0,0)$$

and for F=2 and parameters $(s_1, M_1, N_1, s_2, M_2, N_2)$ under the extended Kanban policy,

$$s_0 = \left(I_1(0), J_1(0), I_2(0), J_2(0), Y_0(0), Y_1(0), Q_0(0)\right)$$
$$= (\min([(s_1 - s_2) - N_1]^+, M_1), N_1, \min([s_2 - N_2]^+, M_2), N_2, 0,0,0)$$

Let $s' = s_0, S_v = \{s'\}, S_u = \phi$.

Step 2: For given parameters, the action a in s' is determined under a given policy, and a subset of states is defined as $S' = \{j; P(j|s', a) > 0\}$

Step 3: Set $S'_u = S_u \cup \{s'\}, S'_v = S_v \cup S' - S'_u$. If $S_v \neq \phi$ select state s' in S'_v. $s = s'$, $S_v = S'_v$, $S_u = S'_u$ and return to step 2. Otherwise terminate the procedure and output S_u as a reachable state space.

If the state space is defined, then the transition probabilities are determined. When the transition matrix P is defined by transition probabilities, the steady state probabilities $\boldsymbol{\pi}$ are given as for given initial probability $\boldsymbol{\pi_0}$

$$\boldsymbol{\pi} = \lim_{n\to\infty} \boldsymbol{\pi_0} \boldsymbol{P}^n.$$

Thus probability $\boldsymbol{\pi}$ is obtained by computing $x_i = \sum_{k=0}^{S_{max}-1} x'_k \times P_{ki}$. r epeatedly until x_i convergences. When $\boldsymbol{\pi}$ is computed, the average cost can be derived by (1).

To derive the set of sub-optimal parameters of base stock policies, first set a parameter s_2 for a single process 2 by local search, and then fix it and derive a parameter s_1 in the two-stage system by local search. The pair of derived parameters is an initial parameter set. For extended Kanban systems there are three parameters for each process, and so for process 2, first set parameters $(s_2, M_2, N_2) = (A, I_{2:max}, J_{2:max})$ where A is enough large. Then fix M_2, N_2 by decreasing s_2 derive optimal s_2. After that, fix s_2, N_2 and by decreasing M_2 find optimal M_2. In the same way determine N_2. Then parameter sets of process 2 is fixed. For process 1 determine parameters in the two-stage system in the same way. After all parameters are computed, start a local search with the derived parameters as an initial parameter set and derive a local optimal parameter set. Of course, it is not assured that the derived local optimal parameter set is optimal, but in results of the following numerical examples it seems that these policies are near optimal policies.

4.2 Data of Numerical Experiments

We first discuss the demand information and policies. Common parameters are given as follows.
$C_{i:max} = 3(i = 1,2)$, $(D_{min}, D_{max}) = (0,3)$, $(L_1, L_2) = (1,2)$, $(T_1, T_2) = (0,1)$, $B_{max} = 2$, $(C_1^I, C_2^I) = (3,6)$, $(C_1^J, C_2^J) = (6,12)$, $(C_1^Q, C_2^Q) = (0,6)$, $(C_1^B, C_2^B) = (0,80)$, $(B_1, B_2) = (0,120)$, $C_l = 1000$, $I_{i:max} = 8$, $J_{i:max} = 10$ (i=1,2), F=2.

The demand has the following distribution, which is a truncated Poisson distribution. The expected value is 1.410196.

$$q_0 = 0.223130, q_1 = 0.334695, q_2 = 0.251021,$$
$$q_3 = 0.191153$$

The probability on production capacity is considered in the following two cases.

$$\alpha : p_3^i = 0.7, p_2^i = 0.2, p_1^i = 0.1, p_0^i = 0$$

$$\beta : p_3^i = 0.7, p_2^i = 0, p_1^i = 0.2, p_0^i = 0.1$$

Here $p_j^i = P(C_i(n) = j), j = 0,1,2,3, i = 1,2.$ Four combinations are considered: αα , αβ, βα, ββ, where for example αβ means that process 1 follows type α whereas process 2 follows type β.

In imperfect ADI, the probability distribution

$$q_{i,j} = P(Y_{n+1}(n+1) = j | Y_{n+1}(n) = i)$$

is assumed in one of the following two sets.

Case 1: $q_{0,0} = 0.753655, q_{1,1} = q_{2,2} = q_{3,3} = 0.8, q_{0,1} = 0.246350, q_{1,0} = 0.164230, q_{1,2} = 0.035775, q_{2,1} = 0.047700, q_{2,3} = 0.152300, q_{3,2} = 0.2.$

Case 2: $q_{0,0} = 0.137817, q_{1,1} = q_{2,2} = q_{3,3} = 0.3, q_{0,1} = 0.862183, q_{1,0} = 0.574789 q_{1,2} = 0.125211, q_{2,1} = 0.166949, q_{2,3} = 0.533051, q_{3,2} = 0.7.$

We note that for both cases when $P(Y_{n+1}(n) = j)$ is set as q_j, the resulting probability $P(Y_{n+1}(n+1) = j)$ is also the same as q_j for j=0,1,2,3. The size of change of demand is stochastically bigger in case 2 than in case 1..

Here we call the base stock policies with and without ADI by Policy B-A and Policy B respectively, and the extended Kanban policies with and without ADI by Policy E-A and Policy E, respectively.

4.3 The Case with Perfect ADI

Table 1 shows the sub-optimal parameter sets and the sub-optimal average cost under policies B, E, B-A and E-A, each of which four combinations of production capacities are considered. As shown, B-A and E-A for F=1 are better than B and A respectively, and are worse than B-A and E-A for F=2 respectively. Policy E is much superior to Policy B. In particular, from detail analysis of numerical results it is found that the inventory cost is much smaller under policy E compared with policy B, by the effect of Kanbans. On the other hand, both policies have almost the same the delivery and backlog costs. When process 2 has more production capacity, the extended Kanban policy has much better performance than the base stock policy.

When perfect ADI is used, under both policies the cost decreases. In particular for F=2, Policy B-A attains good performance and is similar to Policy E-A for all production capacity cases. Thus perfect ADI is more effective under the base stock policy than under the extended Kanban policy.

Table 1: Results under perfect ADI

Policy		s1	M1	N1	s2	M2	N2	average cost
B	αα	10			8			74.54638
	αβ	10			9			89.61155
	βα	11			8			84.62245
	ββ	12			9			98.50113
E	αα	10	5	3	9	8	4	67.58925
	αβ	11	6	3	10	8	6	87.08353
	βα	11	6	5	9	8	4	77.05313
	ββ	12	6	5	10	8	6	94.94034
F=1								
B-A	αα	8			6			67.19144
	αβ	9			7			85.42022
	βα	9			6			78.65704
	ββ	10			7			94.69493
E-A	αα	8	5	4	6	8	4	63.82044
	αβ	9	5	3	8	8	6	83.64551
	βα	9	6	6	6	8	4	74.86397
	ββ	10	6	6	8	8	6	92.63474
F=2								
B-A	αα	6			4			59.77750
	αβ	7			6			80.83257
	βα	8			4			73.31085
	ββ	8			6			91.20191
E-A	αα	6	6	4	4	8	4	59.55409
	αβ	7	5	4	6	8	6	80.51865
	βα	8	6	7	4	8	4	72.86934
	ββ	8	6	6	6	8	6	90.55633

4.4 The Case with Imperfect ADI

For imperfect advance demand information, cases 1 and 2, policies B-A and E-A are compared when F=2. The results are shown in Table 2.

For case 1, the suboptimal parameter sets are the same as perfect ADI, but the average cost is greater under both policies. In particular, when both processes have production capacity distribution α, the time-variant property of ADI worse the average cost. In particular, the backlog cost is increased under imperfect ADI. That is, the change in time of demand information mostly influences backlogs.

Compared with case 1, case 2 has worse performance because of more variance of ADI. Case 2 has the same parameters as case 1 under policy B-A, whereas sub-optimal E-A policy increases the amount of base stocks with smaller Kanbans. In fact, under E-A policy the number of products in front of each process is greater and the number of products after processes is smaller than those under B-A policy. Therefore, under E-A policy more products are in each process, but Kanbans make the parts wait for process under imperfect information. As a result, the difference of average costs between E-A and B-A policies is greater under case 2 compared with case 1. Thus when ADI is imperfect, appropriate numbers of Kanbans may be more useful than in the case of perfect ADI.

Table 2: Results under Imperfect ADI

Policy		s1	M1	N1	s2	M2	N2	average cost
Case 1								
B-A	αα	6			4			61.25221
	αβ	7			6			81.74380
	βα	8			4			74.26519
	ββ	8			6			92.03674
E-A	αα	6	6	4	4	8	4	60.67295
	αβ	7	5	4	6	8	6	81.26103
	βα	8	6	7	4	8	4	73.34775
	ββ	8	6	6	6	8	6	91.20015
Case 2								
B-A	αα	6			4			64.10062
	αβ	7			6			83.98064
	βα	8			4			79.19906
	ββ	9			6			64.50480
E-A	αα	6	6	4	4	8	4	62.80729
	αβ	7	5	3	6	8	6	83.30052
	βα	8	6	6	5	8	4	74.41647
	ββ	9	6	5	7	8	6	93.14097

5. CONCLUSION

In this paper, a two-stage production and inventory system with ADI are considered. Base stock and Extend Kanban policies are applied and the system under each policy is formulated as a Markov chain. Performance of these policies with and without ADI is evaluated by numerical examples. When ADI is applied, the difference of both policies is smaller, but when the fluctuation of imperfect ADI increases, Kanbans still have a positive effect for controlling production.

More numerical comparison and sensitivity analysis are needed to confirm properties of these control mechanisms and effects of ADI under these controls. In addition, theoretical comparison will be needed under several control mechanisms with and without ADI. Comparison with other production control policies and optimal policies which can be computed by Markov decision processes is also left for future research.

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