A Spare Parts Allocation Problem for Multi-Level Systems

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Abstract. In this paper, we consider a spare parts allocation problem for multi-level systems with two-echelon repair system. The system consists of multi-units (components and modules) and has a multi-level structure. The (S-1, S) inventory policy is considered to order spare parts and the system availability and life cycle cost are considered as the optimization criteria. We determine the line replaceable units (LRUs) and these maximum spare parts storage levels S in each echelon site that minimize the life cycle cost and satisfy the target system availability. The near-optimal solutions are obtained by a genetic algorithm and simulation. We investigate the effect of parameters on the near-optimal solutions in numerical examples.

Keywords: Availability, Multi-Level, Multi-Echelon, Spare parts

1. INTRODUCTION

Recently most systems which have many units (components and modules) with multi-level structure are used in many fields such as military, manufacturing and so on. These systems generally have many functions to carry out their own missions. The system failure can result in great economic loss and mission failure. Therefore, it is important to maintain high system availability. The system availability is influenced by the reliability and maintainability of components and modules. The maintainability is related to maintenance time of the failure unit and the maintenance time is influenced by capability of supply maintenance resources such as technicians, equipment and spare parts. The maintainability is improved by reducing maintenance time and supplying maintenance resources immediately. Therefore, improving the maintainability can lead to high system availability.

In multi-level systems, the one way to restore the failed system is to replace the failed unit with new one. This unit is called line replaceable unit (LRU) that is the unit which is replaced with new one to restore the failed system at operation sites. Assembly and disassembly are carried out during replacement procedure. The number of assembly and disassembly for replacing the failed LRU is decided by the level of LRU in the multi-level system. As the level of LRU moves from the bottom to the top level of the system, the number of assembly and disassembly is decreased. Thus, the LRU at low levels requires more long replacement time than the LRU at top level. However, the price of LRU is also increased as the level of LRU increases. Therefore, there is trade-off between replacement time and the price of LRU.

In order to replace the failed LRU, it is important to supply its spare parts immediately at the start of replacement procedure. Spare parts is stored in the storage site and is replaced with the failed one for restoring the failed system. We need appropriate spare parts to replace the failed systems within reasonable time period, and also should order the spare parts periodically. Therefore, In order to improve the system availability, the optimization problems determining LRUs and these inventory levels are considered jointly.

There are many existing papers about spare parts

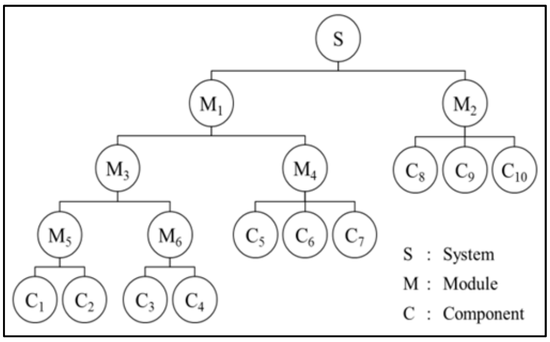

Figure 1: The example of a multi-level system

problem based on multi-echelon inventory systems. We introduce recent research results. Costantino et al. (2013) studied the allocation problem for LRUs and SRUs (subset of LRU). A two-echelon system is considered and each echelon has storage site and maintenance site. The objective is to find the inventory level of LRU and SRU for each storage site for minimizing backorders satisfying the target availability of system. Budget constraint is also considered and a marginal analysis is used to solve the problem. Lu and Yang, (2012) studied the allocation problem of repairable spare parts in three-echelon system. The objective is to obtain the inventory level of each echelon to maximize the supply availability under the spare parts cost limit. The marginal analysis is used to obtain the optimal solutions. Sun and Zuo (2010) studied the allocation problem of aircraft spare parts in two-echelon system. The objective is to determine an optimal spare parts level in each echelon to maximize the aircraft availability with minimum life cycle cost. Wang et al. (2012) considered the resources allocation problem in multi-echelon support system. The objective is to find the optimal inventory level in each echelon to maximize the support probability under cost constraint. Yun et al. (2012) dealt with a PM and spare parts problem for a rolling stock system. Optimization criteria are the system life cycle cost and system availability. A genetic algorithm and simulation are used to find near optimal PM intervals and inventory level of spare parts.

In this paper, we consider a spare parts allocation problem in multi-level systems with two-echelon repair system. The objective is to determine the LRUs and these maximum inventory levels in each echelon to minimize the life cycle cost and to satisfy the target system availability.

This paper is organized as follows. In section 2, we define the system and two-echelon repair system. Optimization approach used to solve our problem is described in section 3. In section 4, we study the numerical examples. Section 5 concludes this paper.

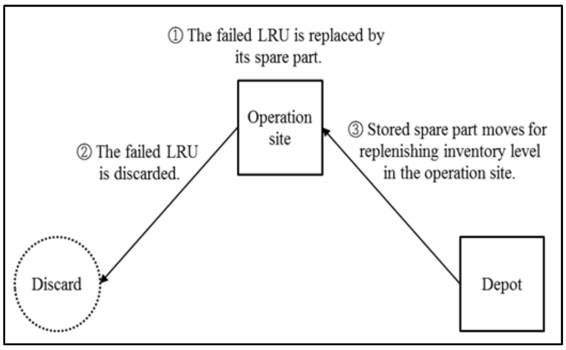

Figure 2: The replacement procedure of failed LRU

2. SPARE PARTS ALLOCATION PROBLEM

2.1 The System and Two-Echelon Repair System

In this paper, we consider a series system that has modules and components with multi-level structure. The module consists of several components or modules. The system consists of several modules. The example of a multi-level structure is shown in Figure 1.

We consider two-echelon repair system which can store and supply the LRUs. The first level echelon is called an operation site and the second level echelon is called by a depot. In the operation site, the systems are operated and if the system is failed the failed LRU is replaced by new one. All operation sites are connected to the depot. In the depot, the spare parts of LRU are manufactured, stored and transported to the operation sites for satisfying the demand of spare parts in the each operation site. The all types of LRU can be stored in the operation sites and depots. Figure 2 shows the replacement procedure of LRUs.

We consider direct lines to select the LRUs. Direct line is defined as a set of units from a component level to the system level in the multi-level system. (Yun and Kim, 2004) For example, the structure in Figure 1 has 10 direct lines which are (C_1-M_5-M_3-M_1-S_1), (C_2-M_5-M_3-M_1-S_1), (C_3-M_6-M_3-M_1-S_1), and so on.

2.2 Assumption

The assumptions in this paper are as follows:

1. Failed system is only restored through replacing failed LRU by its spare part.
2. All the failed LRU are discarded.
3. (*S*-1, *S*) inventory policy is considered to order the spare parts of LRUs in operation site and depot.
4. The number of technicians and maintenance support equipment is enough.
5. The only one unit can be selected to LRU in one direct line.

Table 1: The Notation used in objective function and constraints

Notation	Definition	Notation	Definition
i	Index of unit (i = 1, 2, …. , I)	M_{di}	Maximum possible inventory level of unit i in depot d
j	Index of component (j = 1, 2, …. , J)	n_{pi}	Inventory level of unit i in operation site p
p	Index of operation site (p = 1, 2, …. , P)	m_{di}	Inventory level of unit i in depot d
d	Index of depot (d = 1, 2, …. , D)	E[LC]	Expected life cycle cost
y_i	unit i is the LRU (y_i = 1) or not (y_i = 0)	E[NR_i]	Expected number of replacements of unit i
LT	System life time (hour)	E[NS_i]	Expected inventory level of unit i per hour
A_T	Target system availability	E[TT_i]	Expected transportation time of unit i per hour
A_S	System availability	RC_i	Replacement cost of unit i
S_j	Set of ancestor units of component j	HC_i	Inventory holding cost of unit i per hour
N_{pi}	Maximum possible inventory level of unit i in operation site p	TC_i	Transportation cost of unit i per hour

3. ALGORITHM FOR OPTIMIZATION

In this section, we give the notation, the objective function and constraints of the optimization problem in which LRU and order policy is determined. Next, we describe the optimization approach using a genetic algorithm and simulation for finding the near optimal solution about our problem.

3.1 Objective Function

The Table 1 gives the notation which is used to solve our problem. The objective function is given by

$$Min\ \ E[LC] = \sum_{i=1}^{N} y_i(E[NR_i]\cdot RC_i + LT\cdot E[NS_i]\cdot HC_i + E[TT_i]\cdot TC_i) \quad (1)$$

Life cycle cost consists of replacement cost $E[NR_i]\cdot RC_i$, inventory holding cost $LT\cdot E[NS_i]\cdot HC_i$ and transportation cost $E[TT_i]\cdot TC_i$ of LRU. The expected values of NR_i, NS_i and TT_i are obtained by simulation. The equations from Eq. (2) to Eq. (6) are the constraints.

$$A_S \geq A_T \quad (2)$$

The Eq. (2) means that system availability must be greater than target system availability.

$$y_i = 0 \text{ or } 1,\ \ \forall i \quad (3)$$

The Eq. (3) means that unit i is LRU or not.

$$y_j + \sum_{k\in S_j} y_k \leq 1 \quad (4)$$

The Eq. (4) means that the only one unit can be LRU in each direct line.

$$n_{pi} \leq N_{pi}\cdot y_i,\ \ \forall p, i \quad (5)$$

$$m_{di} \leq M_{di}\cdot y_i,\ \ \forall d, i \quad (6)$$

The Eq. (5) means that the inventory level of unit i in the operation site p must be less than the maximum inventory level in the operation site p.

3.2 Genetic Algorithm and Simulation

In order to find the near optimal solution, we use a genetic algorithm and simulation. In general, the genetic algorithm has several advantages. First, the genetic algorithm handles multiple solutions in total solution space. Second, the genetic algorithm is less complex than the other algorithms. Third, the operators of genetic algorithm have flexibility. Therefore, the user can modify the operators easily based on the problem. Alternative solutions are generated by the genetic algorithm and we evaluate the solutions using simulation. The optimization procedure using genetic algorithm and simulation as follows:

Step 1 Input data about site, system, unit and spare parts
Step 2 Generate initial solutions
Step 3 Crossover and mutation
Step 4 Evaluate solutions by using simulation
Step 5 Selection
Step 6 Check the generation is the last one.
  6.1 If the generation is not last then go to Step3. Otherwise, go to Step 7.
Step 7 Estimate system availability and life cycle cost by using simulation

Table 2: Input data of components, modules and system

Unit	Failure time	Replacement time	Unit price	Additive cost	Replacement cost	Transportation cost (per hour)	Inventory holding cost (per hour)
S	-	LN(2.20, 0.5)	2,604	10	2,614	0.045	0.075
M_1	-	LN(4.40, 0.5)	1,543	12	1,555	0.027	0.044
M_2	-	LN(7.50, 0.5)	825	13	838	0.014	0.024
M_3	-	LN(8.75, 0.5)	908	15	923	0.016	0.026
M_4	-	LN(12.50, 0.5)	495	16	511	0.009	0.015
M_5	-	LN(20.00, 0.5)	330	18	348	0.006	0.010
M_6	-	LN(17.50, 0.5)	495	19	514	0.009	0.015
C_1	Exp(1,000)	LN(40.00, 0.5)	100	22	122	0.002	0.003
C_2	Exp(1,500)	LN(45.00, 0.5)	200	23	223	0.004	0.006
C_3	Exp(1,000)	LN(35.00, 0.5)	150	21	171	0.003	0.005
C_4	Exp(1,500)	LN(50.00, 0.5)	300	22	322	0.006	0.009
C_5	Exp(1,000)	LN(30.00, 0.5)	150	18	168	0.003	0.005
C_6	Exp(1,500)	LN(25.00, 0.5)	200	19	219	0.004	0.006
C_7	Exp(1,000)	LN(30.00, 0.5)	100	18	118	0.002	0.003
C_8	Exp(500)	LN(25.00, 0.5)	200	15	215	0.004	0.006
C_9	Exp(1,000)	LN(20.00, 0.5)	250	16	266	0.005	0.008
C_{10}	Exp(1,000)	LN(15.00, 0.5)	300	16	316	0.005	0.009

Step 8 Check the target system availability is satisfied

8.1 If the target system availability cannot be satisfied then go to Step 2. Otherwise, go to Step 9.

Step 9 Obtain the statistic values of solution

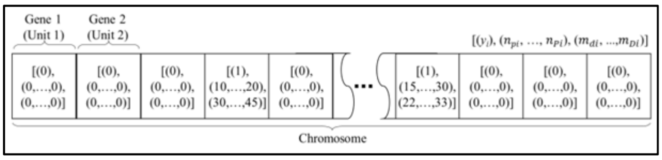

Figure 3: The example of the solution representation

An example of the solution representation is shown in Figure 3. Each gene in the chromosome represents a unit in the system structure. The gene has three values, y_i, n_{pi} and m_{di}. y_i is binary-valued variable and '1' represents that unit i is LRU. n_{pi} and m_{di} are non-negative integer values and they represent the inventory level of unit i in the operation site p and depot d. At the beginning of the genetic algorithm, initial solutions are generated at random. Generally, there are three operators which are called crossover, mutation and selection. In the crossover operator, one-point and two-point crossover are used. However, we cannot use the one-point or two-point crossover because these kinds of crossover operations can break the direct line rule in this paper. There are several LRUs which can exist simultaneously in a direct line using one-point or two-point crossover operator. Therefore, we select two chromosomes (parent 1 and parent 2) from population randomly. We select the LRU randomly and find units which cannot be LRU due to selected LRU from parent 1

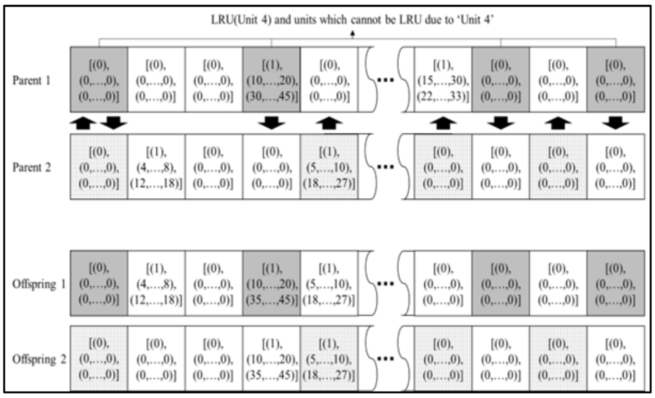

Figure 4: The example of considered crossover

and 2 to generate offspring. Figure 4 shows an example of crossover in this paper. In the mutation operator, a unit which is not LRU is selected randomly. We set it to be LRU and allocate inventory levels of the unit in operation sites and depots. In the selection operator, elite selection is considered and the value of the objective function is used for fitness evaluation.

4. NUMERICAL EXAMPLES

We consider the system structure of Figure 1 in numerical examples. The system has multi-level structure which consists of 10 components and 6 modules. Exponential and lognormal distributions are assumed for time to failure and replacement times. Table 2 shows input data of each unit in the system. The replacement cost is the sum of unit price and additive cost. The additive cost is related to disassembly and assembly of the unit. As the unit

Table 3: Optimal LRUs and these inventory levels of each target system availability.

Unit	Target system availability 0.80		Target system availability 0.85		Target system availability 0.90	
	LRU	Inventory level [operations site 1, 2, 3, 4, 5 / depot 1, 2]	LRU	Inventory level [operations site 1, 2, 3, 4, 5 / depot 1, 2]	LRU	Inventory level [operations site 1, 2, 3, 4, 5 / depot 1, 2]
S						
M_1						
M_2			✔	[4, 2, 6, 8, 4 / 3, 9]	✔	[6, 3, 9, 12, 6 / 1, 3]
M_3					✔	[2, 1, 3, 4, 2 / 7, 21]
M_4					✔	[11, 5, 16, 22, 11 / 3, 9]
M_5	✔	[2, 1, 3, 4, 2 / 15, 45]	✔	[3, 1, 4, 6, 3 / 14, 42]		
M_6	✔	[7, 3, 10, 14, 7 / 2, 6]	✔	[14, 7, 21, 28, 14 / 3, 9]		
C_1						
C_2						
C_3						
C_4						
C_5	✔	[10, 5, 15, 20, 10 / 13, 39]	✔	[2, 1, 3, 4, 2 / 20, 60]		
C_6	✔	[2, 1, 3, 4, 2 / 14, 42]	✔	[4, 2, 6, 8, 4 / 17, 51]		
C_7	✔	[13, 6, 19, 26, 13 / 8, 24]	✔	[11, 5, 16, 22, 11 / 13, 39]		
C_8	✔	[4, 2, 6, 8, 4 / 12, 36]				
C_9	✔	[5, 2, 7, 10, 5 / 11, 33]				
C_{10}	✔	[5, 2, 7, 10, 5 / 6, 18]				

Table 4: Optimal life cycle costs of different target system availability.

Target system availability	Replacement cost	Inventory holding cost	Transportation cost	Life cycle cost
0.80	24,202,736	848,121	8,506	25,050,857
0.85	46,401,877	769,530	15,756	47,187,163
0.90	74,159,911	672,395	25,328	74,857,634

level moves from the bottom to the top in the system structure, the unit price is increased. However, the additive cost is decreased because lower level unit needs more complicate disassembly and assembly for replacing the unit. The inventory holding cost and transportation cost are affected by the volume of unit. As the unit level moves from the bottom to the top in the system structure, the volume is increased. In order to store and transport the high volume unit, the inventory holding and transportation cost are increased. In case of the replacement time, the number of disassembly and assembly of unit affects the replacement time. Therefore, the low level unit needs more time than the higher level unit during the replacement. We consider the two-echelon repair system which is described in Figure 5. The maintenance system consists of 5 operation sites and 2 depots. Two operation sites are supported by the depot 1 and three operation sites are supported by the depot 2. Operation sites have different multi-unit systems. We assumed all transportation time of spare parts is a random variable and follows identical exponential distribution with mean 10

The parameters of a genetic algorithm are as follows: population size (100), crossover rate (0.7), mutation rate (0.3) and the number of generation (50). The simulation time is 20 years (175,200 hours) and the number of replication is 10 times.

We carried out the sensitivity analysis with different target system availabilities which are 0.80, 0.85 and 0.90. Table 3 shows the result of sensitivity analysis. As the target system availability increases, we can find that the level of LRU is increased because shorter replacement time leads to increase the target system availability. Therefore, the life cycle cost is also increased as the replacement, inventory holding, transportation cost increase. Table 4 shows the life cycle cost and target system availability.

5. CONCLUSION

In this paper, we deal with a spare parts allocation problem for multi-level systems with two-echelon repair system. The system availability and life cycle cost are considered as optimization criteria. Decision variables are the LRUs and these maximum inventory levels in operation sites and depots. We use a genetic algorithm and simulation in order to find the near optimal solution to minimize the life cycle cost. The genetic algorithm is used to generate alternative solutions and simulation is used to evaluate the alternative solutions. In the numerical examples, we carried out sensitivity analysis with three different target system availabilities. The result shows that the level of LRU is increased as the target system availability increases. For further studies, we will consider inventory policy of depot (*s*, *S*) and the repair of failed LRU.

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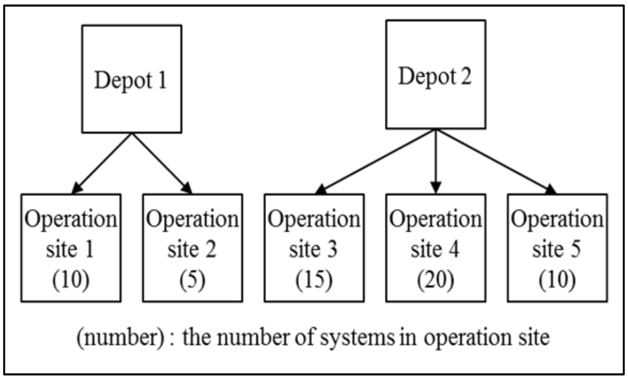

Figure 5: The two-echelon system for numerical example

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