Expected Severity Model for FMEA under Weibull Failure and Detection Time Distributions with a Common Shape Parameter

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Abstract. When a failure cause is detected and corrected before the failure itself occurs, there will be no other effect except the correction cost. But, if its cause is detected after the failure actually occurs, its effects will become more severe depending on the duration of the uncorrected failure. This situation is not addressed properly by the traditional FMEA, although it is a popular technique in industries to evaluate failure risks. In this paper, the severity of a failure effect is modeled as a function of undetected time duration of failure. Three types of severity function are considered in the model; constant, linear, and quadratic. Assuming the failure occurrence time and its corresponding failure cause detection time to have Weibull distributions with a common shape parameter, the expected severity is derived for each failure cause. Based on the expected severity, a risk evaluation metric for each failure cause is proposed with an illustrative example.

Keywords: FMEA, Risk Priority Number, Expected Severity, Risk Evaluation Metric

1. INTRODUCTION

Since the RPN(Risk Priority Number) of the FMEA (Failure mode and effect analysis) has several drawbacks as a risk metric, there have been many improvement efforts for better evaluation of the risk of a failure. For detailed discussions, see Liu et al. (2013) In most practical situations, a failure occurs after at least one of its causes has occurred. And there is a time gap between occurrences of a failure cause and the failure itself. The detection and corrective action on the failure cause may take place even later. Some work considers the effect of time on the failure risk. For example, Rhee and Ishii(2003) introduced a life cost-based FMEA for analyzing design alternatives of a particular system with Monte Carlo simulation. Kwon et al. (2011) proposed a time dependent expected loss model for a given mission period, assuming a homogeneous Poisson process for occurrence of failures and causes. Few studies, however, incorporate the effect of delayed detection and correction of a failure, considering the role of time in risk evaluation for FMEA. If a failure or its cause is left uncorrected for a longer time, the severity of the effect will become larger and the corresponding risk will also increase. For example, the effect of a failure incurring leakage of a toxicant or a radioactive substance will become more severe as the time elapses without fixing the failure.

There are only a few studies considering this situation. Kwon et al. (2013) presented an optimal monitoring policy under a time dependent model with a quadratic loss function for the unfulfilled mission period. Jang et al.(2016)

suggests risk evaluation in FMEA when the failure severity depends on the detection time. Jang et al.(2016) proposed a hierarchical time delay model for risk evaluation in FMEA assuming exponential distributions for occurrence and detection times of failures and corresponding causes.

The above works assume that the time distributions are exponential for simplicity. In practical situations, a failure is more likely to occur as the time elapses if its pre-occurred cause is not eliminated or corrected. For example, consider a mechanical shaft seal of a pump. Poor lubrication (root cause) may cause a very high frictional heat on the seal face. And the alternating local heating and cooling of the seal face may cause small, radial, thermal cracks. If the pump system is operated continuously without fixing the poor lubrication problem, the cracks will grow bigger and bigger as time elapses, which eventually results in leakage (failure) of the pump. In this situation, an increasing failure rate is a more reasonable assumption than a constant failure rate.

In this paper, we consider the situation where a failure occurs only after at least one of its causes has occurred. And it takes some time to detect (identify and correct) the cause. We assume Weibull probability law with a common shape parameter for occurrence and detection time distributions. Three types of the severity function are considered, i.e., constant, linear and quadratic. The model is constructed in section 2 and a risk evaluation metric (REM) is defined in section 3. An illustrative example is provided in Section 4 with some discussions and conclusion is followed in Section 5.

2. THE EXPECTED SEVERITY MODEL

2.1 The Time-oriented Failure Mechanism

Suppose that an item is exposed to random events (failure causes) leading to a failure. And the failure occurs in time if no corrective action is taken on its pre-occurred cause. Then the occurrence process of failure can be described by the elapsed time T from the failure cause occurrence to the actual failure occurrence. There may also be a time delay D from the failure cause occurrence to its detection. We admit detection to include identification and corrective action for the failure cause. This situation is depicted in Fig. 1. Thus, when an item is exposed to random events leading to failure, a stochastic model may be constructed to appropriately describe the failure occurrence, detection and its severity.

2.2 The Severity Function of a Failure

The effect of a failure cause will not be so much severe if it is detected before the actual failure occurs. On the other hand, it will be very much severe if the failure cause is detected after the actual failure occurs. Under the situation of a failure incurring leakage of a toxicant or a radioactive substance, the result may be even disastrous if detection of the failure cause is delayed. Thus, we may reasonably assume that the severity of a failure effect is a function of T and D.

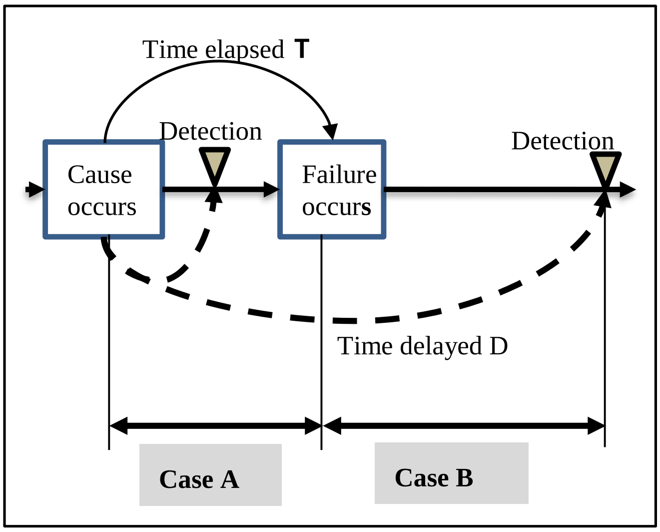

Figure 1. The Time-oriented Failure Mechanism

We consider three types of severity function; constant, linear and quadratic. For the constant severity function, we assume that a constant cost or loss is incurred depending on failure occurrence. A constant cost will be expended to fix the failure cause if it is detected before the failure itself occurs. If it is detected after its corresponding failure occurs, a far bigger but constant amount of losses will be incurred. Thus, the constant severity function is defined as

$$S = \begin{cases} a, & 0 < D < T \\ a + b, & T < D. \end{cases} \tag{1}$$

For the linear severity function, we assume the loss of the failure increases proportionally to the delayed time of identifying and fixing its cause. Thus, the linear severity function is defined as

$$S = \begin{cases} a, & 0 < D < T \\ a + b(D - T), & T < D. \end{cases} \tag{2}$$

For the quadratic severity function, we assume the loss of the failure increase as a quadratic function of the delayed time of identifying and fixing its cause. Thus, the quadratic severity function is defined as

$$S = \begin{cases} a, & 0 < D < T \\ a + b(D-T)^2, & T < D. \end{cases} \quad (3)$$

2.3 The Expected Severity

It will be reasonable to evaluate the risk attributable to a failure cause based on its corresponding severity and its occurrence rate. The occurrence rate of a failure cause will be considered later and only the severity is considered here. Each severity function of (1), (2), and (3) is a random variable that depends on the failure occurrence time T and the failure cause detection time D.

If the probability distributions of T and D are given, the expected severity can be derived. We assume T and D follow the Weibull probability law with the same shape parameter once a failure cause has occurred. The probability density and distribution functions of T and D are

$$f_T(t) = \beta\lambda^\beta t^{\beta-1} e^{-(\lambda t)^\beta}, \quad t > 0, \quad (4)$$

$$F_T(t) = 1 - e^{-(\lambda t)^\beta}, \quad t > 0, \quad (5)$$

$$f_D(u) = \beta\mu^\beta u^{\beta-1} e^{-(\mu u)^\beta}, \quad u > 0, \quad (6)$$

$$F_D(u) = 1 - e^{-(\mu u)^\beta}, \quad u > 0. \quad (7)$$

For the constant severity, its expected value can be easily derived as

$$E(S) = a + b\frac{\lambda^\beta}{\lambda^\beta + \mu^\beta}. \quad (8)$$

Before deriving the expected values for the linear and quadratic severities, we must first obtain some integration results. Denote the gamma function and the upper incomplete gamma function by $\Gamma(s)$ and $\Gamma(s,x)$, respectively, that is,

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t}\, dt, \quad (9)$$

$$\Gamma(s,x) = \int_x^\infty t^{s-1} e^{-t}\, dt. \quad (10)$$

By integration by part, we get the following results:

$$\int_t^\infty u f_D(u) du = \int_t^\infty u\beta\mu^\beta u^{\beta-1} e^{-(\mu u)^\beta} du$$

$$= t e^{-(\mu t)^\beta} + \frac{1}{\mu\beta}\, \Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right). \quad (11)$$

$$\int_0^\infty \Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right) f_T(t) dt = \Gamma\left(\frac{1}{\beta}\right)\left\{1 - \mu\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{1}{\beta}}\right\}. \quad (12)$$

$$\int_t^\infty u^2 f_D(u) du = \int_t^\infty u^2 \beta\mu^\beta u^{\beta-1} e^{-(\mu u)^\beta} du$$

$$= t^2 e^{-(\mu t)^\beta} + \frac{2}{\mu^2\beta}\Gamma\left(\frac{2}{\beta}, (\mu t)^\beta\right). \quad (13)$$

$$\int_0^\infty t^2 e^{-(\mu t)^\beta} f_T(t) dt = \int_0^\infty t^2 e^{-(\mu t)^\beta} \beta\lambda^\beta t^{\beta-1} e^{-(\lambda t)^\beta} dt$$

$$= \lambda^\beta \left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{1+\frac{2}{\beta}} \Gamma\left(1 + \frac{2}{\beta}\right). \quad (14)$$

$$\int_0^\infty \Gamma\left(\frac{2}{\beta}, (\mu t)^\beta\right) f_T(t) dt = \Gamma\left(\frac{2}{\beta}\right)\left\{1 - \mu^2\left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{\frac{2}{\beta}}\right\}. \quad (15)$$

$$\int_0^\infty t f_T(t) \Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right) dt$$

$$= G(\lambda, \mu, \beta) - \mu\Gamma\left(\frac{2}{\beta}\right)\left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{\frac{2}{\beta}}, \quad (16)$$

where $G(\lambda, \mu, \beta) = \int_0^\infty e^{-(\lambda t)^\beta} \Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right) dt$.

Using these results, we obtain the expected values for the linear and quadratic severities as

$$E(S) = a + b\Gamma\left(1 + \frac{1}{\beta}\right)\left\{\frac{1}{\mu} - \left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{1}{\beta}}\right\}, \quad (17)$$

$$E(S) = a + b\left\{\frac{1}{\mu^2}\Gamma\left(1 + \frac{2}{\beta}\right) - \frac{2}{\mu\beta} G(\lambda, \mu, \beta)\right\}, \quad (18)$$

respectively, See the appendices for detailed derivations of (17) and (18).

3. RISK EVALUATION

3.1 The Risk Evaluation Metric

Since the FMEA is used as a prevention-oriented technique, the risk linked with each failure cause is necessary to be evaluated. We incorporate the occurrence rate of each failure cause with the expected severity of the corresponding failure into the newly defined risk metric, i.e., the REM (risk evaluation metric). The REM of a failure cause is simply defined by the mathematical product of its occurrence rate τ and its corresponding expected severity, that is,

$$REM = \tau E(S). \quad (19)$$

A failure may be incurred by several causes with different failure rates and each cause again usually has different occurrence rate. Also, it requires different time to detect (identify and correct) each failure cause. Thus, to examine the whole structure of the overall failure mechanism for an item, a worksheet like the FMEA sheet will be very useful. Table 1 shows an REM worksheet which is designed by modifying the traditional FMEA worksheet slightly to fit our purpose. Notice that Table 1 provides an illustrative REM worksheet assuming that the failure mode F_i has three root causes, which is not always the case. Also Table 1 does not provide the specific calculation procedure for REM. Actually, the REM worksheet is a summary sheet for risk evaluation of the overall failure causes.

3.2 Distribution Parameters and Severity Type

For practical use of the REM worksheet in the industrial field, the distribution parameters λ, β, μ of failure time and failure cause detection time must be estimated first. If sufficient data is available, the statistical method such as maximum likelihood estimation may be used. The previous knowledge based on experience or expertise may also be incorporated into the statistical methodology using Bayesian approach. In fact, the traditional FMEA also assumes that some information is available on the occurrences and detections of failures and their causes. If there is no information, the number of 1~10 cannot be assigned to occurrence and detection of each failure cause in the FMEA sheet.

Table 1. REM Worksheet

Failure			Effect		Cause				REM		
Failure mode	Failure Time		Severity coefficient		Root Causes	Detection Time		Occurrence rate (τ)	Constant severity	Linear severity	Quadratic severity
	λ	β	a	b		μ	β				
…	…	…	…	…	…	…	…	…	…	…	…
F_i	λ_{i1}	β_{i1}	α_{i1}	b_i	C_{i1}	μ_{i1}	β_{i1}	τ_{i1}	REM_{i1}	REM_{i1}	REM_{i1}
	λ_{i2}	β_{i2}	α_{i2}	b_i	C_{i2}	μ_{i2}	β_{i2}	τ_{i2}	REM_{i2}	REM_{i2}	REM_{i2}
	λ_{i3}	β_{i3}	α_{i3}	b_i	C_{i3}	μ_{i3}	β_{i3}	τ_{i3}	REM_{i3}	REM_{i3}	REM_{i3}
…	…	…	…	…	…	…	…	…	…	…	…

In some situations, accelerated life testing may be employed. In statistics or reliability engineering, there are many approaches to parameter estimation available, which we are not going deeper here.

The type of the severity function and its cost parameters may be rather easier to determine. If a failure cause is detected before its corresponding failure occurs, the cost of corrective action for that cause is the only contribution to severity. Thus, the constant a can be determined considering the correction cost for each failure cause. Next, the coefficient b and the appropriate type of severity function can be estimated if the total amount of losses due to each failure can be calculated at two time points. Suppose that a failure is detected immediately upon its occurrence and the fire fight action against this failure and corrective action for eliminating its root cause are taken without any delay. The total amount of losses in this situation will determine the coefficient b. Next, the total amount of losses is estimated for the case where there is a time delay between the failure occurrence and its detection and corrective actions taken. The type of severity function can be determined by comparing the one with the other.

Finally, the occurrence rate τ of each failure cause is determined considering the total number of occurrences during the system life time. For example, if the system with the targeted item is expected to operate for 10 years, then τ is the expected number of occurrences of each failure cause for 10 years.

4. AN ILLUSTRATIVE EXAMPLE

4.1 An Example

Consider a mechanical shaft seal for pumps with rotating shafts. A pump with a through-shaft is very difficult to be completely sealed. And it is a challenge to the entire pump industry to minimize leakage. According to the technical documents of Grudfos(2009), 39% of the pump system failure is attributable to the shaft seal failure. With countless variants of shaft seals, its most basic form combines a rotating part with a stationary part. When properly designed and installed, the rotating parts rides on a lubricating film of only 0.00025mm in thickness. If this film becomes too thick, the pumped medium will leak. On the other hand, if it becomes too thin, the friction loss increases and the contact surface overheat, triggering seal failure.

Typical failure modes for mechanical shaft seals include lubrication failure, contamination failure, degrading and wear, installation failure, and system failure. Only the lubrication failure is examined here. It has four possible causes; dry running, exposure to excessive pressure /

temperature, too low viscosity, and excessive heat dissipation.

Given $\lambda, \beta, a, b, \mu,$ and τ, Table 2 provides the numerical values of REM for each failure cause. The numerical figures of $\lambda, \beta, a, b, \mu,$ and τ are given only for illustration purpose and not based on real data. And the quadratic severity function is assumed to be appropriate. In this example, the risk attributable to the cause "Heat dissipation" is the most significant.

4.2 Effect of Improvement

If improvement actions are taken to reduce the risk, the distribution parameters $\lambda, \beta, \mu,$ and τ will be affected. To get some insights on the most effective improvement action, the REM values are compared for two different values of each $\lambda, \beta, \mu,$ and τ in Table 3. Note that all the other parameters remain fixed when the effect of improvement of one parameter is analyzed. The level of improvement is assumed to be twice as compared with its original value. And the set of the parameters with the largest REM is taken as the baseline for comparison.

Table 3 shows that the shape parameter β has the most significant effect on REM. But it may be closely related with the failure mechanism which may be very difficult to improve. The scale parameter μ of detection time has the next significant effect on REM which may be relatively easy to improve. The scale parameter λ of failure time has the third significant effect on REM. By improving the reliability of the item, λ can be reduced. Compared with other parameters, the last parameter τ has a smaller effect on REM.

Table 2. REM Worksheet for the Illustrative Example

Failure			Effect		Cause				REM
Failure mode	Failure Time		Severity coefficient		Root Causes	Detection Time		Occurrence rate (τ)	Quadratic severity
	λ	β	a	b		μ	β		
Lubrication Failure	0.1	3	20	100	Dry running	0.5	3	5	102
	0.05	2	10	100	Excessive pressure	0.25	2	3	93
	0.04	1.5	15	50	Low viscosity	0.2	1.5	2	163
	0.02	1.2	15	50	Heat dissipation	0.1	1.2	2	1500
…	…	…	…	…	…	…	…	…	…

To take improvement action, the effectiveness as well as expenses should be considered. Keeping the improvement cost in mind, the most effective action will be preferred. Table 3 gives some insights on improvement direction.

Table 3. Effect of Improvement Actions

Parameter	Value		Improved REM	Difference from 1500
	original	changed		
λ	0.02	0.01	350	1150
β	1.2	2.4	39	1461
μ	0.1	0.2	100	1400
τ	2	1	750	750

5. CONCLUSION

Under many practical situations in the industrial field, the effect of a system failure becomes more severe if its undetected time or delayed time of corrective action is longer. The proposed model tries to take this practical situation into account. But it still has several limitations to describe the real situations appropriately.

First, the assumption for the detection time of a failure cause is not so much realistic. It is assumed to have a Weibull distribution with the same shape parameter as the failure time. This is to get a simple solution but not to be likely in the real situation. Moreover, the detection time distribution may change into a totally different context after a failure actually occurs. Besides, a failure cause is usually detected by periodic inspection or monitoring of the system. So the detection time may not be a continuous random variable.

Second, the expected severity of a failure cause reflects the total losses and corrective cost under the condition that it has already occurred. Thus, to get a right metric of risk evaluation, the probability of the failure cause occurrence may be better to be multiplied to the expected severity. The occurrence rate instead is used in the proposed model, because the probability cannot be determined if the time duration is not specified. The occurrence rate of a failure cause can also be different as to the length of baseline time duration. For example, if the occurrence rate 1 is applied when one year's duration is assumed as the baseline duration, it will be 10 when 10 year's duration is the base. It is not so easy to determine which duration is appropriate for the baseline even in this simple case.

There may be several other drawbacks that should be addressed. At its infant stage of study on time-linked risk evaluation, future works are expected for further refinement or expansion, reflecting diverse industrial situations.

APPENDIX A. Derivation of (17)

Using the equations (11) and (12), we obtain Formula (17) as follows:

$$E(S) = a + b\int_0^\infty \int_t^\infty (u-t) f_D(u) f_T(t)\,du\,dt$$

$$= a + b\int_0^\infty f_T(t)\left\{\int_t^\infty u f_D(u)du\right\}dt - \int_0^\infty t f_T(t)\left\{\int_t^\infty f_D(u)du\right\}dt$$

$$= a + b\left[\int_0^\infty f_T(t)\left\{te^{-(\mu t)^\beta} + \frac{1}{\mu\beta}\Gamma\left(\frac{1}{\beta},(\mu t)^\beta\right)\right\}dt - \int_0^\infty t f_T(t) e^{-(\mu t)^\beta}dt\right]$$

$$= a + \frac{b}{\mu\beta}\int_0^\infty \Gamma\left(\frac{1}{\beta},(\mu t)^\beta\right) f_T(t)dt$$

$$= a + \frac{b}{\mu\beta}\Gamma\left(\frac{1}{\beta}\right)\left\{1 - \mu\left(\frac{1}{\lambda^\beta+\mu^\beta}\right)^{\frac{1}{\beta}}\right\}$$

$$= a + b\Gamma\left(1+\frac{1}{\beta}\right)\left\{\frac{1}{\mu} - \left(\frac{1}{\lambda^\beta+\mu^\beta}\right)^{\frac{1}{\beta}}\right\}.$$

APPENDIX B. Derivation of (18)

Using the equations (11), (13), (14), (15) and (16), we obtain Formula (10) as follows:

$$E(S) = a + b\int_0^\infty \int_t^\infty (u-t)^2 f_D(u) f_T(t)\,du\,dt$$

$$= a + b\left\{\int_0^\infty f_T(t)\left[\int_t^\infty u^2 f_D(u)du\right]dt - 2\int_0^\infty t f_T(t)\left[\int_t^\infty u f_D(u)du\right]dt + \int_0^\infty t^2 f_T(t)\left[\int_t^\infty f_D(u)du\right]dt\right\}$$

$$= a + b\{A - 2B + C\}$$

$$= a + b\left[\Gamma\left(1+\frac{2}{\beta}\right)\left\{\frac{1}{\mu^2} - \left(\frac{\mu^\beta}{\mu^\beta+\lambda^\beta}\right)\left(\frac{1}{\mu^\beta+\lambda^\beta}\right)^{\frac{2}{\beta}}\right\}\right.$$

$$- 2\left\{\left(\frac{\lambda^\beta}{\lambda^\beta+\mu^\beta}\right)\left(\frac{1}{\lambda^\beta+\mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(1+\frac{2}{\beta}\right) + \frac{1}{\mu\beta}\int_0^\infty e^{-(\lambda t)^\beta}\Gamma\left(\frac{1}{\beta},(\mu t)^\beta\right)dt - \frac{1}{\beta}\left(\frac{1}{\lambda^\beta+\mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(\frac{2}{\beta}\right)\right\}$$

$$\left. + \left(\frac{\lambda^\beta}{\lambda^\beta+\mu^\beta}\right)\left(\frac{1}{\lambda^\beta+\mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(1+\frac{2}{\beta}\right)\right]$$

$$= a + b\left\{\frac{1}{\mu^2}\Gamma\left(1+\frac{2}{\beta}\right) - \frac{2}{\mu\beta}\int_0^\infty e^{-(\lambda t)^\beta}\Gamma\left(\frac{1}{\beta},(\mu t)^\beta\right)dt\right\}$$

$$= a + b\left\{\frac{1}{\mu^2}\Gamma\left(1+\frac{2}{\beta}\right) - \frac{2}{\mu\beta}G(\lambda,\mu,\beta)\right\}$$

where

$$A = \int_0^\infty f_T(t)\left[\int_t^\infty u^2 f_D(u)du\right]dt$$

$$= \int_0^\infty f_T(t)\left[t^2 e^{-(\mu t)^\beta} + \frac{2}{\mu^2\beta}\Gamma\left(\frac{2}{\beta},(\mu t)^\beta\right)\right]dt$$

$$= \int_0^\infty t^2 e^{-(\mu t)^\beta} f_T(t)dt + \frac{2}{\mu^2\beta}\int_0^\infty \Gamma\left(\frac{2}{\beta}, (\mu t)^\beta\right) f_T(t)dt$$

$$= \lambda^\beta \left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{1+\frac{2}{\beta}} \Gamma\left(1 + \frac{2}{\beta}\right)$$

$$+ \frac{2}{\mu^2\beta}\Gamma\left(\frac{2}{\beta}\right)\left\{1 - \mu^2\left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{\frac{2}{\beta}}\right\}$$

$$= \Gamma\left(1 + \frac{2}{\beta}\right)\left\{\lambda^\beta\left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{1+\frac{2}{\beta}} + \frac{1}{\mu^2} - \left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{\frac{2}{\beta}}\right\}$$

$$= \Gamma\left(1 + \frac{2}{\beta}\right)\left\{\frac{1}{\mu^2} - \left(\frac{\mu^\beta}{\mu^\beta + \lambda^\beta}\right)\left(\frac{1}{\mu^\beta + \lambda^\beta}\right)^{\frac{2}{\beta}}\right\},$$

$$B = \int_0^\infty t f_T(t)\left[\int_t^\infty u f_D(u)du\right]dt$$

$$= \int_0^\infty t f_T(t)\left[t e^{-(\mu t)^\beta} + \frac{1}{\mu\beta}\ \Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right)\right]dt$$

$$= \int_0^\infty t^2 e^{-(\mu t)^\beta} f_T(t)dt + \frac{1}{\mu\beta}\int_0^\infty t f_T(t)\Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right)dt$$

$$= \left(\frac{\lambda^\beta}{\lambda^\beta + \mu^\beta}\right)\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(1 + \frac{2}{\beta}\right)$$

$$+\ \frac{1}{\mu\beta}\int_0^\infty e^{-(\lambda t)^\beta}\Gamma\left(\frac{1}{\beta}, (\mu t)^\beta\right)dt$$

$$- \frac{1}{\beta}\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(\frac{2}{\beta}\right)$$

$$= \left(\frac{\lambda^\beta}{\lambda^\beta + \mu^\beta}\right)\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(1 + \frac{2}{\beta}\right) +\ \frac{1}{\mu\beta}G(\lambda, \mu, \beta)$$

$$- \frac{1}{\beta}\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(\frac{2}{\beta}\right),$$

and

$$C = \int_0^\infty t^2 f_T(t)\left[\int_t^\infty f_D(u)du\right]dt = \int_0^\infty t^2 f_T(t)e^{-(\mu t)^\beta}dt$$

$$= \int_0^\infty t^2\beta\lambda^\beta t^{\beta-1}e^{-(\lambda t)^\beta}e^{-(\mu t)^\beta}dt$$

$$= \lambda^\beta\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{1+\frac{2}{\beta}}\int_0^\infty y^{\frac{2}{\beta}}e^{-y}dy$$

$$= \lambda^\beta\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{1+\frac{2}{\beta}}\Gamma\left(1 + \frac{2}{\beta}\right)$$

$$= \left(\frac{\lambda^\beta}{\lambda^\beta + \mu^\beta}\right)\left(\frac{1}{\lambda^\beta + \mu^\beta}\right)^{\frac{2}{\beta}}\Gamma\left(1 + \frac{2}{\beta}\right).$$

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