Determining the Optimal Correlated Multistate Resource Assignment with Maximal Network Reliability Using a Hybrid GA-TS Algorithm

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Abstract. From a perspective on quality management, network reliability maximization is very important for system supervisors. Despite the numerous studies related to network reliability maximization, no study takes correlated failure into account. This paper focuses on determining the optimal multistate resource assignment to maximize network reliability with correlated failures, where a correlated failure of a multistate resource may be from a large-scale disaster or routine maintenance and thus affects the network reliability. A hybrid algorithm integrating a genetic algorithm (GA) and a tabu search (TS) is developed to solve the addressed problem, in which the network reliability associated with a resource assignment is evaluated in terms of a correlated binomial distribution model and minimal paths. A Taiwan academic network is utilized to demonstrate the computational efficiency of the proposed hybrid algorithm by comparing it with GA and TS.

Keywords: network reliability maximization; multistate resource assignment; correlated failure; hybrid GA-TS

1. INTRODUCTION

Owing to the possibility of failure, partial failure, and maintenance, many real-life systems such as electric power systems, computer systems, and transportation systems may be characterized as stochastic-flow networks (SFNs) composed of both sets of arcs and nodes to analyzing its performance (Lin et al., 1995; Lin, 2001). Network reliability is one of major performance indices of a system. Network reliability is defined as the probability that d units of demand are successfully transmitted from a source node to a sink node. Such a probability can be evaluated in terms of minimal path (MP) (Lin et al., 1995; Lin, 2001; Lin et al., 2013), where an MP is a path whose proper subsets no longer connect the source and sink nodes.

From a perspective on quality management, network reliability optimization is the goal for most of system supervisors. Different network reliability optimization problems such as multi-commodity allocation (Hsieh and Lin, 2006), flow assignment (Liu et al. 2008), transportation assignment (Xu et al., 2009), and resource assignment (Lin et al., 2013) have been studied over the past few years, where Lin et al. (2013) discussed how to assign resources to maximize reliability for SFNs. Nevertheless, these studies do not discuss network reliability optimization with the consideration of correlated failures. In the real world, a large-scale natural disaster or a human-made disaster such as weapons of mass destruction may cause the assigned resources to generate correlated failures. Lin et al. (2012) indicated correlated failures have a substantial negative impact on the network reliability. The correlated failures for a network depends on both the geographical distance of the location of the arc to the epicenter of the large-scale disaster and the intensity of the disaster event (Rahnamay-Naeini et al., 2011). The correlated failures for a network depends on both the geographical distance of the location of the arc to the epicenter of the large-scale disaster and the intensity of the disaster event (Rahnamay-Naeini et al., 2011).

This paper focuses on searching the optimal resource assignment to maximize the network reliability with the consideration of correlated failures. A hybrid algorithm integrating a genetic algorithm (GA) (Lin and Yeh, 2010) and a tabu search (TS) (Bilgin and Azizoğlu, 2009) is developed to solve the addressed problem, in which the network reliability associated with a resource assignment is evaluated in terms of a correlated binomial distribution model and minimal paths. A Taiwan academic network is utilized to demonstrate the computational efficiency of the proposed hybrid algorithm by comparing it with GA and TS.

2. PROBLEM FORMULATION

A network is denoted by (**N**, **A**), where **N** represent represents a set of nodes and $\mathbf{A} = \{a_i \mid i = 1, 2, \ldots, n\}$ represents a set of n arcs. Assume there are q resources available to be assigned. Let $\boldsymbol{Y} = (y_1, y_2, \ldots, y_n)$ be a resource assignment, where y_i indicates the index of a resource assigned to arc a_i for i =1, 2, ..., n. If resource #j is assigned to arc a_i , then $y_i = j$. Each multistate resource is combined with ω_j binary-state components and owns M_j states including $h_{j,1}$, $h_{j,2}$, …, h_{j,M_j} with a probability distribution, where $h_{j,l}$ represents the lth capacity of resource #j and $M_j = \omega_j + 1$. Therefore, the network (**N**, **A**) associated with any resource assignment $\boldsymbol{Y}$ should be an SFN and is described by a capacity vector $\boldsymbol{X} = (x_1, x_2, \ldots, x_n)$, in which x_i denotes the current capacity of a_i. Any capacity vector $\boldsymbol{X}$ is feasible associated with resource assignment $\boldsymbol{Y}$ if and only if it meets the following constraint.

$$x_i \le h_{y_i, M_{y_i}},\ i = 1, 2, \ldots, n. \tag{1}$$

Constraint (1) means the current capacity x_i should not exceed the maximal capacity of resource #j assigned to arc a_i. Let $\boldsymbol{\Omega}_{\boldsymbol{Y}}$ be the set of $\boldsymbol{X}$ which satisfy constraint (1) and can successfully transmit d units of demand from the source node O to the sink node D. Accordingly, network reliability is defined as the probability that d units of demand can be successfully delivered from O to D, i.e. $R_d(\boldsymbol{Y}) = \sum_{\boldsymbol{X}\in\boldsymbol{\Omega}_{\boldsymbol{Y}}} \Pr(\boldsymbol{X})$, where $\Pr(\boldsymbol{X}) = \Pr\{x_1\} \times \Pr\{x_2\} \times \ldots \times \Pr\{x_n\}$. The addressed problem is formulated as follows.

$$\textit{Maximize } R_d(\boldsymbol{Y}) = \sum_{\boldsymbol{X}\in\boldsymbol{\Omega}_{\boldsymbol{Y}}} \Pr(\boldsymbol{X}) \tag{2}$$

Subject to

$$y_i = j,\ j \in \{1, 2, ..., q\} \text{ for } i = 1, 2, ..., n, \text{ and} \tag{3}$$

$$y_i \ne y_u,\ i, u \in \{1, 2, ..., n\} \text{ for } i \ne u. \tag{4}$$

Constraints (3) and (4) mean that each resource can be assigned to at most one arc and each arc must contain exact one resource. Note that $R_d(\boldsymbol{Y}) = 0$ if $\boldsymbol{\Omega}_{\boldsymbol{Y}} = \varnothing$. Then, the optimal resource assignment $\boldsymbol{Y}$ with maximal network reliability is determined by maximizing the objective (2). To solve the addressed problem, the following assumptions are made:

I. No resource is assigned to any node.
II. Flow in (**N**, **A**) must satisfy the flow-conservation law (Ford and Fulkerson, 1962).
III. The capacities of resources are statistically independent.

3. CORRELATED BINOMIAL DISTRIBUTION

The correlated failure causes the identical physical lines of a resource assigned to an arc fail in a statistically dependent manner. A correlation denoted by ρ_i is a value which represents the degree of correlated failures between each pair of physical lines and takes the value from the interval [0, 1]. According to the correlated binomial distribution proposed by Lin et al. (2012), the probability distribution of arc a_i can be represented as

$$\Pr\{x_i = h_{j,\gamma+1}\} = \frac{1}{\beta_i} C_{\gamma}^{\omega_j} \left(\mu_j \beta_i\right)^{\gamma} (1-\mu_j\beta_i)^{\omega_j-\gamma}, \text{ and} \tag{5}$$

$$\Pr\{x_i = h_{j,1} = 0\} = 1 - \frac{1}{\beta_i} C_{\gamma}^{\omega_j} \left(\mu_j \beta_i\right)^{\gamma} (1-\mu_j\beta_i)^{\omega_j-\gamma}, \tag{6}$$

where γ represents the number of physical lines of resource η_j are reliable and $\beta_i = (1+\rho_i(1\text{-}\mu_j)/\mu_j)$ is a simplified parameter.

4. RELIABILITY EVALUATION ALGORITHM

A reliability evaluation algorithm (REA) applied to calculate the network reliability for a resource assignment is developed based on MP (Lin et al., 1995; Lin, 2001; Lin et al., 2013) and shown as follows:

Step 1. Consider the influence of correlations to evaluate the probability distribution of each resource of resource assignment $\boldsymbol{Y}$.

Step 2. Find all F under resource assignment Y satisfying the following constraints.

$$f_v \le \min\{h_{y_i}(M_{y_i}) \mid a_i \in P_v\},\ v = 1, 2, \ldots, m, \quad (7)$$

$$\sum\nolimits_{v: a_i \in mp_v} f_v \le h_{y_i}(M_{y_i}),\ i = 1, 2, \ldots, n, \text{ and} \quad (8)$$

$$\sum\nolimits_{v=1}^{m} f_v = d\,. \quad (9)$$

Step 3. Transform each feasible F into X via the following equation.

Step 4. Compare the X generated from Step 3 with each other to obtain al d-MP.

Step 5. Evaluate network reliability of resource assignment $\boldsymbol{Y}$, $\Pr\{\bigcup_{i=1}^{b}\{X \mid X \ge X_i\}\}$, by using the Recursive Sum of Disjoint Products (RSDP).

5. GA-TS ALGORITHM

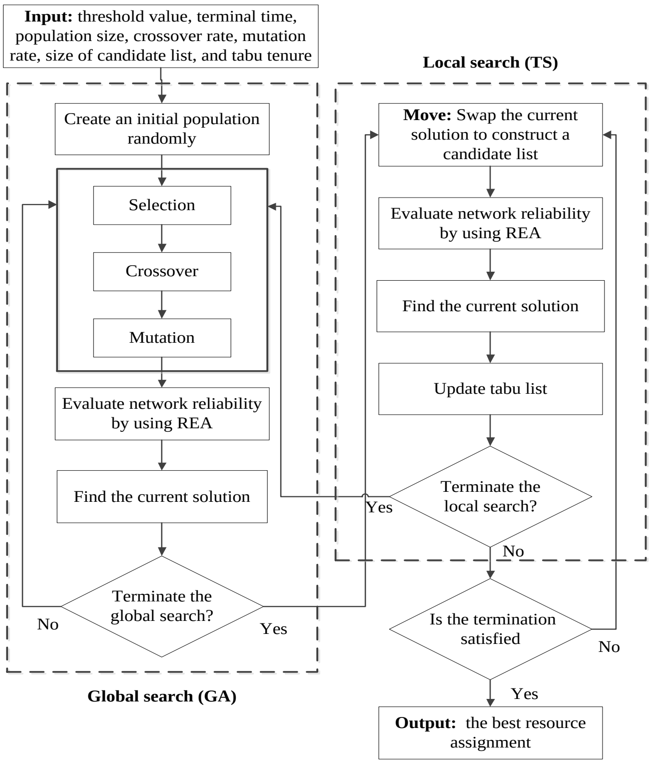

Figure 1: The procedure of HGTA.

In general, GA performs well at exploring the different regions to diversify the global search, and TS is good at performing deeper exploitation to intensify the local research. For owning the global search and local search capabilities, GA and TS are hybridized to achieve a better balance between exploration and exploitation. We propose a hybrid GA and TS algorithm, called HGTA, in which TS is represented as the main base search mechanism and GA is utilized strengthen the diversification ability.

In the proposed HGTA, GA is initially executed to obtain a good solution for TS. Then, TS is utilized to search for the optimal solution. However, if the current best solution is not improved for several iterations, then HGTA is changed to execute GA, in which the candidate list will be transformed to the population. Similarly, if the current best solution from GA is not improved significantly, then HGTA is changed to execute TS. The procedure of HGTA is repeated until the termination condition is satisfied.

GA and TS are used interchangeably in HGTA based on the following equation (Li and Deng, 2012). If the related rate of the network reliability in two recent iterations is not bigger than the threshold value ε, the HGTA is changed to execute TS (resp. GA).

$$\frac{\left|R_d(\boldsymbol{Y})_{current} - R_d(\boldsymbol{Y})_{previous}\right|}{\left|R_d(\boldsymbol{Y})_{previous}\right|} \le \varepsilon. \quad (10)$$

Before execute the HGTA, several parameters including threshold value (ε), terminal time (T), population size (θ), crossover rate (P_c), mutation rate (P_m), size of candidate list (φ), and tabu tenure (π), and and data including demand, set of MPs, resources, and correlations must be given. Moreoer, a solution in HGTA is equivalent to $\boldsymbol{Y}$. Figure 1 depicts the procedure of the proposed HGTA.

6. NUMERICAL EXPERIMENT

In this section, we compare the computational efficiency of the proposed HGTA with GA and TS by Taiwan academic network (TANET) (Lin and Yeh, 2010). TANET is a backbone network connecting all educational and academic organizations in Taiwan. Figure 2 illustrates the topology and indicates the correlations resulted from a disaster. A set of resources (transmission lines) is ready to be assigned. Each resource comprises several identical OC-18 (Optical Fiber 18) or OC-36 (Optical Fiber 36) lines (physical lines). Each OC-18 (resp. OC-36) line has two states: reliable or failed. If the OC-18 (resp. OC-36) line is reliable, then the OC-18 (resp. OC-36) line provides 0 bps (bits per second); otherwise, providing approximate 1 (resp. 2) Gbps (Gigabits per second). Since all resources are provided by several suppliers, they have not only different capacities but also various OC-18 (resp. OC-36) lines with different reliabilities. Once the resources are assigned to the arcs of the network, the probability distributions of the assigned resources will be affected by the correlations of the arcs. The capacity and probability data of 100 available resources are given in Table 1.

The parameters of HGTA are $\theta = 200$, $P_c = 0.8$, $P_m = 0.05$ (Lin and Yeh, 2010), $\pi = 100$, $\varphi = 200$ (Bilgin and Azizoğlu, 2009), and $\varepsilon = 10^{-5}$, and $T = 3600$ sec. All algorithms are programmed in the MATLAB programming language and executed on a personal computer with 64-bit Windows 10, Intel Core i7-4712MQ CPU 2.3GHz, and 8GB RAM.

Table 1: The capacity and probability data of 100 resources.

η_j	Capacity[a]	Probability[b]	η_j	Capacity	Probability
1	2	0.9469	**51**	2	0.9160
2	3	0.9239	**52**	4	0.9240
3	1	0.9707	**53**	1	0.9769
4	1	0.9015	**54**	3	0.9519
5	2	0.9361	**55**	2	0.9661
6	1	0.9868	**56**	2	0.9654
7	3	0.9823	**57**	1	0.9300
8	1	0.9171	**58**	4	0.9415
9	3	0.9356	**59**	4	0.9712
10	2	0.9655	**60**	1	0.9893
11	3	0.9466	**61**	2	0.9963
12	4	0.9802	**62**	1	0.9687
13	3	0.9092	**63**	2	0.9839
14	3	0.9221	**64**	3	0.9758
15	3	0.9527	**65**	3	0.9954
16	1	0.9828	**66**	1	0.9345
17	3	0.9997	**67**	3	0.9384
18	2	0.9792	**68**	3	0.9707
19	4	0.9500	**69**	2	0.9171
20	2	0.9139	**70**	2	0.9962
21	1	0.9776	**71**	4	0.9099
22	1	0.9858	**72**	2	0.9427
23	2	0.9746	**73**	4	0.9154
24	1	0.9447	**74**	4	0.9371
25	2	0.9622	**75**	4	0.9078
26	4	0.9776	**76**	2	0.9543
27	2	0.9449	**77**	2	0.9571
28	3	0.9174	**78**	4	0.9024
29	1	0.9821	**79**	2	0.9679
30	2	0.9799	**80**	1	0.9140
31	2	0.9847	**81**	1	0.9005
32	4	0.9017	**82**	2	0.9171
33	2	0.9826	**83**	2	0.9952
34	3	0.9955	**84**	4	0.9932
35	1	0.9344	**85**	4	0.9955
36	2	0.9076	**86**	1	0.9397
37	1	0.9814	**87**	1	0.9905
38	4	0.9375	**88**	2	0.9901
39	2	0.9086	**89**	1	0.9250
40	2	0.9879	**90**	4	0.9247
41	2	0.9754	**91**	3	0.9036
42	2	0.9741	**92**	2	0.9710
43	3	0.9226	**93**	2	0.9338
44	2	0.9921	**94**	3	0.9301
45	1	0.9455	**95**	3	0.9381
46	4	0.9118	**96**	1	0.9987
47	3	0.9512	**97**	2	0.9080
48	1	0.9653	**98**	1	0.9043
49	3	0.9708	**99**	1	0.9290
50	3	0.9270	**100**	2	0.9891

a. Number of the optical fibers.

b. Reliability of each optical fiber.

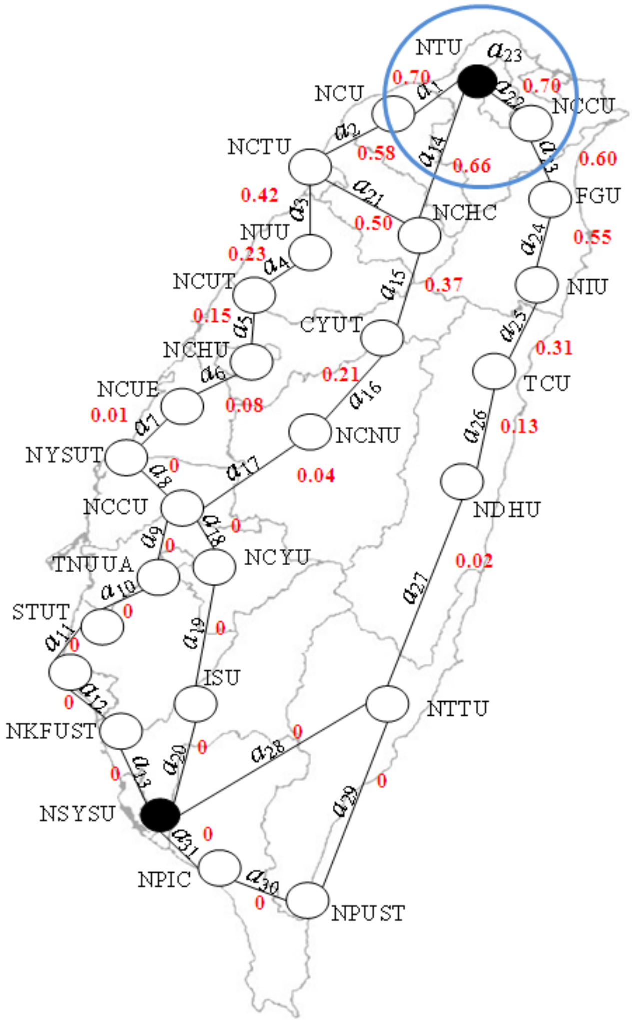

Figure 2: TANET (Lin and Yeh, 2010).

Table 2: The experimental results of the TANET for $d = 6$.

Algorithm	**Largest maximal reliability**	**est resource assignment**	**Average maximal reliability**
GA	0.96839	(68, 43, 18, 70, 83, 40, 88, 46, 15, 75, 52, 74, 90, 85, 12, 84, 26, 47, 73, 32, 63, 34, 49, 17, 7, 38, 65, 19, 2, 44, 9)	0.95373
TS	0.98098	(92, 41, 31, 100, 33, 47, 94, 32, 19, 75, 52, 73, 90, 85, 84, 12, 26, 58, 74, 59, 10, 65, 70, 34, 17, 7, 64, 15, 55, 69, 29)	0.97036
HGTA	**0.98445**	(68, 47, 70, 63, 33, 11, 67, 32, 78, 73, 75, 90, 74, 84, 85, 12, 26, 19, 59, 71, 76, 17, 34, 65, 7, 49, 64, 13, 2, 16, 80)	**0.98240**

Considering d = 6 Gb, each algorithm is executed for 10 times to obtain the largest and average maximal reliabilities. The experimental results are concluded in Table 2. It is obvious that the proposed HGTA performs better on the largest and average maximal network reliabilities than GA and TS for TANET. Observing Figure 3, initially GA performs better than TS, but TS performs better than GA from approximate 360th second. Such a result demonstrates that the proposed HGTA integrates the advantages of TS and GA.

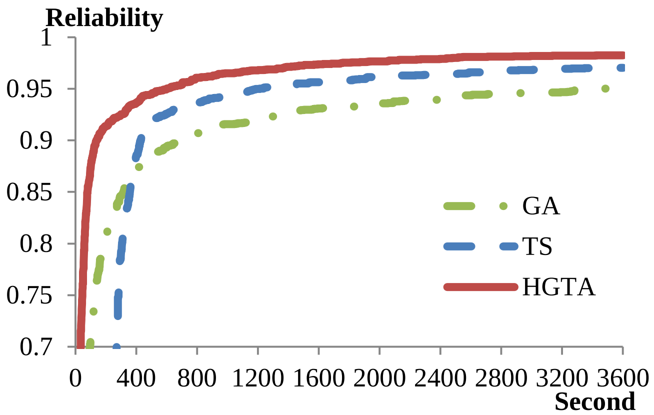

Figure 3. The comparison of the algorithms for the TANET.

7. CONCLUSIONS

Several researches studied the issue of network reliability optimization, yet most of them didn't consider the frequently observed phenomenon of correlated failures of arcs in a network. This study focuses on maximizing network reliability for SFN with correlated failures so that the kind of network reliability problem can be more consistent with the situation of reality. We propose HGTA which adopts TS as the main base search mechanism for executing the local search and applies GA to complement TS by reinforcing the global search capability for solving the addressed problem. The TANET experiment demonstrates that HGTA has not only better quality of obtaining the maximal network reliability but the better computational efficiency for finding the optimal resource assignment than TS and GA. Obviously, HGTA is an efficient and stable algorithm for solving the addressed problem.

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